

Faculty of Commerce - Cairo University



LIFE INSURANCE

Compiled by
Dr. MOHAMED GOUDA HOZAIEN
Professor of Risk Management and Insurance

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Faculty of commerce
Cairo University

ACKNOWLEDGMENT

To my lovely sons:

Abd El-Rahman

Youssef

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CHAPTER 1

FUNDAMENTALS OF
RISK AND INSURNCE

CHAPTER OBJECTIVES

After studying this chapter, you should be able to:

1- Explain the concept of risk.

2- Contrast pure risk with speculative risk.

3- Identify and distinguish between loss, peril, and hazard.

4- List the four methods of handling risk.

5- Define insurance and explain how it handles risk.

6- Distinguish between chance of loss and degree of risk.

7- Show how insurance differs from gambling.

8- Summarize the relationships between insurance, the law of large numbers, and risk.

RISK

Risk means uncertainty about future loss or, in other words, the inability to predict the occurrence or size of a loss. Will we have an accident or illness requiring hospital care? If so, when will it happen? Should we set aside some money just in case it does happen? How much money will we need? Of course we don't know the answers to these questions, because we cannot predict the future with any certainty that is the nature of risk. Ours is a world of count less risks. a world in which losses of many kinds happen suddenly and unexpectedly.

Risk can be classified as either pure or speculative. **Pure risk** can result only in loss or absence of loss. A building will have a fire or it will not; a car will be stolen or it will not be. **Speculative risk** has a third possible outcome: gain. Gambling creates a speculative risk. A person who bets on ball games may either lose, break even, or win. Business ventures involve many speculative risks. When Walt's Waterbeds opens a new store, Walt knows he is taking a risk. He makes his decision in the belief that it will result in a profit rather than a loss.

The distinction between pure and speculative risk is important because usually only pure risks, situations in which there is no chance of profit, can be insured. The possibility of loss generally cannot be insured when there is a corresponding possibility of gain, as there is in speculative risk. The owner of a valuable painting faces both pure and speculative risks. The painting could be stolen. This possibility is a pure risk, because if the painting is stolen the owner will lose, but if it is not stolen the owner's position will simply remain unchanged. The owner will not profit from the mere absence of theft. However, the market value of the painting could decline if works of its type became less popular. This risk is speculative because there is another side to the coin, the possibility that the painting will rise in value. If it does rise, the owner will have a gain instead of a loss. The risk of theft is insurable, but the risk of declining market value, because it is a speculative risk, is not insurable.

Types of pure risk

LOSS, PERIL, AND HAZARD

Several commonly used words have rather precise meanings when they are used in connection with risk and insurance. It will be helpful to identify three such words: loss, peril, and hazard.

Loss

In insurance, a loss is an unexpected reduction or disappearance of economic value. This is a narrower definition than that frequently used. Because insured losses are unexpected, they do not include the wearing out or normal depreciation of property, nor do they include damage intentionally done to property by its owners. Also, as insured losses are confined to economic value, the loss of purely sentimental value such as that resulting from the destruction of personal photographs or souvenirs is not covered by insurance.

There are four principal types of losses: loss of property, loss of income, loss associated with legal liability claims, and loss due to unexpected expenses.

1- All of us are familiar with the first type of loss, loss of property. It includes the cost of repairing or replacing things like automobiles, jewelry, or clothing that have been stolen or have been damaged by fire, collision, or vandalism.

2- For many people income loss could have more serious consequences than property loss, as the ability to work and earn an income is the most valuable asset that most of us have. The loss of this asset can be far more costly than the loss of our physical possessions. Loss of income can result from sickness, accidental

injury, or death, among other causes. Business firms are exposed to this loss too. They lose income if their buildings or equipment are damaged seriously enough to force a temporary closing of the business.

3- The third type of loss stems from legal liability claims. These claims, based on the laws of negligence, are described in Chapter 4. A homeowner may be sued by someone who trips and falls over a toy left lying on the sidewalk. If such an accident happens, the owner will have to incur the costs of defending against the lawsuit and may have to pay a sum of money to the injured person. Legal liability claims can also result from automobile, boating, or hunting accidents, and from almost any other kind of activity. Business firms are subject to liability claims from numerous sources, including people injured on their premises and customers injured by using their products. A multimillion dollar case resulting from the explosion of a Ford Pinto's fuel tank is a well-known example.

4- Unexpected expenses, primarily for medical services, are the final type of loss. Each year many families are faced with huge bills from doctors, hospitals, or nursing homes.

Peril

A peril is the cause of a loss. Commonly insured perils include collision, fire, theft, collapse, explosion, and illness. A single peril can cause more than one type of loss. The collapse of two skywalks in the Kansas City Hyatt Regency Hotel in 1981 was a dramatic example. One hundred thirteen people were killed and 186 were injured. Insurance companies paid many millions of dollars (1) for building repairs, (2) to replace income lost by the hotel during the months it was closed after the disaster, (3) to settle lawsuits, and (4) for medical bills. In less spectacular fashion, automobile collisions frequently create each of these four types of loss also.

Hazard

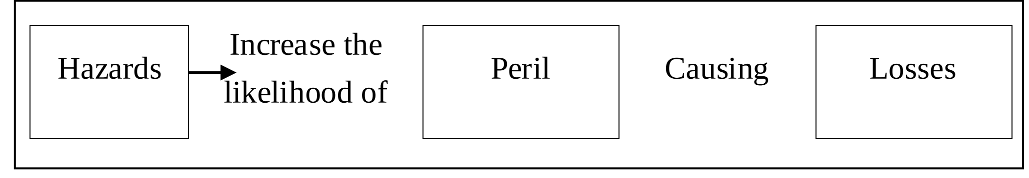
A hazard is a condition that increases the likelihood of loss due to a particular peril. Poor automobile brakes are a hazard making loss due to the collision peril more likely. There are three kinds of hazard: physical, moral, and morale.

1- Physical hazards are tangible characteristics of whatever is exposed to loss. Examples include the poor brakes just mentioned, slippery floors, and dry forests.

2- A moral hazard exists when the insured person is one who may dishonestly cause or exaggerate a loss. Insurance companies try to avoid insuring in situations where there is evidence of moral hazard. A person with a record of arrests for arson would have a hard time getting fire insurance! As a matter of fact, such a person might be turned down for other kinds of insurance as well. Insurance companies figure that a person who would make one type of fraudulent insurance claim would be likely to make others too.

3- A morale hazard exists when the presence of insurance causes the insured person to be indifferent to loss. This indifference can result in extreme carelessness, such as by people who, instead of taking reasonable care of their property, say "It's insured, so why should I worry?"

The relationship between hazards, perils, and losses is illustrated



DEALING WITH RISK

One of the interesting things about risk is the way that people react to it. If the risk is a speculative one and the amount at stake is not too high, taking a chance may be enjoyable. It may be fun to bet on a ball game or a horse race, especially if we think we have a good chance of winning. Also, we don't mind making what seem to be good business decisions, even though there is a possibility of suffering a loss. However, in the case of pure risk, because there is nothing to be gained and perhaps much to lose, the usual reaction is a feeling of uneasiness and insecurity. Generally speaking, individuals and businesses dislike pure risk and if very many dollars are involved they look for a way to minimize the risk's effects or to avoid them completely. Basically, there are four ways to deal with risk. They are risk avoidance, loss control, risk retention, and risk transfer.

Risk Avoidance

Sometimes risks can be avoided by not doing whatever produces them. People who are worried about being killed in a plane crash can practice risk Avoidance by not flying. The obvious drawback of this approach is that it requires one to forego the convenience of air transportation. Risk avoidance sometimes is practical, though. Many people choose to avoid the risks involved in hang gliding or skydiving, and some decide against motorcycling for similar reasons. We probably can think of other risks that people or businesses intentionally avoid. In most cases, however, there is no practical way of avoiding risk; it must be handled in some other way.

Loss Control

The goal of loss control is to reduce the total amount of loss. Loss control is not an alternative to the other methods of handling risk but is used in addition to one or more of them.

The total amount of loss is a function of loss frequency and loss severity. That is, the total depends on how many losses occur and how big they are. Accordingly, there are two aspects of loss control: loss prevention and loss reduction. Loss prevention activities are aimed at reducing loss frequency; loss reduction measures are designed to limit the severity of the losses that do occur. Lightning rods are loss prevention devices. They direct lightning bolts to the ground, thus preventing fires and reducing loss frequency. Fire alarms and sprinkler systems are loss reduction devices. They do nothing to prevent fires; instead they limit the severity of fires after they begin. Some loss control measures, like automobile brakes, are aimed at both preventing losses and reducing the severity of those that occur.

Risk Retention

Risk retention is practiced when risks are retained (kept) by the people or organizations exposed to them. Risks may be retained either deliberately or unintentionally.

Sometimes risks are unintentionally retained out of ignorance. A property owner who buys an inexpensive home insurance policy may not realize that it doesn't insure against damage caused by water leaking from a broken pipe, for instance. Without intending to do so, the property owner is retaining the risk of such damage. Another reason for unintentional risk retention is underestimating the likelihood of loss the "it can't happen to me" attitude. The risk of income interruption due to death or disability is frequently retained for this reason. Scarcely any of us really believe we are immortal or not subject to serious illnesses or accidents. But because these are not pleasant things to think about we tend to tell ourselves that they won't happen to us. As a result, we may retain the risk without actually reaching a decision to do so, even though it would be more sensible to use one of the other risk handling methods.

In other cases risks may be retained unintentionally, or at least unwillingly, because there is no other way of handling them. A family may realize that its main income supply would be cut off by the death of one of the parents, but if that person is in very poor health and is unable to obtain life insurance there may be no alternative to retain the risk.

Intentional risk retention may involve losses that are too small to justify handling in any other way. If we break a comb or lose a cheap ballpoint pen, we just buy another. Sometimes risk is retained intentionally because there seems to be very little chance that a loss will occur. People who live on high ground usually retain the risk of flood loss because they figure the chance of a flood is too slim to justify buying flood insurance or taking any other action.

Risk Transfer

Risk transfer, the fourth method of dealing with risk, occurs when risk is shifted to someone else. The usual way of doing this is to transfer the risk to an insurance company, but there are other, noninsurance transfers. A school collects a refundable $20 "breakage fee" from all students taking chemistry classes and deducts the cost of any laboratory equipment broken by each student before returning the fee at the end of the semester. The effect is to transfer part of the risk of equipment breakage from the school to the students. Notice that two other methods of dealing with risk are also involved. First, the school retains the risk of equipment breakage in excess of $20 per student. Second, giving the students a financial stake in the matter probably motivates many of them to use the equipment more carefully. Thus, the breakage fee also serves as a loss control measure.

The most important and widely used means of transferring risk is through insurance. We now turn our attention to that approach.

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INSURANCE

Insurance may be defined as a system of handling risk by combining many loss exposures, with the costs of the losses being shared by all of the participants. The term loss exposures refers to the objects that are subject to loss. In auto insurance the loss exposures are autos; in life insurance they are lives.

Imagine a group of one hundred people each of whom owns a stereo system. To simplify things, assume that each system is worth $1,000. The owners realize that their stereos could be stolen or destroyed by fire. To eliminate the risk of losing the money they have invested in their equipments, each of them buys an insurance policy. An insurance policy is a legal contract under the terms of which an insurance company agrees to pay for stated losses. In this case, the policies cover the stereos for the perils of theft or fire for a period of one year.

The price of an insurance policy is called a premium, and the premium for each of these policies is $25. That is the amount the insurance company collects from each of the participants (commonly called "policyholders" or "insureds") to pay for stolen or destroyed stereos, plus its costs of doing business. The company has been insuring stereo equipment for many years and has found that an average of two out of every one hundred of the units it insures are either stolen or destroyed by fire each year. By charging the one hundred owners $25 for the policies, the company will accumulate a fund of 100 x $25, or $2,500. If the loss experience of this group is like that of persons the company has insured in the past, there will be two losses and the company will pay 2 x $1,000, or $2,000, for them.

What about the other 500? It is an essential part of the arrangement, because it will pay the company's operating expenses. Out

of the $500 difference between its income from premiums and its payment for losses the company must pay sales commissions to the people who sell the policies, salaries, office expenses, energy costs, taxes, and so forth. The $500 also includes a safety margin in case there are more than two losses or in case the company's other expenses turn out to be greater than expected. Finally, if all goes well there may be something left over as profit for the company and its owners.

What does such a system achieve? What does insurance do for the stereo owners? It relieves them of risk. That is, it removes the possibility of financial loss of the types and amount covered by their policies. It does this by a process of sharing. The losses that a few policy holders suffer are shared by all of the participants by means of the premiums that all of them pay. In other words, each stereo owner substitutes a relatively small but known expense ($25) for the possibility of incurring a much larger loss ($1,000). And this is feasible because the insurer is able to estimate the total amount of loss. Because the company can predict the amount rather accurately, it is able to calculate the premium that each owner must pay to cover his or her share of the total cost. To understand how this can be done, we now examine the process more closely.

HOW INSURANCE HANDLES RISK

Insurance handles risk by transferring it to an insuring organization. An explanation of this process involves three other concepts: chance of loss, degree of risk, and the law of large numbers.

Chance of Loss

Chance of loss, sometimes called probability, can be defined as the probable number of losses out of a given number of loss exposures. This concept was used in the example of insurance for stereo equipment when the insurance company estimated that two out of every one hundred

stereos would be stolen or destroyed during the year. Another illustration of chance of loss is betting on the flip of a coin. Because there is an equal chance of winning or losing, the chance of loss in this case is one out of two. If we were predicting the suit of a card to be drawn at random from a full deck, the chance of loss would be three out of four.

Insurance companies usually have to estimate chance of loss on the basis of what has happened in the past. Because it is important that their estimates be as accurate as possible, the companies accumulate huge amounts of data on the exposures they insure. In later chapters we will see how they use records of automobile losses, death rates, hospital costs, and so forth in setting insurance premiums.

Degree of Risk

We should be careful not to confuse chance of loss with degree of risk. **Degree of risk** is the extent of uncertainty about future losses; it is the extent to which losses are unpredictable. If losses can be predicted quite accurately, there is a small degree of risk regardless of what the chance of loss may be. For instance, if losses are certain not to happen (the chance of loss is zero), the degree of risk is zero because there is no uncertainty. But notice that if it is certain that a particular number of losses will happen, the degree of risk again is zero because there is no uncertainty in this situation either. The chance of loss differs in these two cases, but the degree of risk is the same. It is the same because in both situations the outcome is known in advance, meaning there is no uncertainty.

The following example also illustrates the two concepts. Two taxicab companies Red Cab Company and Blue Cab Company have fifty cabs apiece. The managers of both are trying to predict how many of their cabs will be destroyed in collisions next year. Checking their records, the

two companies find the numbers of cabs destroyed during the last five years (Table 1-1). For the five year period, both companies have lost an average of three cabs per year. The chance of loss for both, on the basis of past experience, is therefore 6% (3 out of 50). Both predict that they will lose three cabs next year. But what about the degree of risk? Are the managers of the two companies equally certain about their predictions? Probably not. Red Cab Company has had between two and four cabs destroyed each year. They feel quite certain that they will lose about three again next year. But Blue Cab's losses have ranged all the way from zero to seven. Its managers throw up their hands. "The number fluctuates tremendously from year to year," they say. "We'll predict three, but goodness only knows how many there actually will be!" The point is that the two firms have the same chance of loss but different degrees of risk. Red Cab's record has led its managers to be relatively certain of their prediction for next year. Blue Cab's officials, being less certain, have a greater degree of risk even though their company has the same chance of loss.

As the basis for predicting next year's losses, both of the cab companies examined their records for the last five years. Why five years rather than one? Because the longer period of experience is a more reliable indication of what is likely to happen in the future. We know this intuitively, but an important principle is at work here, the law of large numbers.

The Law of Large Numbers

The law of large numbers is the key to the functioning of insurance. Application of the law makes it possible for insurance companies to handle risks like that of the Blue Cab Company and countless other individuals and businesses. **The law of large numbers** is a mathematical principle stating that as the number of exposures is

increased the actual results tend to come closer to the expected results. We can illustrate the principle by flipping a coin and predicting whether it will come up heads or tails. If we bet a dollar that it will be heads, our chance of loss is 1 in 2; we have a 50% chance of winning a dollar and a 50% chance of losing a dollar. How certain or uncertain are we of the outcome? Assuming that it is a fair flip and a properly balanced coin, we are as uncertain as we can be. Risk is at a maximum. We will either win a dollar or lose a dollar, but there is no basis for predicting which will be the case.

Now assume that the coin is going to be flipped 1,000 times. Each time we will bet a dollar that it will land heads up. The chance of loss is unchanged; the expected result is that there will be 500 heads and 500 tails. But how certain or uncertain are we of the total outcome? Previously we were completely in the dark. Our prediction that the coin would land heads up was going to be either absolutely correct or dead wrong. Now the situation has changed. There is less risk because it is practically impossible that we will either win 100% or lose 100% of our 1,000 bets. We are confident that the result will not be far from the expected result, as determined by the underlying chance of loss. If, for instance, the result turns out to be 520 heads and 480 tails (or vice versa), we shall have missed our prediction of 500 by only 20, an error of only 4%. If we were to bet numerous times on fewer or more flips than 1,000 (say 100 or 10,000), we would find that the law of large numbers would continue to hold true: the greater the number of exposures, the closer the outcome is likely to be to the underlying chance of loss. In other words, the law of large numbers tells us that increasing the number of loss exposures decreases the risk with regard to the total outcome.

There are many other examples of the law of large numbers. Consider how it is illustrated by each of the following:

1- Which would you rather predict, the grade average of a single student chosen at random, or the grade average of the entire junior class?

2- Which can be predicted more accurately, the team batting average of the St. Louis Cardinals after the first game of the season, or the composite batting average of the entire National League at the end of the season?

3- A national organization makes predictions of the total number of traffic fatalities in the United States during a holiday weekend. The prediction is not broken down on a state-by-state or countyby-county basis. Why not?

Insurance and the Law of Large Numbers

Insurance reduces risk by combining many individual loss exposures. Of the law of large numbers, the insurance company is then able to predict the total loss with reasonable accuracy.

The process of combining a large number of exposures and predicting aggregate rather than individual losses is essential. If an insurer covered Tom Hyde's $1,000 stereo and no others, it would be as uncertain of the as Tom had been. Either it would make a $1,000 loss payment or make no loss payment (ignoring the possibility of partial loss as we lier). Tom's risk would simply have been transferred to the insurer pany. But, if the company insures many other stereos in addition to the situation is different. The company then is in a position compare person who bets on 1,000 flips of a coin. Because of the law of large numbers the insurance company's prediction of the total amount of loss payments for all the stereos it insures will be fairly close to its actual payments.

Notice that the insurer still is no better able to predict whether Tom Hyde will have a loss than Tom is. But, after Tom's risk is

transferred by means of the insurance policy, the insurance company really isn't concern whether or not he or any other particular one of its many policy hold loss. It has predicted the total amount of loss, and the premiums the company has charged are based on that prediction. Its only concern now is whether the total amount it pays for losses will be more or less than the total amount that it predicted.

The predictions of insurance companies are never precisely correct. Sometimes actual losses paid for a year are considerably above or below expected losses. One reason is that even the largest companies may not cover a sufficient number of exposures of a given type. The theoretical point at which predictions would become precise is when the number of exposures becomes infinitely large. If a coin were flipped an infinite number of times the outcome would be exactly that of the underlying chance of loss, half heads and half tails. But flipping a coin an infinite number of times, of course, is not possible, nor is it possible to insure an infinite number of cars or lives or houses. Another reason why combining many exposures does not completely eliminate risk (from the insurer's viewpoint) is that we live in a world of constant change. For instance, changes in the average cost of garage repairs affect future loss developments in auto insurance, as do changes in gasoline supplies, traffic law enforcement, and highway maintenance. These and many other factors influence the chance of loss in unpredictable ways. In spite of such limitations, however, insurance companies usually can predict their aggregate loss payments with reasonable accuracy. They are able to do so because of the law of large numbers.

Insurance and Gambling

Is purchasing an insurance policy the same as gambling? Is a person who insures a $1,000 stereo for a $25 premium simply betting that there will be a loss? Is the insurance company betting that there won't be?

In some ways insurance does seem like gambling. Both involve an exchange of money based on the occurrence of a future event, and in both situations the amount payable by one of the parties may be greater than the amount payable by the other. One difference, of course, is that insurance involves pure risk whereas gambling is a speculative risk. That is, insured risks present only the possibilities of losing or not; in gambling one also has the chance of coming out ahead of the game. Another difference, and a very important one, is that gambling creates risk, but insurance transfers an existing risk to someone else. For example, until people bet on certain horses, they have no financial stake in the outcome of a race. Placing bets puts them in a position in which they can win or lose, depending upon which horses they bet on. Gambling thus creates a risk that did not previously exist. In contrast, any person who has a loss exposure (for instance, owns an automobile or a house) already is in a position where he or she may incur financial loss. The car may crash, the house may burn, and so forth. Risk always exists before insurance is purchased. Insurance relieves a person of risk rather than creating a new risk. In this respect, insurance is the opposite of gambling.

Insurance and Risk

Earlier, we defined insurance as a system of handling risk by combining many loss exposures so that the costs of unexpected losses are shared by all participants. We now should have a good idea of why that is done, how it is done, and what the results are.

The reason for insuring is to rid oneself of uncertainty about a possible financial loss or, in other words, to rid oneself of a risk. The method used is risk transfer. The insurance buyer transfers the risk to an insurer that promises to reimburse the insured if a loss occurs. The insurer estimates its total losses and expenses and charges each policy holder share of the total in the form of the premiums for their policies. The

insurer is able to price its policies with reasonable accuracy because it is dealing with the total cost of many exposures and because, given the law of large numbers, the degree of risk decreases as the number of exposures increases. The result is that policyholders replace the possibility of a relatively large loss with the payment of a much smaller expense, the premium.

More than one hundred years ago farmers in the eastern and midwestern states established a number of organizations to insure farm properties against fire and lightning. Operating in just one or a few rural counties, many of these organizations at first covered only a few dozen farms. Losses were infrequent; one company had none at all during a three-year period and another's only payment one year was $150 for a horse killed by lightning. The usual procedure was to collect little or no premium until the need arose, and then to charge each member with his or her share of the loss. Clearly, that was a form of insurance. Risk was transferred, a number of loss exposures were grouped together, and costs were shared, even though the financing was on a pass-the-hat basis rather than through premiums paid in advance. Insurance today is essentially the same. It is still a system of handling risk by combining exposures and sharing costs.

IMPORTANT TERMS

risk	loss control	insurance
pure risk	loss frequency	loss exposure
speculative risk	loss severity	insurance policy
loss	loss prevention	premium
peril	loss reduction	chance of loss
hazard	risk retention	degree of risk
risk avoidance	risk transfer	law of large numbers

KEY POINTS TO REMEMBER

1- **Risk**, meaning uncertainty about future loss, may be either pure or speculative. Pure risk can result only in loss or in no loss; speculative risk can result in either loss, no loss, or gain. Ordinarily, only pure risks can be insured.

2- **In insurance**, a loss is an unexpected reduction or disappearance of economic value. Insured losses include loss of property, loss of income, loss associated with legal liability claims, and loss due to unexpected expenses.

3- **The cause of a loss**, such as fire or theft, is called a peril.

4- **Hazards** are conditions that increase the likelihood of loss due to a particular peril. There are three types of hazard: physical, moral, and morale.

5- There are four basic methods of handling pu**re risk: risk avoidance, loss control, risk retention**, and risk transfer. Insurance is one type of risk transfer.

6- **Insurance** is a system of handling risk by combining many loss exposures, with the costs of the losses being shared by all of the participants.

7- Loss exposures are units that could sustain losses.

8- **An insurance policy** is a legal contract by the terms of which an insurance company promises to pay for stated losses.

9- **An insurance premium** is the price of an insurance policy.

10- **The chance of loss** is the probable number of losses out of a given number of loss exposures. It should be distinguished from degree of risk, which is the extent of uncertainty about future losses.

11- **The law of large numbers** is a mathematical principle stating that as the number of exposures is increased the actual results tend to come closer to the expected results.

12- **Insurance reduces risk** by combining many loss exposures.

REVIEW QUESTIONS

1) Why is risk an important concept?
2) Give an example of each of the four methods of handling risk.
3) What is the difference between loss prevention and loss reduction?
4) Give an example of each of the four types of loss.
5) What is the difference between a peril and a hazard?
6) Give an example of each of the three types of hazard.
7) Distinguish between chance of loss and degree of risk.
8) Why is the law of large numbers vital to insurance?

DISCUSSION QUESTIONS

1) Which of the following are perils and which are hazards?

   (a) Flood (b) Careless driving
   (c) Theft (d) Badly worn automobile tires

2) Some things can be either perils or hazards. (a) How could this be true of hail? (b) Of illness?
3) Several illustrations of the law of large numbers are given in this chapter. Can you think of others?
4) Changes in such things as technology, inflation, and law enforcement can affect future loss developments. How might changes in each of these affect the amount of automobile insurance loss payments?

5) Does the fact that many people enjoy gambling mean that the risk of financial loss is not undesirable after all?

6) Gambling creates risk of financial loss and insurance eliminates such risk. Can you think of instances in which the opposite would be the case?

7) In a discussion of the nature of risk and insurance, one person said that insurance eliminates risk. Another person retorted that insurance doesn't eliminate risk, but does transfer it and reduce it. A third person insisted that the first speaker was looking at insurance from the insured's viewpoint and the other was looking at it from the insurer's viewpoint. Who do you think was right?

8) A person who knows nothing at all about insurance asks how it can be possible to insure an $80,000 house for only $150 a year. Based upon what you have learned about the general nature of insurance, what would your response be?

CHAPTER 2

THE INSURANCE INDUSTRY

CHAPTER OBJECTIVES

After studying this chapter, you should be able to:

1- List and explain the reasons for the four features of insurable risks.

2- Outline the major fields of insurance.

3- Identify the principal insurance operations.

4- Show that insurance is affected by current events of various kinds.

FEATURES OF INSURABLE RISKS

Insurance is not always available as a method of handling risk. That is, some risks are insurable but others are not. We already have seen that only pure risks can be covered by insurance. Gambling and other speculative risks are not insurable. Beyond that, what are the features distinguishing insurable from non-insurable risks?

The question is a difficult one because insurance is not an abstract science that conforms strictly to establish rules and principles. It is a living, dynamic business operated by people who are capable of doing imaginative and unusual things. Therefore, exceptions can be found to most generalizations about insurance, and it would be misleading to list features that insured risks "absolutely must have." At the same time, however, most insured risks have certain characteristics not found in risks that are generally not insured. These characteristics, which can be thought of as the ideal features of insurable risks, are:

1. There are many similar loss exposures.
2. Losses are definite, measurable, and important.
3. Losses are accidental.
4. Catastrophic loss is very unlikely.

There Are Many Similar Loss Exposures

A large number of similar exposures is necessary. First, a reasonable estimate of the chance of loss can then be made, based upon information about past experience. Premiums then can be set at the proper level. Second, by insuring many similar exposures it is more likely that an insurer's actual loss experience will be close to the predicted loss experience. Even if the insurance is properly priced, an insurer cannot safely cover only a few units of a given class. A fire insurance company that insures only a few buildings would be bearing too much risk; it would have less risk if it covered a great many buildings.

How many loss exposures are enough? Premiums often are based on the experience of hundreds of thousands of policies over a period of several years. Statistical agencies compile the loss data of many insurers, giving each company a broad experience base for predicting future losses. As to how many exposures a given company believes it must insure, the answer depends on how much uncertainty it is willing and able to bear. Probably many companies would not undertake to insure a new class of exposures (motorcycles or sailboats, for instance) unless they felt they could write at least several thousand policies within a year or two. Some companies would be more venturesome; others would be more conservative.

Clearly, there are exceptions, cases in which insurance is written without there being a large number of similar exposures. Lloyd's of London is well known for handling such things. A few years ago Lloyd's wrote a policy to cover the possible capture of Scotland's legendary Loch Ness monster. The policy insured a scotch whisky firm which, as an advertising gimmick, was offering $2 million if the monster was caught. The policy would reimburse the firm if it became necessary to pay the reward. Lloyd's of London wrote the policy for a premium of $6,000. Obviously, as there are not many loss exposures similar to this one (some people say there is no monster at all!), cases like this are exceptions. Lloyd's of London is both willing and able to insure certain exposures that most other organizations would regard as uninsurable. But, even though there are exceptions, the existence of many similar loss exposures is generally regarded as a necessary element of insurability.

Losses Are Definite, Measurable, and Important

The second characteristic of insurable risks is that the insured losses are definite, measurable, and relatively important.

A definite loss is one that is unmistakable. This usually means that the insured party can establish when, where, and why the loss occurred. For example, on September 9th the house located at 411 Elm Street was damaged by explosion caused by a leaking gas line. If covered losses were not definite, too many disputes between policyholders and insurers would be likely to develop. Consider a policy that pays a weekly income if the insured person is disabled because of accident or sickness. Losses due to accident usually are definite. If an insured breaks her leg while skiing at 9:30 A.M. on January 20th, her disability begins that day. But what if the insured just doesn't feel up to par and decides not to go to work some Monday morning? Is she disabled for as long as she says she doesn't feel like working? What does disability mean? If the insured cannot return to her former job but takes a less strenuous one instead, is she still disabled? Disability due to sickness would be difficult to insure if the policies did not define the covered loss carefully. Sometimes they define it as the inability of the insured to perform his or her regular occupation. Inclusion of a waiting period also helps make this loss more definite. Payments might begin with the third week of disability, for instance. This stipulation makes it more likely that claims will be submitted for genuine disabilities only.

Losses are measurable when their dollar amount is easily determined by a method agreed upon in advance. For disability income insurance, losses are made measurable by stating in the policy the amount ($300 per week, for instance) that the company will pay. An example of a loss that is not measurable is the sentimental value that people may attach to certain property. Consider the ring that had been your great-grandmother and that was made from gold mined by your great-grandfather during the California Gold Rush. Your family would hate to have this ring stolen or destroyed. They treasure it highly, not so much for its intrinsic worth as for its place in the family's history. In other words, the ring's value is largely a sentimental value and its insurable value may be quite small. Sentimental value is subjective and hard to measure. Unless the owners and the insurer agree upon an amount in advance and state it in the policy, sentimental value is not insurable.

Insured losses must also be important. That is, there must be the possibility of a fairly large financial loss. Insurance against damage to a cheap ballpoint pen, for instance, is not feasible. In the first place, of course, no one would buy it; it wouldn't be worth the bother. Second, even if it were available, it would be too expensive. The costs of selling the policy, preparing it, processing the premium, and handling claims would probably require something like a $15 annual premium to cover an 89 cent pen. Potentially large loss is necessary in order for insurance to be economically feasible.

Losses Are Accidental

Another feature of insurable risks is that the covered losses are accidental, meaning that the losses are unintended and unexpected by the policyholder. Thus, fire insurance does not cover arson by the owner, and auto policies likewise exclude injury or damage caused intentionally by the insured. Also, loss that is due solely to normal depreciation of property is not insurable. If the only loss to your car is a reduced market value because of normal wear and aging, your auto insurance won't buy you a new one! In such a case the loss is unintentional, but it is not unexpected.

Catastrophic Loss is Very Unlikely

Insurance companies operate on the assumption that not too many of their policyholders will suffer losses at the same time. They assume that losses will be unrelated, independent occurrences and that loss payments will be a fairly predictable and reasonable percentage of premium income. For some risks this assumption cannot safely be made. Unemployment is an example. A severe depression can put so many people out of work that an insurance company providing unemployment compensation benefits would quickly go broke. Only the government, with its power to tax and to borrow practically unlimited sums of money, can insure this risk.

Each of the features of insurable risks, including this one, is viewed differently by different insurers. Some companies are large enough or venturesome enough to cover exposures that others would regard as catastrophic. One company might insure thousands of buildings in an area subject to severe hurricane damage, whereas another might be reluctant to cover more than a few buildings in that area.

Through the strength and ingenuity of the insurance industry, protection is now provided for a wide range of risks. For other risks, however, one or more of the features of insurability are judged to be lacking, and insurance therefore either is limited or is not available at all.

THE FIELDS OF INSURANCE

This section furnishes an overview of the various kinds of insurance. It is intended to help provide a context within which the specific types of insurance will be studied in subsequent chapters.

In order to visualize the entire span of insurance, it is helpful to divide it into parts and then examine each part individually. There are several ways in which the various kinds of insurance can be classified for this purpose. One method is to divide insurance between that protecting individuals and that protecting businesses and other organizations. A second way is to classify insurance as either voluntary or involuntary, depending upon whether or not it is required by law. Third, insurance can be divided between the types that protect against loss of income (such as by death, disability, or unemployment) and the types that pay for damage to property. Finally, insurance can be classified on the basis of whether it is provided by private insuring organizations or by the government. Using the last of these approaches, we look first at private insurance and then at insurance that is provided by governmental units.

Private Insurance

Private insurance is that which is furnished by private (nongovernmental) insuring organizations. it consists of three major fields: life insurance, health insurance, and property-casualty insurance.

LIFE INSURANCE

Life insurance companies write two types of coverage: life insurance and annuities. Both types relate to people's uncertainty about how long they will live, and to the financial implications of that uncertainty. Life insurance deals primarily with the risk of dying while others are still financially dependent upon the insured person, such as while other members of a family are relying upon the earnings of a parent. Upon the death of the insured person, a life insurance policy pays a stated amount to a designated individual, called the beneficiary.

Life insurance policies deal with the financial risk associated with a short life span. Life annuities, on the other hand, concern the risk of living to an old age. In this case the financial problem is that of outliving one's income, as with people who retire from work and then live long enough to use up all of the funds they had saved while they were employed. A life annuity insures against this possibility by guaranteeing an income to the insured person for as long as he or she lives.

HEALTH INSURANCE

The field of health insurance deals with two principal types of loss. The first is the expense of medical treatment. This may include doctor and hospital bills, the cost of medicines, private nursing care, and so forth. The other loss that can be handled by health insurance is the income that an insured person is unable to earn during a period of disability.

Health insurance losses may be triggered by either of two perils, accident or illness. Some policies cover only accidents, but many health insurance contracts cover losses caused by either accident or illness.

The fields of life and health insurance are connected by a dotted line. This indicates a close relationship between these fields. Although some insurers, including Blue Cross and Blue Shield, specialize in health insurance, a great deal of private health insurance is provided by companies whose principal business is life insurance.

PROPERTY-CASUALTY INSURANCE

The last of the three major fields of the private insurance business is property-casualty insurance (sometimes called property-liability insurance). It is made up of property insurance and casualty insurance. These two areas are further divided into "lines" or classes of coverage.

The distinction between property insurance and casualty insurance and the further division into more specific lines of coverage is largely due to state laws that no longer exist. Until the laws were changed (during the 1940s and 1950s) each insurance company was permitted to write only certain lines or groups of related lines of coverage. Companies licensed to write fire insurance and other property lines could not engage in the various casualty lines and vice versa. The laws now permit what are called "multiple-line" operations, enabling a single company to provide any or all lines of property-casualty insurance. In fact, it can do so in a single policy if it wishes. As a result, the old-time divisions within the property-casualty field have become less distinct. They are still used, however, to identify the various parts of this field.

Property insurance. The property insurance field is divided into marine and non-marine lines.

Marine insurance is a broad area divided into ocean marine and inland marine. Ocean marine, one of the oldest forms of insurance, covers ships and their cargo, both on the high seas and on inland waterways. Inland marine insurance grew out of ocean marine, originally to cover goods being carried on land to and from ocean ports. Today inland marine insurance covers cargo being shipped by air, truck, or rail. In addition, the field has expanded to include a variety of other risks that in some way relate to transportation.

The non-marine part of the property insurance field covers damage to practically all kinds of property other than those considered to be part of the marine field. These properties range from common household articles to huge computer systems, from private residences to giant skyscrapers. At one time the only damage covered was that done by fire or lightning, but over the years protection has been added for numerous other perils. Today properties can be insured for practically any peril, including wind, hail, explosion, riot, vandalism, and earthquake. The policies may cover either direct or indirect loss. Direct loss refers to loss that is the immediate, direct result of physical damage to the covered property. Indirect loss is the loss of income or the extra expenses that result from a direct loss.

The most rapidly growing part of the nonmarine property insurance field is multi-peril insurance, meaning policies that cover a variety of perils. Applying to either personal or business exposures, some of these policies (including homeowners policies) bridge the gap between property insurance and casualty insurance by including both types of protection.

Casualty Insurance. The distinction between property insurance and casualty insurance, never very clear, has become less distinct since the advent of the multiple-line laws. Casualty insurance can include almost any line of non-life insurance other than those identified as part of the property insurance field. The best way to describe this field is by listing the lines of coverage that it includes.

Automobile insurance is the largest of the casualty lines in terms of total premium volume. Cover ages available under automobile policies include protection against legal liability claims, payment of medical expenses, and payment for theft of or damage to insured automobiles.' In states with "no-fault" laws, additional benefits including lost wages are provided.

General liability policies cover a wide variety of business and professional liability exposures. They protect people and organizations ranging from the owner of a corner hardware store to huge multinational corporations against financial loss due to legal liability claims. Also part of the general liability lines are policies providing liability insurance for physicians, dentists, and other professional practitioners.

Worker's compensation insurance furnishes payments that employers are legally required to provide to employees who are injured on the job. The payments compensate for wages lost by disabled workers and for medical expenses. Because this protection is required by law and because the insurance is provided by state agencies in some states, workers" compensation could be classed as a type of social insurance. However, most of it is provided by private insurers, and it is generally considered to be a form of casualty insurance.

Suretyship is a form of non-insurance risk transfer. A surety is one who guarantees to fulfill a contract if the person who is principally obligated fails to do so. An example is the co-signer of a loan agreement. If a lender isn't willing to bear the risk of default by a certain borrower, the borrower may have to provide a co-signer. The co-signer becomes the borrower's surety. That is, the co-signer promises to pay back the loan if the borrower does not do so. The lender then has transferred the risk of default to the co-signer. Although individuals can act as a surety and frequently do so as co-signers, insurance companies usually provide this service in other situations. Surety contracts are called bonds rather than policies, and a distinction is made between fidelity bonds and surety bonds. Fidelity bonds reimburse employers for loss caused by employee

dishonesty. Protection is provided both for elaborate embezzlemeets and for simple employee theft of goods or money. Surety bonds, on the other hand, guarantee the performance of a specified act such as the construction of a building. If a building contractor fails to complete the work as specified, the insurance company protects the owner against loss resulting from the contractor's failure to comply with the contract.

Miscellaneous casualty is the name given to various other lines of casualty insurance. This includes policies covering such things as burglary, robbery, forgery, boiler explosion, glass breakage, aircraft liability, and aircraft damage.

Government Insurance

Government insurance is that which is written by governmental agencies. It may be either compulsory or voluntary.

Insurance provided by the government on a compulsory basis is called social insurance. In a later chapter we examine the nature of social insurance more closely and make some interesting comparisons between it and private insurance. The major components of social insurance in this country are social security and unemployment compensation.

SOCIAL SECURITY

The technical name for what is commonly called social security is Old Age, Survivors, Disability, and Health Insurance. A federal program, it is operated by the Social Security Administration and is financed by special taxes paid by employers, employees, and self-employed persons.

Since its creation in 1935, social security has expanded rapidly and now supplies an important part of the financial security of almost all Americans. The major benefits are income for retired persons, income for survivors of deceased workers, income for disabled workers and their families, and hospital and medical benefits for persons age 65 and above.

UNEMPLOYMENT COMPENSATION

In the section of this chapter that described the features of insurable risks, we noted that private insurers are not in a position to furnish unemployment insurance. The government, however, can and does write this coverage. Acting upon the impetus of a federal law, each of the states has developed an unemployment compensation program. Workers who are laid off from their jobs receive weekly income payments, usually for a maximum of twenty-six weeks. Special taxes paid by employers finance the system.

OTHER GOVERNMENT INSURANCE

The state and federal governments provide a variety of other forms of insurance in addition to social security and unemployment compensation. Most of the other forms are offered on a voluntary basis and are either the same as or similar to the kinds of protection furnished by private insurers. Examples include flood damage insurance and life insurance for military personnel.

INSURANCE OPERATIONS

Thus far, we have examined the theoretical nature of insurance and have seen that it is one of several methods that can be used for handling risk. We also have learned what types of risks are insurable and have reviewed the kinds of protection that are furnished by private and government insurers. But how does the insurance system, particularly the private part of it, operate? In other words, who does what in order to make the system function? In a few short paragraphs, this section summarizes the operational side of the insurance business. It is not intended to be a complete description; instead, it is designed to introduce some important concepts and to provide a working vocabulary that will be useful in connection with following chapters. Then, in the concluding chapters (Part V) of the book, we return to these topics and take a closer look at each of them: marketing, pricing, underwriting, loss adjusting, and company management.

Marketing

Marketing is the process of determining what consumers want and then directing the flow of goods and services from producer to consumer in order to satisfy those wants. Three factors are important in marketing insurance: (a) product design, (b) competitive pricing, and (c) customer service. An example of the first of these, product design, is the development of package policies. These are contracts that combine in a single policy various kinds of protection that previously had been available only through the purchase of two or more separate policies. Another example of improved product design is the recent development of shorter, easier-to-understand policies.

Competitive pricing is much more prevalent in insurance than many people realize. That is, most people believe that all companies charge about the same price for their policies, but in many cases there actually is a great range of prices. Perhaps one reason for this misconception is that few people believe they know enough about insurance to shop for it effectively, and therefore they make no effort to do so. Well-informed consumers know what forms of insurance protection they need and save money by knowing how to buy it.

Customer service is also an important aspect of insurance marketing. Insurance buyers, whether individuals or organizations, need assistance in identifying their insurable risks and determining the kinds and amounts of coverage that are appropriate. After policies are written they must be kept up to date as the policyholders' needs change; when losses occur, customer service means prompt and fair payment in accordance with the terms of the policy.

The persons most directly responsible for insurance sales and service are the agents and brokers. Insurance agents are men and women who are authorized to sell policies on behalf of one or more insurance companies. In life insurance, agents customarily represent a single

insurer; in the property-casualty business many agents sell for a number of companies. Agents usually receive a commission, which is a percentage of the premiums that are paid for the policies they sell.

Unlike agents, who represent one or more insurance companies, brokers represent insurance buyers. Insurance brokers usually concentrate on serving commercial clients, arranging for business firms to be covered by one or more insurers. Although they legally represent buyers rather than insurers, brokers receive a selling commission from the insurers just as agents do. (For the sake of brevity, we customarily use the term "agent" unless it is important to distinguish between agent and broker).

Agents are licensed by the states where they reside. Applicants for licenses must demonstrate a basic knowledge of insurance by passing written examinations. Through continuing study and experience, agents can develop the skills needed to furnish their services at a high level of professional competence.

Pricing

As we know, the price of an insurance policy is its premium. Premiums are based upon (a) rates and (b) exposure units. An insurance rate is the price per exposure unit. Exposure units are the measuring units used in insurance pricing. Premiums therefore are determined by multiplying rates by the number of exposure units. To illustrate, in life insurance the exposure unit is the number of thousands of dollars of coverage. If a policy provides $20,000 of coverage and the rate is $15 per thousand, the policy's premium is $15 x 20, or $300 per year.

Most insurance rates are class rates, meaning that the loss exposures to be insured are classified on the basis of certain characteristics and all of the loss exposures in each classification are subject to the same rate. The rating classifications in life insurance, for instance, are based on the age and sex of the insured persons. All 25-year-

old males who are in good health are in the same classification. Those who are not of that age or sex are in different classifications and are subject to different rates. In automobile insurance the rating classifications are based on several characteristics of each driver, including age, sex, place of residence, and whether or not the car is used in business.

Insurance pricing is designed to meet two principal objectives. First, the company's premium income (together with income from other sources) must be sufficient to pay its policyholders' losses and meet company expenses. If a particular company's losses and expenses total $100 million for the year, it is essential for that sum to be available. Second, the total cost must be distributed fairly among the various policyholders. Each insured should pay his or her fair share. That is the reason for using rating classifications. Older people are charged higher rates for life insurance than younger people, because a higher percentage of older insured die in any given year. For automobile insurance teenaged drivers are charged higher rates than middle-aged drivers because a higher percentage of the former have accidents in any given year. In either case the result is believed to be a fairer sharing of the insurance company's total costs than would be the case if all persons were charged the same amount.

Specialists professionally trained in the techniques of insurance pricing are called actuaries. They develop the rates and rating systems that insurers use.

Underwriting

Perhaps you have heard of someone who has had difficulty in securing automobile insurance after a series of accidents or motor vehicle violations. Or you may know of someone who could not buy more life insurance after developing a serious health problem. Cases like these reflect underwriting, the process by which insurers decide which loss

exposures to accept and how to insure them. Company-employed underwriters receive applications for insurance from agents and brokers, develop information about the prospective insured, evaluate the information, and make the underwriting decisions.

The objective of the underwriting process is to select those applicants for insurance that meet the company's standards of acceptance. In other words, the underwriter's job is to avoid insuring too many below-average risks. Unless this is done, "adverse selection" will result, meaning that too many of the company's insured will have losses and the company may not have enough income to cover all of its costs.

Underwriters are also responsible for seeing that the proper rate classifications and policy forms are used for each contract that is issued. In addition, they may be involved in reinsurance, a process by which one insurance company insures part of its risks with another company (the reinsure). By reinsuring a portion of the insurance which it accepts, a company limits the amount it will have to pay for any one loss or group of losses.

Claims Adjusting

When insured losses are reported, policyholders are entitled to receive fair payment for them. It is important, however, that payment not for fake or exaggerated claims. Losses are paid with the premium supplied by the entire group of policyholders, and if excessive loss are made the policyholders' premiums are wasted, requiring premiums rates and premiums to be increased in the future.

The process of investigating, evaluating, and settling claims is called claims adjusting. Chief responsibility for this process lies with an insure adjusting department. In many cases most of the work is done by stuff adjusters employed by the company. In some situations insurance

comp. the services of independent loss adjusting organizations rather than using their own staff adjusters. In property-casualty insurance small fire, wind, collision losses frequently are handled by the agents who sold the policies covering the losses.

To obtain maximum value from their insurance, consumers should acquaint themselves with their policies. They should know what losses are covered and what procedure to follow when a loss occurs.

Company Management

Several thousand insurance companies operate in the United States. either stock companies (corporations) or mutual companies (a type of nonprofit organization.(

Nearly 2 million people are employed in the insurance business. about one-third are engaged in insurance sales. The other two-third are engaged in a great variety of occupations. These include underwriting, claims, actuarial science, investment analysis, accounting, computer programming systems analysis, data processing, personnel administration, secretarial, and general management. The insurance work force is rather divided between men and women. Although most of the women do secretarial and clerical work of various kinds, increasing numbers of management positions are held by women.

Many of the large companies operate on a nationwide basis, and offices located throughout the country. An even larger number of insurers operate in only one or a few states.

Insurance companies are subject to several forms of regulation by governments. Many of the regulations are designed to safeguard the stability of the companies and thus to make sure that they will be able to pay the claims that their policyholders are entitled to receive. Other state regulations are intended to prevent the use of discriminatory or excess) deceptive policy wording, or unfair sales practices.

THE CHANGING WORLD OF INSURANCE

Before Pope John Paul II made his historic tour of Great Britain in 1982, millions of dollars were invested in preparations for the visit. Costly measures were taken for special security, transportation, and communications facilities, outdoor stages were constructed for open-air masses, and souvenir manufacturers turned out hundreds of thousands of items bearing the Pope's picture and the dates of his expected visit. If for some reason the visit had been called off, much of this investment would have been lost. The people and organizations providing the funds therefore sought to transfer this risk by purchasing what is called nonappearance insurance. More commonly used to protect the promoters of rock concerts and similar events, the coverage would reimburse investors if the trip was canceled for reasons beyond their control. The rate for which insurers initially offered the coverage was 2.5% of the amount insured. Buyers paid $25,000 for $1 million of protection, for instance. Then, twelve months before the scheduled visit, the Pope was seriously wounded by a terrorist who attacked him in St. Peter's Square. When he recovered, plans for the visit were resumed. However, nonappearance policies purchased at this point cost 10% of the sum insured. Later, as the Pope's recovery continued, the rate declined to around 5%. Then, just a few weeks before the visit, Argentina occupied the British-owned Falkland Islands and fighting erupted in the South Atlantic. Again, fears rose that the trip would be canceled, and again the price of newly issued nonappearance policies soared. The rate reached as high as 20% a few weeks before the visit took place. These rate fluctuations, of course, reflected the insurers' charging estimates of the likelihood that the tour would be canceled, whether due to the Pope's health, concern for his safety, escalation of the Falkland Islands conflict, or some other reason.

This episode illustrates an important aspect of insurance: its need to adapt to an ever-changing world. To do so requires that protection be provided for a wide variety of risks, many of which reflect recent developments that may range from international terrorism to the latest technological advances. The point is further illustrated by insurance for the following:

1- Liability protection for a manufacturer of genetically engineered microbes that will be tested in humans and then mass-produced for commercial use.

2- Insurance of the $90 million spent by a television network for rights and construction costs to cover Olympic games. (NBC was insured against such a loss when the United States withdrew from the 1980 Moscow Olympics to protest the Soviet Union's invasion of Afghanistan.)

3- Kidnap and ransom insurance for a U.S. construction firm that is building a giant hospital complex in a terrorist-plagued Third World nation.

4- Coverage for the construction, launching, and operation of privately owned communications satellites.

5- 5.Insurance against pollution emanating from a dump site for hazardous chemical wastes.

6- Insurance of a $1 billion oil well drilling platform located in the North Atlantic where it is exposed to 100-mile-per-hour winds and 50-foot waves.

Ventures like these are not undertaken without adequate insurance protection. Unless risks can be insured, investments will not be made. As the head of a large insurance firm observed, "Without insurance, much that is critical and is now happening simply wouldn't be underway in a free society. No space shuttle, no nuclear plants at all, no North Sea oil."[2]

Thus, we see that insurance is a dynamic field, constantly responding to the needs of our rapidly changing world. In the weeks ahead, watch for news events with significance for insurance. The most obvious will be disasters like major fires, windstorms, and plane crashes. Other news stories will have equally important but less apparent implications for insurance. They may include reports of new government regulations, medical treatments, construction materials, energy supplies, communications technology, or leisure products. Consider, too, the impact upon insurance of trends in political philosophy, social relationships, or ethical standards. As we continue our study of risk and insurance, it will become increasingly apparent that the need to deal with change is a continuing challenge to the insurance industry and the people who operate it.

IMPORTANT TERMS

private insurance	direct loss	unemployment
life insurance	indirect loss	compensation
life annuity	multi-peril insurance	package policy
health insurance	casualty insurance	insurance agent
property-casualty	automobile insurance	broker
insurance	general liability	insurance rate
property insurance	workers' compensation	exposure unit
marine insurance	insurance	class rate
ocean marine	suretyship	actuary
insurance	miscellaneous casualty	underwriting
inland marine	social insurance	adverse selection
insurance	social security	reinsurance

KEY POINTS TO REMEMBER

1) Not all risks are insurable. Insurable risks generally have four characteristics: (a) there are many similar loss exposures, (b) losses are definite, measurable, and important, (c) losses are accidental, and (d) catastrophic loss is very unlikely.
2) The three major fields of private insurance are (a) life, (b) health, and (c) property-casualty.
3) Both life insurance and annuities deal with the financial risks associated with living and dying.
4) Two types of loss can be covered by health insurance: the cost of medical treatment and income lost while insured persons are disabled.
5) Property insurance, one part of the property-casualty field, is divided into marine and non-marine insurance. Marine insurance consists of ocean and inland marine; the rest of the property insurance field includes fire and allied lines plus multi-peril insurance.
6) Property insurance can be written to cover either direct or indirect loss.
7) Included in casualty insurance are automobile, general liability, workers' compensation, suretyship, and miscellaneous casualty coverages.
8) Government insurance may be either compulsory or voluntary. Insurance written by the government on a compulsory basis is called social insurance. The principal kinds of social insurance in this country are social security and unemployment compensation.

9) Insurance is marketed on the basis of (a) product design, (b) competitive pricing, and (c) customer service. Most of the customer service is provided by agents and brokers.

10) Agents represent one or more insurance companies and sell policies on their behalf. Brokers represent insurance buyers and arrange for insurance on their behalf.

11) An insurance rate is a price per exposure unit. Premiums are determined by multiplying rates by the applicable number of exposure units.

12) Underwriting is the process by which insurers decide which applicants to accept and how to insure them. Underwriters seek to avoid adverse selection, which means insuring too many belowaverage risks.

13) Through the process of reinsurance one insurance company can transfer part of the insurance it has written to another insurer.

14) The goals of claims adjusting are to pay valid claims promptly and fairly and to avoid paying fake claims or exaggerated amounts.

15) Numerous occupations are involved in the operation of insurance companies and agencies. About one-third of the people employed in insurance are in sales work; the other two-thirds perform a wide variety of jobs.

16) The needs for insurance protection are constantly evolving, requiring continuous change by the insurance industry.

REVIEW QUESTIONS

1) Why is the existence of many similar exposures a characteristic of most insurable risks?
2) (a) What is the difference between a definite loss and a measurable loss? (b) Why are these characteristics important?
3) (a) What is meant by "accidental" loss? (b) Why is it important?
4) How are life insurance and annuities related to each other?
5) In property-casualty insurance, what are multiple-line operations?
6) Distinguish between direct loss and indirect loss.
7) List the major lines of property-casualty insurance.
8) Define social insurance.
9) What are package policies?
10) What are class rates?
11) What are the principal goals of insurance pricing?
12) What is the main objective of underwriting?
13) What is the relationship between claims adjusting and rate levels?

DISCUSSION QUESTIONS

1) Do the following risks have each of the usual features of insurability? (a) Automobile collision (b) Burglary from a camera store (c) Shoplifting from a department store (d) Wind damage by hurricane (e) Style change that would diminish the value of a clothing store's inventory
2) In a great many cases, when reonle think of careers in insurance, they think only of sales work. Why do you suppose they have this attitude, given that twice as many of those who work in insurance are in jobs other than sales?
3) (a) Looking at the various fields of private and social insurance, which ones appear to affect you most directly? (b) Are there any that do not affect you at all, either directly or indirectly?
4) Can you add any recent or unusual risks to the list in the concluding section of the chapter?
5) Explain the statement, "Without insurance, much that is critical and is now happening simply wouldn't be underway in a free society."
6) What current news events have important implications for insurance?

CHAPTER 3

RISK MANAGEMENT

CHAPTER OBJECTIVES

After studying this chapter, you should be able to:

1- Summarize the concept of risk management.

2- Outline the activities included in the risk management process.

3- Give examples of the four methods of handling risk.

4- Compare the risk retention techniques used by large organizations with those of families and individuals.

5- Show how the selection of risk handling methods is influenced by risk characteristics.

6- Discuss the rules of personal risk management.

THE RISK MANAGEMENT CONCEPT

Risk management is the systematic and efficient handling of pure risks. It deals with all pure risks, whether or not they are insurable, and uses whatever risk handling methods are most appropriate. These methods include but are not limited to insurance.

Risk management is a relatively new concept, having gained little acceptance before the 1960s. Most of its development has occurred in very large corporations, some of which spend millions of dollars each year on insurance and other methods of risk handling. In many of the largest firms, **risk handling** is now the work of specialists called risk managers. Although they rely heavily on insurance, they regard it as one of several tools that are available to them. This approach, in fact, is one of the important ways in which today's corporate risk manager differs from yesterday's corporate insurance buyer. The latter's responsibility was limited to administering the corporation's insurance program, making sure that all of the conventional insurance policies were properly prepared and maintained. In contrast, the modern corporate risk manager uses insurance only when other techniques are not preferable. Insurance is an important method of risk handling and is used extensively even by large corporations, but in the modern view it is not automatically presumed to be the best method in every situation.

Although the techniques of risk management were originally developed to deal with the risks of giant corporations, one does not have to employ a professional risk manager in order to use them. The risk management approach is as applicable to the Jones family or to Pete's Delicatessen as it is to General Motors.

THE RISK MANAGEMENT PROCESS

The risk management process is the set of activities that are followed in applying the risk management concept. The process, in other words, is a method of dealing with risks systematically and efficiently rather than on a hit-or-miss basis. Four activities are involved:

1. Risk identification
2. Risk evaluation
3. Selection of risk handling methods
4. Administration of the program

These four activities follow one another in logical order and can be thought of as a series of steps, but the process does not end when the fourth "step" is completed. New risks frequently arise, and existing ones constantly are being altered. Risk management therefore is a continuous process.

Risk Identification

The sources of possible loss must be recognized before anything can be done about dealing with them. Accordingly, the first phase of the risk management process is identifying the risks to which one is exposed.

Families and individuals are subject to each of the four types of losses: property losses, income losses, legal liability claims, and unexpected expenses. Property losses may involve the residence and its almost limitless assortment of contents ranging from furniture and appliances to sports equipment and musical instruments. In addition, there may be a car and perhaps a boat or some other recreational vehicle. All of this property is exposed to loss from numerous perils. An effort should be made to identify all of the important categories of property and all of the perils to which they are exposed. Major income loss may befall a family

because of death, disability, or unemployment. Although it is unpleasant to dwell on such possibilities, successful risk management demands that they be recognized and dealt with realistically. Legal liability claims are probably most likely to result from a family member's use of an automobile, but other possible sources of liability should also be identified. These include claims brought by people accidentally injured in the family's residence. Lawsuits also may stem from various personal activities including hunting, boating and athletics. The major risk associated with the final type of loss, unexpected expenses, is the possibility of unusually large bills for health care.

It is difficult to generalize about the risks to which families are exposed, because their possessions and activities are so varied. The important point is that a conscious effort should be made to recognize the possible sources of loss. This requires an awareness of pure risk and its importance as well as an alertness to new risks that develop as people acquire additional property and engage in new activities.

Risk Evaluation

The next step of the process is to estimate the risks' importance. Risks are evaluated on the basis of the severity and frequency of the losses associated with them. As indicated earlier, loss severity means the size of the potential losses; loss frequency refers to the probable number of losses. The two concepts, in other words, relate to the questions, How big? and How often? Of the two, loss severity is more crucial because a very large loss, although unlikely to happen, could be a financial catastrophe if it did occur.

For most families the risks with the greatest potential severity are those of long-term disability, early death of a parent, and major legal liability claims. Although such events happen only rarely, they can have a staggering financial impact. In contrast are low severity risks like the loss

or destruction of inexpensive items of property. Losses of that nature can rather easily be paid for out of a family's regular income.

Selection of Risk Handling Methods

In our survey of risk and insurance fundamentals in Chapter 1, four methods of dealing with risk were identified: risk avoidance, loss control, risk retention, and risk transfer. We now examine the ways in which these methods are selected and applied in the risk management process.

RISK AVOIDANCE OR REDUCTION

In the context of risk management, it is helpful to consider the four methods of dealing with risks in two categories. The first two methods are ways to avoid or reduce risk; the other two methods are ways to finance the effects of the risks that remain.

Risk Avoidance. This method is used when the risk is known to be a serious one and when not too much is given up by avoiding it. Risk avoidance is feasible more often in business situations than it is for personal risks. For instance, manufacturers sometimes halt production of items that have been found to result in numerous lawsuits by injured consumers. Several sporting goods companies, including MacGregor, Spalding, and Wilson, dropped football helmets from their lines of athletic equipment for this reason. Perhaps you have noticed that many motel swimming pools no longer have diving boards. This is another case of avoiding the risk of lawsuits for personal injuries. Some multinational corporations refrain from operating in countries where anti-American riots may result in damage to their property or where unfriendly governments may expropriate their assets. If a firm has important activities in such a country, however, it may decide that avoiding the risk would be too costly.

Individuals also make use of the risk avoidance technique. But like corporations, individuals in many cases find that the cost of giving up a risk is too high. For this reason, millions of people choose not to avoid the risks that they know are associated with heavy cigarette smoking. And although many of us rather easily refrain from the risks of white water river canoeing or from those of traveling in areas where tropical diseases flourish, we would find that the costs of avoiding the risks of property ownership or of automobile travel would be far too high. For individuals, as for business firms, the other risk handling methods usually are more appropriate.

Loss Control. The potential impact of the risks that cannot be avoided usually can be reduced through loss control programs. By lowering loss frequency and severity, loss control pays off in several ways. First, it has major humanitarian and ecological values; it can save lives, health, energy, and resources. Second, reducing losses avoids the consequences that insurance cannot pay for: inconvenience, delay, emotional strain, disruption of family plans, and other nonfinancial results of accidents, fires, illnesses, and so forth. Third, controlling losses can reduce the cost of one's future insurance protection. Finally, those who don't take steps to control losses may not be able to obtain insurance. Heavy drinkers or people who are extremely overweight, for instance, may find it difficult to buy life insurance. Intelligent risk management therefore always includes actions designed to preserve lives, limbs, and property.

All of us can learn a great deal about loss control by observing the measures taken by business firms and other organizations. Notice the efforts of schools and theaters to prevent fires, or of jewelry stores to prevent loss by burglary or theft. In the case of trucking firms, loss control programs focus on (a) driver selection, (b) driver training and supervision, and (c) vehicle maintenance. In the driver selection process,

the health condition and accident record of persons applying for jobs are checked. New drivers then are trained in safe and efficient operation of their vehicles, and their driving practices are observed by loss control specialists who follow the firm's trucks in unmarked cars. Vehicle maintenance programs include frequent and regular inspections and servicing.

In one state more than two thousand tavern owners are finding that efforts to control their losses pay off by reducing lawsuits and lowering their insurance premiums. Acting through a trade association, the tavern owners attend risk management seminars where, among other things, they have learned that bar stools with backs and arm rests are much safer than the conventional type of stool. In addition, the tavern owners have been urged to train their employees to be "crowd control engineers" rather than "bouncers." "Whenever you try to toss someone out," one owner said, "you know there is going to be a claim." The "crowd control engineers" are instructed to call police when trouble develops, instead of trying to eject the troublemakers themselves.'

Most businesses devote more effort to preventing employee injuries than to controlling any other kind of loss. The measures taken include such things as machine guards; factory ventilation and lighting; noise control; instruction in safe working methods; control of toxic materials; plant safety meetings; safety contests; and first aid facilities. Those who are responsible for employee safe ty programs collect and analyze information about their accidents and workers' compensation claims. They try to identify accident trends and to spot problems regarding particular procedures, employees, supervisors, types of machines, or departments. Such analysis, loss control specialists say, should not concentrate on major injury cases alone. Instead, all accidents should be examined, including those that resulted in little injury or no injury at all. This is because accident frequency is more controllable than

accident severity. One investigation showed that out of every 330 industrial accidents, 300 will involve no injury, 29 will cause a minor injury, and only 1 will result in a fatality or major injury. As one safety director put it, "What you are aiming to do in this job is to prevent those 329 minor accidents from becoming that one fatality or serious injury." In other words, by studying all of the firm's accidents, risk managers maximize their knowledge of accident causes, trends, and patterns. Then, by adopting appropriate loss control measures, they strive to curtail all future losses, both major and minor.

RISK FINANCING

After all reasonable efforts have been made to avoid risks or to reduce them, action must be taken to pay for those that remain. In general, two methods of risk financing are available: risk retention and risk transfer. In other words, losses can either be paid by the party that sustains them or they can be transferred probably by insurance to someone else. Frequently, some combination of the two methods is used.

Risk Retention. As a tool of risk management, risk retention usually means retaining the risk of small losses rather than transferring the risk to a non-related insurer. The purpose is to save money by reducing insurance costs. In the past this method received far less attention than it deserved. Professional risk managers now are using several interesting techniques to reduce costs by retaining more of their firms' risks. Some very large corporations retain "small" losses up to $100,000 or more; losses above that amount are either insured or handled in some other way. Families and individuals can practice risk retention in essentially the same way by using deductible clauses. A deductible clause is a policy provision stating that a specified amount will be subtracted from covered losses, the insurance paying only the amount in excess of the amount subtracted. Both automobile and home insurance policies ordinarily contain deductible clauses.

The risk retention plans of large corporations take numerous forms. These include deductibles, self-insurance plans, and captive insurers. Self-insurance is a type of risk retention in which an organization with many similar loss exposures sets aside a fund for the payment of future losses. An example is a firm that provides a group health program for its one thousand employees The program pays certain of the expenses incurred by the employees and their families for hospital and surgical services. Because many exposure units are involved, the total covered expenses incurred by the group remain about the same from year to year. The cost to fully insure the program would be $1.2 million per year. But instead of buying the insurance, the firm has established a self-insurance plan. Covered hospital and surgical benefits are paid from a special self insurance fund set up for this purpose. To protect itself against the possibility that benefit payments for a particular year may be much greater than predicted and provided for, the firm buys a special kind of insurance policy. Called an aggregate stop-loss policy, it sets $1.5 million as the maximum amount the firm will pay for health costs during the year; any benefits in excess of that amount will be paid by the insurance company. Because the chance of substantial payment under this policy is slim, its cost is only $20,000 a year.

In effect, the stop-loss policy is a group health insurance contract with a $1.5 million deductible. This firm is combining self-insurance with a deductible insurance policy to finance its group health program. Notice, however, that there are two important differences between true self-insurance and the conventional use of deductible clauses (such as on private passenger automobile policies). First, self-insurance requires the combination of many exposure units so that the total amount of losses can be predicted within a reasonable range. Second, a fund must be set aside in advance for paying the anticipated losses. Without these two elements the use of deductibles may be an appropriate risk retention technique, but it does not constitute self insurance.

A captive insurer is an insurance company formed primarily to insure the risks of its parent organization. Premiums for insurance protection are paid to the captive by the parent organization. In return, covered claims are paid just as they would be by a regular insurance company. It is estimated that about 1,400 captive insurers exist. Examples include insurance subsidiaries owned by Exxon, Colgate-Palmolive, Levi Strauss, and the University of California.

Essentially, a captive insurer is an extension of the self insurance technique. The organizations that use either of these methods expect to reduce their total insurance costs; a saving of 20 or 30% is a typical goal. The principal sources of savings are reductions in administrative expenses and in loss payments. Administrative expenses include claims handling costs and sales commissions, as well as the general overhead costs of operating a commercial insurance company. When captive insurers are used, certain tax advantages may also be realized. Self-insuring organizations and owners of captive insurers also expect to save money through effective loss control programs that have the effect of reducing the amounts paid for covered losses. An important point in this regard is that the incentive for loss control is increased when loss payments come from an insured's own funds (or those of a subsidiary captive insurer) rather than from a non-related insurance company.

Risk Transfer. The usual way to transfer risk, as we know, is to buy insurance. Risk sometimes is transferred in other ways, however. A retail florist that sells its delivery vans and contracts with a local delivery service thereby transfers to the delivery company the risks involved in operating its own vehicles. A home builder that subcontracts some of its activities to excavation, plumbing, and electrical contractors also transfers some important risks.

Today's corporate risk managers seek to avoid all unnecessary insurance purchases. In other words, their objective is to rely upon insurance only to the extent that is necessary after making effective use of the other risk handling methods. This does not mean that insurance is unimportant or is little used by large organizations. On the contrary, it continues to be the most important and widely used method of dealing with major business risks. This is indicated by the fact that General Motors Corporation spends about $200 million a year for property-casualty insurance alone.2 Insurance is at least equally important in handling non-business risks. In fact, families and individuals probably rely more heavily upon insurance than business firms do.

In making a choice between insurance and the other risk handling methods, the loss frequency and loss severity of the particular risk are important. If for purposes of analysis we assume that both characteristics are either "high" or "low," there are four possible combinations.

Risks of the first type have both low loss frequency and low loss severity, meaning that losses seldom occur and when they do happen they are small. Insurance of such risks is neither necessary nor economical; risk retention is preferable. Risks with high loss frequency and low severity include the possibility of breaking dinnerware or small hand tools. Retention and loss control are the best methods of handling those risks. Fortunately, most people are not confronted by risks of the third type, when losses are both frequent and severe. For those who are, private insurance is extremely expensive and perhaps not available at all. An example would be the risk of early death for alcoholics or heroin addicts. Most people would agree that such risks should be dealt with by loss control or risk avoidance.

The last combination, low loss frequency and high severity, is the one for which insurance is ideally suited. The risk of legal liability is one

that has these characteristics. Because loss severity is high, the risk is very important; the consequences of an uninsured loss could be severe. But because loss frequency is low, the cost of insuring this risk is not too great. Risks with low loss frequency and high severity can be financed economically by transferring them to an insurance company.

Administration of the Program

The final step is to administer the risk management program on a ca basis. For large organizations this is a substantial task.. The risk manager of the University of California has eleven people on his staff. They oversee risks that involve 111,000 employees, 140,000 students, 9 campuses, 5 hospitals, 2 nuclear research centers, and about $5 billion in property values. The annual risk management budget has been reported to be $45 million.3 On a far more modest scale, all of us as individuals have our own risk management programs to administer. This requires buying insurance policies, taking action to control losses, and participating in the settlement of losses. Proper administration of the program also means keeping it up to date. Our lives are in a state of constant change, and as our lives change so do the risks we face. Arrangements that were made for handling our personal risks a year or two ago may be out of date today. Automobile policies, for instance, should be revised to reflect any differences in address, additional drivers, or changes of cars. Insurance of homes and personal property should be revised as property values rise or additional property is acquired. Additional life insurance should be purchased to keep pace with rising income levels and added family responsibilities. It is easy to put off making such changes or to forget them altogether. A regular review, perhaps at the beginning of each year, helps keep a program up to date.

RULES OF PERSONAE RISK MANAGEMENT

In the handling of personal risks, important decisions must be made concerning the risks to insure and how to insure them. Four basic rules can help families and individuals make these decisions in a manner that is consistent with the risk management concept.

1. Insure the major risks first.
2. Don't insure small losses.
3. Shop before buying.
4. Get help from a good agent.

Insure the Major Risks First

Major risks are those with high potential loss severity the ones that could cause serious financial harm to the persons who face them. Such risks should be insured first, and only after that is done should the insuring of less important risks be considered. Less important risks never should absorb premium dollars that could be used to buy more adequate coverage for the major risks.

To illustrate, some home insurance policies cover certain losses that others don't. Are the broadest (and most expensive) policies always better? Not necessarily. For some people a limited and less costly policy will cover the major risks, and the money saved can be used to buy more adequate life or health insurance. Should flood insurance be purchased in situations where the odds against flood destroying a house are 100 to 1? This is a major risk that should be insured, even though the likelihood of serious damage is slight. If the insurance is properly priced, its cost will be low. Remember that buying insurance makes the most sense when the expected loss frequency is low and when loss severity can be high. Another example concerns life insurance. Should parents buy life insurance to cover the lives of their children? The major risk to be

handled by life insurance is loss of a family's principal source of income. If the father is the main income earner, then the family's life insurance premium dollars should be spent to insure his life first. Only if and when that major risk is adequately insured should parents consider insuring their children.

Don't Insure Small Losses

The second rule, like the first, deals with what things to insure. It is to insure small losses even if they occur frequently. Insuring small losses essary, wastes money, and drains away funds that can be spent more other insurance. Many people ignore this rule. They believe the "isn't doing any good" unless covered losses occur; they think the insurance is that which is likely to pay for the most losses. This explains the popularity of accident policies that pay a few dolla minor cuts and bruises that occur so frequently. It also is why me prefer a small deductible amount on their automobile damage insurance.

Insuring small losses is uneconomical for two reasons. First, as injuries collisions and so forth are bound to happen, premiums that cover them must be high simply to reimburse policyholders Second, the premiums must also allow for the overhead costs of ha numerous small claims. If it costs $20 just to process a small claim, an insurer must spend a total of $45 to pay a $25 loss. It therefore makes sense to pay small losses out of one's own pocket, perhaps by use of policy deductibles, and to buy insurance to cover large losses only.

Shop Before Buying

Many people have the mistaken impression that all insurance companies charge the same prices. Although it is true that some insurers charge card" prices, there is a wide range of prices in most lines of cove survey of twelve automobile insurers in one city showed that for a pa

policy five of them were charging $544, while the other seven offer the same policy to the same person for various prices ranging as low as $318.

Some people may be reluctant to compare prices before buying they feel intimidated by insurance and by insurance agents. Consume ably would shop more wisely for insurance if they had what it is, how it is priced, and how it is sold.

Get Help From a Good Agent

Good insurance agents provide risk management counseling and advice. They offer valuable suggestions concerning the kinds and amounts of protection that are most appropriate for their clients. In life insurance, for example, they use a process called "programming" to design a plan of protection that meets the specific needs of the particular family. Agents who use this approach are professionals in the sense that they use their expert judgment to provide the services that are in the best interests of their clients. These services may be more important than minor differences in premiums. That is, it is in the consumer's best interest to buy from a good agent, even though somewhat lower cost policies may be available elsewhere. Unfortunately, many people do not realize how valuable the services of their insurance agents can be and fail to make full use of them. The importance of this point will become more clear as we continue our study of risk and insurance.

IMPORTANT TERMS

risk management	risk avoidance	deductible clause
risk manager	loss control	self-insurance
loss severity loss frequency	risk retention	captive insurer

KEY POINTS TO REMEMBER

1) Risk management is the systematic and efficient handling of pure risks.
2) Large organizations employ specialists called risk managers to handle their risks.
3) The risk management process involves four activities: (a) risk identification, (b) risk evaluation, (c) selection of risk handling methods, and (d) administration of the program.
4) Risks are evaluated with regard to their loss severity and loss frequency.
5) Loss severity is the size of the potential losses.
6) Loss frequency means the probable number of losses.
7) Modern risk management uses the most efficient and economical of the four methods of handling risks: (a) risk avoidance, (b) loss control, (c) risk retention, and (d) risk transfer.
8) A deductible clause is a policy provision stating that a specified amount will be subtracted from covered losses, the insurance paying only the amount in excess of the amount subtracted.
9) Insurance is the principal method of handling important insurable risks. Risk managers minimize costs by using other methods whenever they are appropriate.
10) Insurance is most suitable for handling risks with high loss severity ar low loss frequency.
11) An important part of the administration of a risk management program is a regular review to keep it up to date.
12) Four rules of personal risk management are: (a) insure the major risks first, (b) don't insure small losses, (c) shop before buying, and (d) get help from a good agent.

REVIEW QUESTIONS

1) (a) List the four steps of risk management. (b) Why is the order in which they are listed a logical order?
2) Name two criteria used in evaluating risks.
3) Why might it be better to retain certain risks rather than to insure them?
4) What is self-insurance and how does it compare with the use of a captive insurer?
5) How can families and individuals practice risk retention? What is the most common method of transferring risk?
6) What is the most common method ofo tranferring risk?
7) For which of the risk characteristics shown in Figure 3-1 is insurance most appropriate and why?
8) List the benefits of personal loss control.
9) (a) The first and second rules of personal risk management are closely related. In what way? (b) What is the difference between these two rules?

DISCUSSION QUESTIONS

1) For which is the risk management concept more valid: large corporations or individuals?
2) How does Figure 3-1 relate to the rules of personal risk management?
3) Pete's automobile collision insurance has a $250 deductible clause. Is it proper to say that Pete is self-insuring for collision damage up to $250?
4) One person says that deductible clauses are used by insurance companies to punish policyholders who have losses. Is that correct?
5) Some people say that insurance does them no good unless it actually pays them for losses. (a) Give an example that illustrates what this means. (b) Is the statement true? Explain.
6) Why do people so frequently violate the second rule of personal risk management? That is, why are they often more interested in insuring small frequent losses than in insuring rare but very large losses?
7) (a) Identify the three most important pure risks that you personally are facing today. (b) What are you (or others) doing about them?
8) Ted's Camera Shop is located on the fringe of the city's business district. The store stocks several lines of very expensive cameras and supplies in addition to its principal high-volume, moderately priced line. The cost of Ted's burglary insurance has climbed rapidly in recent years, and Ted is looking for ways to hold it down. In what specific ways might at least three of the methods of risk management be helpful to Ted?

CHAPTER 4

LEGAL PRINCIPLES IN INSURANCE

CHAPTER OBJECTIVES

After studying this chapter, you should be able to:

1- Explain the essential elements of insurance contracts.

2- Discuss three aspects of the doctrine of utmost good faith.

3- Summarize the indemnity principle.

4- Show how the indemnity principle is supported by insurable interest, loss measurement, subrogation, and "other insurance" provisions.

5- Identify the liability risk and explain its relationship to the laws of negligence.

6- List and illustrate the four elements of negligence.

7- Describe the principal legal defenses against negligence.

THE POLICY AS A CONTRACT

"Buying insurance isn't much fun," says Sam Policyholder. "You can't slam the doors or kick the tires. It doesn't look good on you, and there's not much to be said for how it tastes or smells. It's not a very good way to impress the neighbors either. It costs a lot of money and all you have to show for it is a piece of paper!"

There is considerable truth in what Sam says, but what he says is misleading too. What his insurance gives him is financial security, and if we could talk with him about it, Sam would have to admit that is important to him and to his family. The piece of paper that he speaks of the policy really isn't insurance. It isn't insurance any more than a diploma is an education. Like education, insurance is intangible; it can't be seen or touched.

An insurance policy is a legal contract. It is comparable to a lease you might sign for an apartment. The lease is a contract under the terms of which you agree to pay rent to the owner and the owner agrees to provide the apartment in return. The lease is probably a rather lengthy document and includes such provisions as the date when you can occupy the apartment, the date when the rent is due, who pays for utility bills and repairs, the length of the agreement, and the conditions under which it can be canceled. The terms of the lease are legally enforceable; that is, if the apartment owner doesn't live up to his or her promises voluntarily, the courts can force the owner to do so. Various law; directly affect the agreement, even though there is no reference to them is the document. For instance, a state law may prohibit the owner from discriminate eating unfairly among the tenants, and a local ordinance may forbid you as tenant to disturb the neighborhood with your stereo.

Like a lease, an insurance policy states the terms of a contract, in this case; contract between a policyholder and an insurance company. As with the lease many of the rights and duties of the parties are not expressed in the contract but are a matter of law. Such law, which provides the basic legal framework o insurance, is the subject of the first part of this chapter.

ESSENTIAL ELENIENTS OF CONTRACTS

To be legally enforceable, all contracts (including insurance policies) must have these four characteristics:

1- There must be an agreement based upon a definite offer by one party and the acceptance of that offer by the other party.

2- The two parties must be legally competent to make a contract.

3- There must be consideration, meaning that both parties must give something of value and both must receive something of value.

4- The contract must have a legal purpose.

Agreement

Insurance contracts generally are made when the insurer accepts the applicant's offer to buy a policy. This is an important point and one that sometimes is misunderstood. One might assume that an insurance company offers to sell insurance and that when a person decides to buy, the contract is effective at once, but that is not correct. If Edna Lewis clips an application for an accident policy from her newspaper, completes it, and mails it to the insurance company, has a contract been made? Assume she tumbles down a stairway a few hours later and breaks a leg. Is she covered? No, because the contract had not been completed. Her submission of the application is considered by the courts to be an offer to buy. There is no agreement between the parties until and unless her offer is accepted by the company.

In the case of life insurance, applications are not valid offers to buy unless they are accompanied by the first premium payment. What if the applicant is killed after he or she applies for the policy (and pays the first premium) but before the application is received at the company's home office? The answer may be supplied by a document known as a conditional receipt. These are used by most life insurance companies today and sometimes are a part of the application form itself. A typical one might say:

If the premium on the insurance herein applied for has been paid to the Company's agent, the insurance as provided by the policy shall be effective from the date of this application PROVIDED that the Company shall approve this application at its Home Office.

Use of the conditional receipt can bring about the strange situation of a company's having either to approve or to disapprove an application for life insurance from a person who is already dead! Is it conceivable that the company nevertheless would issue the policy and pay the claim? You might suppose not, but several company officials to whom the author has put this question have said they would lean over backward to find the application acceptable. Their explanation is that, first, it is the f:air thing to do and, second, paying the claim would probably be less costly to the company than the adverse publicity that would ensue if it were declined.

The subject of offer and acceptance is much simpler in property-casualty insurance. In that field, applications can be legal offers to buy without being accompanied by any premium payment. (As a matter of company practice, however, advance premium payment is sometimes required.)

Property-casualty insurance agents are authorized by their companies to accept offers from applicants. This point is important,

because it means that agreement between an applicant and an agent fulfills a major requirement of a legal contract. Insurance coverage can be put into effect immediately without waiting for approval by the company's home office. Furthermore, in this case neither the offer nor the acceptance has to be in writing. When Jack Sanders buys a new motor boat he phones his agent, describes the boat, and discusses insurance coverage for it. When he and the agent agree on the amount and type of protection, the agent says "OK, Jack, I am binding the coverage." From that moment, a legal and enforceable contract exists. In a few days, Jack will receive the written policy and a bill for the premium. In the meantime, insurance is effected through what is called a binder. A binder is a temporary insurance contract that is in effect until replaced by a formal, written policy. The binder does not have to be in writing, but it is good business practice for the agent to make a memorandum of it if the policy is not being prepared immediately. If there is likely to be a delay before the policy is delivered, the agent or the company may prepare a written binder setting forth the essentials of the contract.

Legal Competence of the Parties

Most people have legal capacity to make insurance contracts; they legal competence (that is, ability) to do so. Minors are the chief exception. In most states minors are now defined as those who are below age 18. Other people who are not legally competent include those who are insurance. Note that the point here concerns the ability to make an insurance contract; it does not affect the individual's status as an insured pers. minors may be covered by their parents' automobile insurance policies or by life insurance policies purchased by their parents or grandparents.

The basis of an insurance company's legal competence is the granted to it by the state where it was organized. In states other than state

a company operates on the basis of licenses issued by the vari insurance departments. The power to revoke a company's charter to operate is the source of the insurance departments' regulatory authority.

The competence of insurance agents to make contracts stems authority granted to them by the companies they represent. In agents must be licensed by the state in order to engage lawfully in the insurance business.

Consideration

The third requisite of legal contracts is the exchange of something known as consideration. The consideration given by the policy course, is the premium that he or she either pays or promises to pay five agents have been known to deliver unsolicited policies in an persuade their prospects to buy. These are not legal contracts, beca is neither agreement nor consideration.

The consideration given by the insurance company is its promise nify the insured according to the terms of the policy in case of loss.

Legal Purpose

Because a contract is a private agreement, we might suppose that it could say whatever the contracting parties want it to say.

However, if it is to be enforced by the courts it must have a legal purpose. The courts will not enforce agreements that are unlawful or are otherwise not in the public interest. This requirement rules out insurance to protect an individual who intentionally injures another party or who intentionally damages another's property. The legal purpose requirement does not prohibit insurance of losses caused by a policyholder's neglect,

however. Indeed, a great many insured fires and accidents are due to simple carelessness on the part of insured persons.

UTMOST GOOD FAITH

Insurance originated as a means of protecting the owners of ships and their cargos. Historians have found forerunners of marine insurance in the records of ancient Greece and Rome. Insurance grew as commerce expanded around the Mediterranean during the Middle Ages and then spread into Northern Europe and the British Isles. Some of the basic legal principles applicable to insurance today-particularly the doctrine of utmost good faith-reflect this heritage.

Marine insurers often were unable to examine the property they were covering. An insurer in London perhaps would cover a cargo of wool that was being shipped from Australia to South Africa in a Dutch ship. The insurer would have to rely upon the statement of the wool merchant that his cargo was in good condition (and not already badly damaged or even at the bottom of the ocean). The insurer also would need to depend on the statement that the ship was seaworthy and properly manned (and not a leaky hulk commanded by a drunken captain). Today the insurer quickly could get information from almost anywhere in the world. But consider the situation during the times before the development of radio and telephone, or even postal service. Insurance was necessary; neither ship owners nor merchants would put to sea without it. But insurers needed a guarantee that they would not be victimized by dishonest insured. The solution was to develop a system of insurance laws and practices which demanded a high degree of honesty and frankness from people dealing with insurance companies. This, the doctrine of utmost good faith, is reflected today in rules of law concerning warranty, representation, and concealment.

Warranty

A warranty is a policy provision making the insurer's responsibility conditional upon some fact or circumstance concerning the risk. A burglary policy on a jewelry store may contain a warranty that the policyholder will keep a burglar alarm system in operation whenever the store is not open for business. If there is a burglary loss at a time when the alarm system is not functioning, the insurer has no obligation to pay.

The old common law of warranty was a very harsh doctrine; it provided that any breach of warranty would void the policy, even if it were immaterial (i.e., unimportant). This point is illustrated by a case in England (the source of American common law) in 1786. A marine insurance policy had warranted that a ship would sail from Liverpool "with 50 hands or upwards." She actually sailed with 46 but picked up 6 more from an island just off the English coast. The ship was bound for Africa and the insurance was to cover its voyage from Africa to the West Indies. She reached Africa safely but was captured and lost sometime later. The court held that the warranty had been breached, and the insurer therefore did not have to pay.

In the early days of life insurance (a hundred or so years ago) policies sometimes contained a provision saying that the application was part of the policy itself and that all statements contained in the application were warranties. This provision made it possible for the companies to void the policies later on for any misstatements in the applications no matter how unintentional or inconsequential they might have been. Insurers no longer can take such action because statements by applicants for life insurance and most other forms of personal insurance protection are now construed as representations (see the next section) and not as warranties. Also, the courts and legislatures have modified the law so that warranties no longer are a trap for the unsuspecting policyholder.

Warranties still are used on some forms of business insurance, such as the burglary policy mentioned earlier. Whenever they are part of an insurance contract it is essential that their terms be complied with.

Representation

A representation is a statement made by an applicant for insurance to supply information to an insurer, or to induce it to accept a risk. Representations may be contained in formal, written applications or they may be oral. The main thrust of the law is that the facts must be as the applicant represents them to be. Misrepresentation (false statement) of a material fact makes the contract avoidable at the option of the company. When Al Kopp applies for life insurance he makes representations concerning his name, age, occupation, present insurance, and marital status. He also answers questions about piloting aircraft and being treated by doctors. If Al says that the only medical treatment he has received during the past five years was for a sprained ankle, whereas he actually has been treated several times for a heart ailment, he has misrepresented a material fact. If he dies of a heart attack one year later and the company then learns the truth, Al's widow will receive only the premiums that Al paid for the policy.

Or consider the case of Ron Kerman. Ron arranges for insurance on his new car, telling the agent he has had no motor vehicle violations or accidents during the last three years. As a matter of fact he has been picked up several times for speeding and reckless driving, and he wrecked his previous vehicle by driving it into a toll booth. A policy is issued, and Ron does it again. He totally wrecks the new car and is sued for $100,000 for injuries to other people. Does Ron's insurance company have legal grounds to refuse to pay the claims? It certainly appears to. Ron lied about important facts when he applied for the policy, and that is material misrepresentation.

Note that the misstatement has to be material. It must be such that if the company had known the true facts it would not have issued the policy on the same terms. Misrepresentation must also be within the knowledge of the applicant. If an applicant says he has no diseases but without knowing it actually has cancer, the statement cannot be used by the insurer as grounds to deny a claim.

Concealment

Concealment, the third aspect of the doctrine of utmost good faith, is the counterpart of misrepresentation. It is the failure of an applicant to disclose material facts. The laws pertaining to concealment, like those that relate to warranty and misrepresentation, stem from ocean marine insurance and from the insurer's need to rely upon the applicant to supply the facts.

Materiality is a requisite of concealment, as it is of misrepresentation. Also, concealment requires intentional withholding of information that the applicant knows is material. In addition, the information must be something that is not readily apparent to the company. It would not be concealment if the owner of a popular bar near a college campus failed to inform her fire insurance company that on Friday and Saturday nights her place is packed with students. The company is expected to realize that fact without being told. But the owner's failure to volunteer the information that she is closing the bar and converting the building to a dry cleaning shop is a concealment that could relieve the company of any obligation if the place burns down later on.

What about information secured by the insurance company? Can't it investigate and find out about the bar owner's plans? The answer is that the insurance company may secure such information or it may not. Whether or not it can and whether or not it does, the applicant has a legal responsibility to tell the truth.

Obviously, there are many borderline cases. Is it concealment in automobile insurance if an insured fails to inform the company when a 16-yearold son or daughter first starts driving? Is it misrepresentation if an applicant claims never to have had an accident when there had been a slight one four years ago? Intelligent insurance buyers do not take chances; instead, they make it a point to convey all such information to their insurers. Although the laws concerning misrepresentation and concealment are less harsh than those relating to breach of warranty, it is important to be truthful when dealing with insurance matters.

THE INDEMNITY PRINCIPLE

Insurance is strongly affected by the indemnity principle, a legal doctrine stating that the function of insurance is to repay (indemnify) insured for their actual losses. Policyholders who have adequate insurance protection are to be restored to the financial position they held before the losses occurred, but they should not profit from the insurance. The intent is to avoid giving people an incentive to cause fires or other "accidents."

The indemnity principle applies most fully to property-casualty insurance. It is less applicable to health insurance and still less to life insurance. The principle is supported by several legal requirements or policy provisions.

Insurable Interest

The indemnity principle requires people to have an insurable interest in whatever they insure. This means that insureds must be in a position to sustain financial loss if the event insured against occurs. Insurable interest is required for all types of coverage, including (with one exception) life insurance.

The insurable interest requirement means, for example, that Frank Mobley cannot buy insurance on John Clark's house. Without this requirement, Mobley might be tempted to insure Clark's house and then firebomb it. Clark's plight would be even worse if Mobley were able to take out life insurance covering his life! The insurable interest rule prevents the use of insurance as a speculative, profit-making device.

The sole exception to the requirement of insurable interest is when people purchase life insurance on their own lives. In spite of the fact that they will not be around to suffer financial loss when the insured event occurs, they are permitted to buy the policies. The insurable interest requirement is ignored because even without it the spirit of the indemnity principle will not be violated. In this case, the insureds will not profit from the loss even though insurable interest is absent! Incidentally, insured persons normally can name whomever they please as beneficiaries, the persons who receive the policy proceeds geneficiaries do not have to have an insurable interest.

There are two types of insurable interest in life insurance. In the first, insurable interest is based upon close family relationships, such as between husband and wife or parent and child. The laws of the various states are neither clear nor consistent as to what other family relationships would suffice. Probably grandparents have insurable interest in the lives of their grandchildren, but it is doubtful that cousins, for instance, can legally insure one another. In any case where dispute arose the final decision would rest with the courts.

The second type of insurable interest in life insurance is based upon a specific financial interest in the insured person. A creditor has an insurable interest in the life of a debtor. If one person lends another $5,000, then the lender can take out that much insurance on the borrower's life. For the same reason, a business firm has an insurable

interest in the lives of key employees who are particularly valuable to the firm. In this situation, as in the case of a loan, the party taking out the policy has a financial interest in the life of the person named in the insurance contract. Coverage of both of these risks by means of life insurance is common today.

In non-life insurance the usual sources of insurable interest are the ownership of property and the obligation to pay debts (such as medical bills). Obviously, property owners will suffer loss if their property is destroyed. Other property relationships that can furnish insurable interest may be less apparent. For instance, a bank holding a mortgage on a house has an insurable interest in the house because the property is the security behind the mortgage. In addition, people who have temporary possession of other people's property have an insurable interest in that property. Such persons are called bailees. A man who owns a parking garage is an example. He is responsible for damage to automobiles caused by the negligence of his employees. Because he is in a position to sustain financial loss if the property is damaged, he can insure it.

Insurable interest in life insurance must exist at the inception of the policy, but it does not have to prevail at the time of a loss. Thus, a wife who owns a policy covering the life of her husband can keep the policy even if they are divorced. In property-casualty insurance the opposite is true; insurable interest does not have to exist when the coverage is put into effect, but it is required at the time of loss. You can insure today an automobile that you expect to buy tomorrow. Of course, you will be unable to recover for any damage to the car today, but setting up the policy in advance guards against delay in getting it into force tomorrow when you will need the coverage.

Loss Measurement

To apply fully the indemnity principle, there must be some basis of loss measurement, a way to determine the amount of payment when a loss occurs. If Ben Stone insures his motor boat for $10,000 when it really is worth only $5,000, he should recover only the latter amount if the boat is destroyed.

The traditional basis of loss measurement in the property insurance field is actual cash value. That is, many policies state that if the insured property is damaged or destroyed the amount of the payment will be the property's actual cash value. Strangely, the policies do not say what actual cash value means; the definition is supplied by the courts and through commonly accepted insurance practices. The most frequently used definition is that actual cash value equals the cost of replacing the destroyed property with new property, minus an allowance for depreciation. Depreciation is the amount by which something declines in value due to its age and use. It is deducted from replacement cost so that the loss payment will be equal to the current value of the property and the insured will not profit from the loss.'

Assume that Barbara's Restaurant owns and occupies a twenty-year-old building in a declining neighborhood. The cost of replacing the building at current prices would be $400,000. If the building has depreciated by 25%, its actual cash value is $400,000 minus 25%, or $300,000. If the building is insured on the actual cash value basis, $300,000 is the maximum payable for a loss, even if Barbara buys more than that amount of insurance.

Automobiles are insured on the same basis, but in this case actual cash value means the market value of similar cars. If an insured's two-year-old Ford is destroyed, the basis of settling the claim will be the going

price of similar two-year-old Fords. As with the coverage on Barbara's Restaurant, measuring the loss in terms of the property's actual cash value helps prevent paying more than the property is worth.

Property insurance policies sometimes cover the full replacement cost of insured property, with no deduction for depreciation. The manner in which dwellings may be insured on this basis is described in a later chapter.

The indemnity principle is not always applied to the measurement of insured losses. Life insurance policies normally pay the total amount of insurance upon the death of the covered person. If a man's life is insured for $100,000 that is the amount the baneficiary will receive; there is no attempt to measure the man's "value" after he dies. Most health insurance policies use the same approach, paying a stated amount for each month of disability or each day o hospitalization, for instance.

Subrogation

Under the laws of negligence, property owners can sue those who damage to their property. If a suit is successful, the property owner is entitled to reimbursement by the negligent party. But let us suppose the pr owner has an insurance policy covering damage to the property. To him or her to recover both from the lawsuit and from the insurance clearly violate the indemnity principle. Subrogation prevents such d recovery. Subrogation is a legal principle that provides that to the extent insurer has paid for a loss, the insurer obtains the right of its policyholder to recover from any third party who caused the loss. Most property insu policies contain a provision reinforcing this principle.

Let's say that on a beautiful fall Saturday afternoon Tom Archer rake pile of leaves in his backyard, starts a bonfire, and then goes into his house and turns on his television set. Becoming engrossed in a football

game forgets his leaf burning. The fire quickly spreads along the leaf-co ground and soon sets ablaze the garage of his next-door neighbor, Bill Carpenter. Before the fire trucks arrive, $1,800 worth of damage is done to the garage. In the absence of subrogation, Carpenter would be able to re, twice, once from his fire insurance and again by suing Archer. Subrog prevents this, because after the fire insurer pays Carpenter, it can sue A: in Carpenter's name. If the suit is successful, the conclusion will be Carpenter will be paid for his loss, his insurance company will be reimbu and Archer, who caused the whole problem, will be left bearing the los

Subrogation is available to insurers only against third parties. An insurance company cannot subrogate against its own policyholders, even for losses due to their own carelessness. Subrogation usually is not applicable to health insurance and never applies to life insurance. If a man who has a large am of life insurance is killed in a plane crash, for example, his family is entitl' any settlement that a court orders the airline to pay; the family's right of recovery is not transferred to the life insurance company.

"Other Insurance" Provisions

"Other insurance" provisions are policy clauses limiting the amount of payment if an insured has other, similar insurance protection. Without such provisions people could get around the other measures that support the indemnity principle by buying two or more policies. The most common "other insurance" provision states that if two or more policies cover the same loss they are to share in its payment on a pro-rata basis. Assume a family insures its house under two policies for a total of $90,000. Company A provides $30,000 under one policy, and Company B provides $60,000 under the other policy. The two companies will share the payment of losses in proportion to the amount of the total coverage which each one furnishes. Company A provides one-third of the coverage and will pay one-third of each loss; Company B provides two-thirds of the coverage and will pay two-thirds of each loss.

Another type of "other insurance" clause states that the coverage provided

by the policy applies only in excess of any other collectible insurance. In this case, if other insurance is available to pay for the full loss, the first policy pays nothing. There are two other types of "other insurance" clauses. One states that the policy obtained earliest provides primary protection, while policies obtained at later dates furnish excess coverage. The remaining type of "other insurance" provision simply prohibits any other insurance coverage of the same kind.

Life insurance contracts never contain "other insurance" provisions. Because there is no satisfactory way to measure the loss in this case, there is no way to apportion it among policies. Each life insurance policy pays the full face amount, regardless of the number of other policies in force.

Health insurance contracts until recent years rarely contained "other insurance" provisions. Now, such limitations are becoming more common, especially on policies covering medical expenses. When these provisions are not included, benefits can exceed the actual loss.

THE LIABILITY RISK

The basic nature of most of the risks we face is rather obvious and uncomplicated. This is true of risks involving property loss, accident or sickness expense, and loss of income, but the liability risk is an exception. The liability risk is based on the laws of negligence, laws that provide that in certain situations a person may have to pay someone else for having injured them or having damaged their property. Those who have such an obligation are said to be legally liable. If they have liability insurance of the proper type and amount, it will pay the money for them. Because this

risk is based on the laws of negligence, some knowledge of those laws will help clarify the nature of the risk and the function of liability insurance. We should realize that these laws are extremely complex and that the following explanation is greatly simplified.

Negligence: A Civil Wrong

Under our system of laws there are two major classes of wrongdoing: criminal wrongs and civil wrongs. Criminal wrongs are public wrongs; that is, they are wrongs for which legal penalties have been established as a means of protecting the public welfare. Although they may be directed against individuals, criminal wrongs (such as theft, arson, or murder) are considered an injury to the general public. As such, they are punishable by the state.

Civil wrongs, on the other hand, involve injury to a specific party only. This class of wrongdoing is not punishable by the state. Instead, the remedy takes the form of court action ("civil action") instituted by the injured party. There are two main categories of civil wrongs: those involving contracts, and torts. Civil wrongs involving contracts are mainly breaches of contract and breaches of warranty. An example would be the failure of a trucking firm to deliver merchandise that it had contracted to haul. Torts, which are civil wrongs not relating to contracts, include such things as libel, slander, assault, trespass, false arrest, and, most important, negligence.

Negligence is failure to use the proper degree of care necessary to prevent injuring other people. (In this context the injury could be either bodily injury or property damage.) A person who negligently injures another may have a legally enforceable obligation to pay a sum of money to the injured party, or, in legal terms, the negligent party may be "legally liable" to pay damages.

Damages are sums awarded by a court to pay for injuries sustained by another party. Ordinarily, damages are **compensatory, meaning** that they are intended to compensate the injured party for the injury. Compensatory damages are composed of special damages and general damages. **Special damages** pay for the injured party's actual expenses, including the cost of property repairs, medical treatment, and lost earnings. **General damages** compensate for losses that are not directly measurable in dollars, such as the injured person's pain and suffering. General damages may also include payment for such things as facial scars or loss of the ability to bear children.

In some cases, courts award additional amounts over and above compensatory damages in order to punish outrageously reckless acts and to deter future conduct of its type. Such awards, called **punitive damages**, were granted very rarely in the past; they now are given more frequently. A spectacular example was a California case in which a jury awarded $125 million in punitive damages against Ford Motor Company for an automobile crash involving a 1972 Pinto. (The amount later was reduced to $3.5 million.)

Although some liability cases result in the payment of enormous sums, many do not. A study of all of the verdicts in the Chicago area during the period 1960 through 1979 showed that plaintiffs (the parties doing the suing) won only slightly over half of the time and that half of the winners received less than $7,900 in 1979 dollars. In the 19,000 cases tried during the period, 157 plaintiffs received "blockbuster" awards totaling $159 million; this amount was 40 percent of the entire sum awarded. Auto accidents accounted for about two-thirds of the trials and for the bulk of the smaller cases. Most of the large awards were made in cases involving products liability or professional liability.3

The legal process through which lawsuits are pressed (and defended against) is a technical one that usually requires both parties to be represented by attorneys. It is an adversary process. Each side is pitted against the other, and the outcome may be affected by the skill of the opposing attorneys, as well as by the facts of the case. The great majority of negligence cases are settled out of court. Only about 2% are litigated to a conclusion; in the others, settlements are reached by negotiation and compromise, but with an eye on the possible outcome if the case were tried in court.

Elements of Negligence

Negligence is made up of four elements, each of which must exist before a court will order the payment of damages. The four elements are (a) the duty to act in a certain manner, a manner that is reasonable and pruclent under the circumstances prevailing; (b) the failure to act in that manner; (c) an injury; and (d) a direct causal relationship between the defendant's carelessness and the plaintiff's injuries, that is, between (b) and (c).

Consider a negligence suit involving an automobile collision. The injured person (Parker) was parked beside a country highway at 11:30 P.M. when his car was struck from behind by a car driven by another person, Driver. Parker, who was sitting in his car, claims that he is totally disabled because of the resulting injury to his back. Parker's attorney is seeking to establish that all of the elements of negligence are present in the case. First, he describes to the court Driver's duty to operate his car in a safe manner, driving at a speed within the legal limit, and keeping sober and alert. Second, he contends that Driver failed to act in such a manner. As evidence, he points to the testimony of the investigating police officer who says there were indications that Driver had been drinking and was exceeding the posted speed limit. Third, Parker's

injuries are described. Evidence is presented concerning the cost to repair his car, the expenses of treating his sore back, and the amount of income he will lose because of his disability. Finally, Parker's attorney argues that the injuries were caused solely by Driver's failure to drive safely. He says that his client was legally parked beside the highway, and he contends that there would have been no collision, and therefore no injuries, if Driver had kept his car unde.

As the case continues, Driver's attorney seeks to disprove some of these contentions. He wants to prove that his client was not negligent. He says that Driver was neither speeding nor intoxicated. He challenges the extent of the injury, raising questions as to how sore Parker's back really is. Anyway, he argues, if Parker really is disabled it is the result of an injury the evening prior to the collision, when Parker fell off a bar stool at a neighborhood tavern. Thus, the attorney for Driver contends that there is not a valid case against his client, because one or more of the elements of negligence cannot be proved. Parker's attorney, incidentally, is hired and paid by Parker. Driver's attorney, assuming Driver carries liability insurance, is furnished by the insurance company. In addition to paying liability judgments against the insured, liability insurance provides and pays the entire cost of defending against suits alleging liability. This aspect of the coverage is important because legal expenses can be very large. If the defense is successful, the insurer may make little or no payment to the plaintiff on the policyholder's behalf; yet, the cost of handling the case may be hundreds or even thousands of dollars.

Defenses Against Negligence

Defendants who are found to have been negligent may still not be legally litable. Several defenses are possible, depending on the circumstances of the case. The most important of these defenses are contributory negligence (or comparative negligence) and assumption of risk.

According to the **contributory negligence rule**, plaintiffs cannot recover if they contributed to their injuries through negligence of their own.

In the previous example, even if Driver had been negligent, he might be able to avoid liability by showing that Parker was partly at fault. Perhaps it could be shown that Parker had not pulled his car entirely off the highway and had not kept his lights on. Another example of the contributory negligence rule would be a jogger who carelessly veers into the path of a moving car and is unable to recover damages even though the driver of the car was also negligent.

An exception to the contributory negligence defense is furnished by the **last clear chance rule**. This rule provides that a negligent injured person, in spite of his or her negligence, still can recover damages if the defendant had a last clear chance to avoid the accident. The rule reflects the principle that each person has a duty to avoid injuring others, a duty that persists even if the others are guilty of contributory negligence. The last clear chance rule is illustrated by a case in which a motorist negligently lets his car run out of gas. The last drop in the tank carries the car halfway across a railroad track, where it is clearly in view of an approaching train. If the engineer has plenty of time to stop before hitting the car but fails to apply the brakes quickly enough, the railroad may be legally liable for the accident.

Strict application of the contributory negligence rule can have the effect of barring payment to a seriously injured person when a minor element of carelessness by that person had contributed to the accident. The rule therefore can have harsh and apparently unfair effects. As a result, a different rule the **comparative negligence rule** has been substituted for contributory negligence in more than half of the states. According to it, negligence by the injured party does not bar recovery, but

instead it reduces the amount of the damages. For instance, a person found to have been responsible for 20% of the negligence may be entitled to 80% of the damages that that person would have received if he or she had not been negligent at all. Thus a comparative negligence rule reduces the payment to a party who was partly at fault, rather than preventing any payment at all.

The **assumption of risk rule** is another defense against negligence. This rule bars recovery if the plaintiff either expressly or by implication accepted the chance of being injured by whatever caused the injury. In the earlier example the defense might argue that Parker assumed the risk when he chose to remain in the parked car under hazardous conditions. The assumption of risk rule has also been used in cases of automobile passengers who willingly accepted the risk of injury when they chose to ride with drivers who obviously were intoxicated.

LAWS AFFECTING LIABILITY FOR AUTOMOBILE ACCIDENTS

Legal liability is determined primarily by the basic rules of negligence have just been summarized. However, various states have modified rules as they pertain to liability for automobile accidents. Two areas is such modifications have been made concern (a) an automobile own responsibility for the use of a car by others and (b) a driver's responsibility to his or her passengers. Another and more sweeping modification has been made by the states that have adopted automobile no-fault laws. Because the next chapter introduces the topic of automobile insurance, we conclude this chapter with a brief examination of these modifications of the laws of negligence. In later chapters, other modifications of the laws of negligence are reviewed, including those pertaining to the liability of landowners and the sellers of products.

Vicarious Liability

By law, there are certain situations in which one party is responsible for the actions of others; such responsibility is known as vicarious liability. One ver, common instance of this concerns employers' responsibility for the acts of their employees. For instance, if part of an employee's job is to operate c vehicle, the employer generally is responsible if other people are injurec because of the employee's carelessness. In such a case, the employee is acting as the employer's agent, making the employer vicariously liable. Thus a trucking company is responsible for accidents caused by the negligence of its drivers.

Some states have adopted the "family purpose doctrine," which makes the owner of a family car responsible for its negligent operation by any member of the family. This doctrine is based on the idea that the head of a family is responsible for providing the family's necessities, including transportation. Rather similar to the family purpose doctrine are laws enacted by a number of states which make the person who signs a minor's application for a driver's license responsible for accidents caused by the minor. In those states the person signing the application (usually a parent) is vicariously liable for the minor's operation not only of the family car, but of other cars as well. About a dozen states have gone beyond the family purpose doctrine and have made the owner of an automobile responsible for its negligent operation by any other person using the car with permission, whether that person is a member of the family or not. These laws are called "permissive use" statutes.

Guest Statutes

According the common law rules of negligence, a driver is obligated only to use reasonable care in operating the car. Some of the states have laws that limit the driver's responsibility, so far as passengers

in the car are concerned. Called "guest statutes," these laws provide that the driver is not liable for injuries sustained by guest passengers unless gross negligence can be proved. Gross negligence is defined as willful and wanton misconduct.

The intent of the guest statutes is to make it more difficult to defraud automobile insurers. Assume that Driver and Rider are friends. If Rider is injured in Driver's car, the two may be tempted to make up a story to show that the injuries resulted from Driver's carelessness. Their purpose, of course, would be to wring a generous settlement from Driver's liability insurer. But gross negligence is harder to prove than ordinary negligence. If there is a guest statute in their state, the two friends will final their scheme much more difficult to carry off.

Guest statutes have been repealed in a number of states during recent years. This action is in line with a trend that has made legal liability easier to establish, a trend that also has made liability insurance more valuable and more costly.

Automobile No-Fault Laws

Since the mid-1960s, controversy has grown concerning the continued use of legal liability as a system of determining who should pay the costs of automobile accidents. As an alternative, various no-fault systems have been proposed. If a pure no-fault system were adopted, the concept of legal liability for auto accidents would be completely abolished. Instead, all motorists would have to carry insurance that would pay for their own injuries. Although none of the states has passed a pure no-fault law, fifteen of them now have partial or modified no-fault laws. In those states lawsuits are not permitted for automobile accident injuries unless the injuries are serious enough to exceed a certain "threshold" level. Less serious injuries are paid for by special no-fault insurance.

Because the laws still permit lawsuits for serious injuries, the liability risk still prevails, even in the states that have no fault laws, and automobile liability insurance continues to be an essential form of protection.

This brief introduction to the no-fault concept is presented at this point in order to indicate its relationship to auto liability insurance, which is the principal topic of the next chapter. A fuller analysis of no-fault, including the arguments pro and con, is presented in Chapter 7.

IMPORTANT TERMS

binder	actual cash value	punitive damages
doctrine of utmost	depreciation	contributory negli
good faith	subrogation	gence rule
warranty	“other insurance”	last clear chance rule
representation	provlslons	comparative negli
concealment	liability risk	gence rule
indemnity principle	damages	assumption of risk
insurable interest	compensatore	rule
beneficiary	damages	vicarious liability
bailee	special damages	guest statutes
loss measurement	general damages	no-fault system

KEY POINTS TO REMEMBER

1. An insurance policy is a legal contract. Some of the rights and obligations of the parties are stated in the contract; others of them are provided by laws and by court decisions.
2. Like other contracts, insurance policies require (a) agreement, (b) legal capacity, (c) consideration, and (d) legal purpose.
3. In insurance the offer is made by the applicant and acceptance is made by the company. Property-casualty agents have authority to accept offers on behalf of their companies. The temporary policies that they provide on this basis are called binders.
4. Insurance policies are contracts of utmost good faith, meaning that the parties to them are held to especially high standards of fairness and honesty.
5. The doctrine of utmost good faith is supported by laws concerning warranty, representation, and concealment.
6. The indemnity principle provides that the function of insurance is to reimburse insureds for their actual losses, but is not to pay in excess of actual losses.
7. The indemnity principle is supported by laws, practices, and policy provisions pertaining to (a) insurable interest, (b) loss measurement, (c) subrogation, and (d) other insurance.
8. The insurable interest rule requires the insured to be in a position to sustain financial loss if the insured event occurs.
9. The traditional measure of property insurance loss is actual cash value, generally meaning replacement cost new minus depreciation.
10. Subrogation transfers to the insurance company that has paid for a loss the policyholder's right to recover from a negligent third party.

11. "Other insurance" provisions prevent recovery in excess of the loss when the insured has mQre than one property-casualty insurance policy.

12. Legal liability stems from negligence. The essential elements of negligence are (a) duty to act, (b) failure to so act, (c) injury, and (d) direct cause. The most common defenses are contributory negligence and assumption of risk.

REVIEW QUESTIONS

1. In the making of insurance contracts, (a) how is agreement reached, and (b) what consideration is exchanged?
2. How does a conditional receipt affect the making of a life insurance contract?
3. What is the doctrine of utmost good faith, and what laws and practices support it in insurance?
4. What is the indemnity principle? What insurance practices and policy provisions support it?
5. Give an example of subrogation and explain its function.
6. Why is insurable interest required?
7. Carson bought a new motion picture theater in 1970 for $300,000. Assuming he has aufficient insurance to cover it, what factors will determine the amount of his recovery if the theater is completely destroyed today?
8. What two principal things does liability insurance do?
9. What must a plaintiff prove in order to recover damages under the laws of negligence?
10. How do contributory negligence, comparative negligence, and assumption of risk relate to legal liability?

DISCUSSION QUESTIONS

1. Why do you suppose life insurance companies don't give their agents binding authority as property-casualty companies do? Can you think of any differences in the nature of the protection that would help explain this?
2. Liability insurance pays for injury that results from the insured's neglect. Should this be permitted? Doesn't it encourage people to be careless?
3. The chapter described how the actual cash value of a damaged restaurant and a damaged car would be determined. Why isn't the former determined in the same manner as the latter?
4. If it is not necessary to apply the indemnity principle in measuring the loss in life insurance, why must it be applied in measuring the loss in property insurance?
5. Why aren't insurance companies permitted to subrogate against their own policyholders if they cause losses through their own carelessness?
6. Why are "other insurance" clauses necessary in view of the premiums that the insurers collect? For instance, if Gloria Williams wants to pay for two $50,000 policies on her $50,000 house, why shouldn't she be able to collect $100,000 if the house is destroyed?
7. A motel guest is drowned in the motel swimming pool. How might the defenses of (a) contributory negligence or (b) assumption of risk be claimed?
8. In filling out an application for a $100,000 life insurance policy, Frank O'Connor stated that he was in good health, had seen a physician only for routine check-ups during the preceding five

years, had a total of $30,000 life insurance in force, and had no other insurance application pending. O'Connor died shortly after the policy was issued. The company refused to pay the policy proceeds to the named beneficiary when it discovered that O'Connor had been treated for high blood pressure five times during the six months preceding his application. Also, at the same time that he applied for his policy, O'Connor applied for a $200,000 policy from another company.

(a) Why do you suppose that the application form asks how much insurance the applicant has and whether or not he has applied for more?

(b) On what legal ground will the company base its refusal to pay the proceeds?

(c) Do you think the company's refusal can be justified legally?

9. John Lennon's record distributor, Warner Brothers, applied for $1 million of life insurance on Lennon's life prior to his murder. What legal principle may have been raised? Do you think the policy was issued?

CHAPTER 5

LIFE INSURANCE FUNDAMENTALS PART ONE

CHAPTER OBJECTIVES

After studying this chapter, you should be able to:

1. State the principal purpose of life insurance and explain how life insurance operates to fulfill this purpose.
2. Compare and contrast several characteristics of life insurance.
3. Identify and describe the three basic kinds of life insurance.
4. Show how two or more kinds of life insurance can be combined in a single policy.
5. Specify several new life insurance policies and indicate how they differ from conventional policies.

Most college students aren't very interested in life insurance. And they don't see why people spend so much money for it. They can understand why during a recent year people spent $46 billion on new cars, for instance, but they don't see why people would spend $41 billion for life insurance during the same year. What would motivate people to spend that much money for something as unexciting as life insurance?

The answer is that by insuring their lives, people insure their future incomes. And future income, especially for young families, is tremendously valuable; the possibility of its being interrupted is a major risk. Consider the importance of Charlier's income to the Newlin family whom we met in the last chapter. You may recall that Charlie and Ruth have a 2-year-old daughter, Julie. They hope to have another child before long. Ruth works part-time and plans to return to a full-time job when the children are older. Until then, she and the children will rely on Charlie's income for their chief support. His income will pay for their housing, food, clothing, transportation, and education. Charlie is earning $20,000 a year now and expects substantial increases during the years ahead. He intends to retire when he reaches 65, 40 years from now. We, of course, don't know that his total earnings will be during that period, but let's say that they average $25,000 a year after taxes. For the 40 years, that would be a total of $1 million. We don't suggest that Charlie needs $1 million of life insurance. We will see later that a much smaller amount than that will serve the Newlins very well. The point is that the ability to earn income is a major asset. Families rely upon income, and the possibility of its being interrupted by death is a major risk, usually their most important one. They buy life insurance as a way of handling that risk. People insure their homes because they are valuable and because the possibility of losing them is an important risk. They insure their live (more specifically, their earning ability) Primarily for the same reasons.

It is not surprising that few students are interested in life insurance, especially if they are unmarried. They are concerned about how they will earn an income, not about whether it may be interrupted. They are interested in buying cars and clothes and airline tickets, not life insurance. But things have a way of changing. Students graduate, find jobs, get married, and start families. Three people out of four are married by age 25. And young married people buy life insurance. Over 70% of all life insurance policies are purchased by people under age 35, and 40% by those 24 or younger. Very few of these people had given a thought to life insurance while they were in school; yet, just a short time later they decided that it was important to them and to their future. This chapter and the next two describe the nature and uses of life insurance. The provisions and costs of various policies are compared, with emphasis on how they relate to the decisions life insurance buyers must make.

CHARACTERISTIC OF LIFE INSURANCE

In several important ways, life insurance differs from other kinds of insurance. Compare it with auto insurance, which, as we know, has these characteristics: (a) policies are written for short terms (one year at the most); (b) policies have no value when they expire; (c) the events that are insured may happen numerous times to the same policyholder; and (d) the policies are contracts of indemnity. In contrast, life insurance has these features: (a) most policies are written for long terms, many for life; (b) most policies have cash values; (c) the event insured-the mortality risk-is unique; and (d) life insurance policies are not contracts of indemnity. These points are summarized by Table 5-1. We will take a look at each of these characteristics of life insurance.

Long-Term Policies

Life insurance policies usually cover long periods, frequently from the time of purchase until the insured reaches age 65 and often longer. One implication of this is that purchase decision are more crucial in life

insurance than in other lines of coverage. It usually is easy to replace one auto or homeowners policy with another if the policyholders change their minds about the coverage they want or if they prefer a different insurer. But, as we will see, replacement of a life insurance policy may interrupt a savings program. It also may require paying higher rates because the policyholder is older than when the original policy was purchased. Worse yet, ad health may make another policy either more costly or impossible to secure. For these reasons, it is especially important to make the right choices when buying life insurance.

Cash Value Policies

Cash values are sums payable to policyholders who choose to discontinue their insurance. The majority of individually purchased life insurance policies provide cash values; auto policies never do. (Some life insurance policies-term policies-do not provide cash values.) Cash values increase year by year; the amounts are shown in the policies. Policyholders also can borrow any amount up to the full cash value of their policies; interest rates for such loans are stated in the policies.

Table 5-1 Comparison of Auto Insurance and Life Insurance

Characteristic	Auto	Life
Long-term policies	No	Usually
Policies have cash value	No	Usually
Covers the mortality risk	No	Yes
Contracts of Indemnity	Yes	No

Because of its accumulation of cash values, life insurance (other than term policies) can be thought of as combining savings with protection, and it often is purchased with that dual purpose in mind. This

is a unique characteristic of life insurance and one that must be understood in order to comprehend the ways in which life insurance can be used.

The Mortality Risk

Life insurance covers the **mortality risk**, the financial uncertainty associated with dying. Other kinds of insurance cover losses that may happen numerous times to the same policyholder; the losses usually are partial and frequently are not very serious. In contrast, the event covered by life insurance is certain to happen-but only once-to every person. The uncertainty. if course, is the time at which it will happen. Basically, life insurance protects against the risk of dying too soon. That is, the primary financial risk from the view point of the family is that death may cut off their source of income while the income still is needed, especially for raising children. This is the principal risk dealt with by life insurance.5

Life insurance transfers the mortality risk to an insurance company. The Insurance company, handling a large number oh similar risks, computes the premiums It must charge on the basis of records of large groups of people. These records are compiled in **mortality tables** that show the probability of living and dying at various ages. Actuaries, mathematical experts who work for insurance companies, have compiled many different mortality tables. Each is based on actual records of the number of people living' and dying at various ages. Experience has shown that mortality data from the past can be relied upon to predict future mortality rates quite accurately.

Table 5-2 shows parts of one mortality table, It starts with a group of 10 million lives and shows the number dying at various ages, the mortality rate per 1,000 at each age, and the number of years of life expectancy at each age. We should realize that the mortality data are

averages; mortality tables are not applicable to any one individual but only to groups large enough to be subject to the law of large numbers. This table assumes that the last survivors of the group live to age 99. Of course, a few people actually live to age SOD and beyond and sonic mortality tables run to a higher age, hut any table must have sortie arbitrary limit.

We can show how a mortality table is used as a basis for life insurance premiums with a simplified example. Assume that 100,000 people, each 20 years old, are to be issued one—year $1,000 life insurance policies. The polities are to have no cash values and are simply to pay $1,000 if' the insured person dies during the year. The table indicates that the insurer can expect 1.79 deaths per 1000 persons of this age. The is the indicates mortality rate for 20-year-olds, the ratio of the number dying during a year to the lumber living at the beginning of the year. With a mortality rate of 1.79, 179 of the 100,000 insured people would die during the year. If the company charged each person $1.79 at the beginning of the year, the company's premium income would be 100,000 × $1.79, or $179,000. This amount would be sufficient to pay $1,000 far each of the policyholders who died during the year. The next year those who survived would be 21 years old, and their average mortality rate would have risen to 1.83. If time policies were renewed, each person would therefore have to be charged $1.83 in order to provide $ 1,000 to the beneficiaries of those who died during lie second sear. In each succeeding year, lie premium rates would continue to rise along mortality rate.

Table 5-2 Standard Mortality Table

Age	*Number Living*	*Number Dying*	*Deaths Per 1,000*	*Expectancy (in years)*
0	10,000,000	70,800	7.08	68.3
1	9,929,200	17,475	1.76	67.78
2	9,911,725	15,066	1.52	66.90
3	9,896,659	14,449	1.46	66.00
4	9.882,210	13,835	1.40	65.10
10	9,805,870	11,865	1.21	59.58
20	9,664,994	17,300	1.79	50.37
21	9,647,694	17,655	1.83	49.46
22	9,630,039	17,912	1.86	48.55
23	9,612,127	18,167	1.89	47.64
24	9,593,960	18,324	1.91	46.73
25	9,575,636	18,481	1.93	45.82
30	9,480,358	20,193	2.13	41.25
40	9,241,359	32,622	3.53	32.18
50	8,762,306	79,902	8.32	23.63
60	7,698,698	156,592	20.34	16.12
70	5,592,012	278,426	49.79	10.12
80	2,626,372	288,848	109.98	5.85
90	468,174	106,809	228.14	3.06
97	37,787	18,456	488.42	1.18
98	19,331	12,916	688.15	0.83
99	6,415	6,415	1,00.00	0.50

Excepts from 1958 Commissioners Standard Mortality Table.

Several important things were omitted from the preceding example. First, the fact the company will invest the premiums it receives and will earn interest on the investment was ignored. Second company operating expenses (such as agents' commissions, salaries, rent, and

taxes) were not taken Into account. The first of these would permit a reduction in the computed rates; the second would require an increase. Another factor that would have to be taken into account is the likelihood hat actual mortality rates will vary Sonic what from the rates predicted oil tile basis of past experience. Any insurer that charged exactly wham, lie mortality table indicated would suffer if it encountered several years of worse-than-average mortality experience. Therefore, a safety margin will be included in the rates that are charged.

Some policies (one-year term contracts) actually ire priced in a hammer similar to the one used in the example. Most are not however, because if that system is used the cost must continue to rise as the policyholders grow older arid the mortality ate climbs. Arid, as Table 5-2 shows, the mortality mate rises steeply after about age 30. At age 40 it's almost twice as high as at age 20. and at age 60 it is more than ten times as high. If the one-year term rate system (the system just described) were used for long-term policies, rates would be very low at first but later would become so expensive that many policyholders would have to consider dropping their policies. Their decisions at this point would be affected by a crucial factor: the state of their health. Many who were in good health would let their policies lapse. But most of those who didn't expect to live much longer would keep on paying the premiums, even though they had become very high. As a result, adverse selection would develop; the group no longer would have average mortality rates. An increasing percentage of the people would be in poor health, and the mortality rate would climb higher, causing more of the healthy members to drop out, causing higher mortality rates, and on and on in a vicious circle. Thus, the one-year-term rate system, with cost step-up year by year, cannot be relied upon for policies providing long-term protection. This has led to development of level premium life insurance.

Most life insurance today is written at a level premium. The premium remains the same over the term of the policy: it is the same at age 60 as it was when a policy was purchased at age 30. The level' premium averages out the mortality cost: it is higher than mortality rates would indicate in the early years of the policy and much less in later years. This practice eliminates the problem oh rising insurance cost for the policyholder. Also, the level premium system is tile source of the savings element of cash value policies. The over-payments (relative to indicated mortality costs) that are collected during the early years are accumulated by the company and invested in bonds, mortgage, and real estate. These funds increase during the term of the policy and are the basis of the increasing cash values.

Figure 5-1 shows how a long-term policy can have a Level annual premium. For a number of years at the beginning the annual cost for the level premium policy is higher than the premium for successive one-year term policies. But the cost of term policies increases .0 keep pace with the rising mortality rate and eventually must climb above the cost of level premium policies. The illustration also shows the source of cash values. As previously stated, the excess payments during the early years of a level premium policy (time shaded area ml Figure 5-1) build a reserve. The reserve, which is used to offset the undercharge in later years when the level premium payments would be insufficient by themselves, is the source of the cash values.

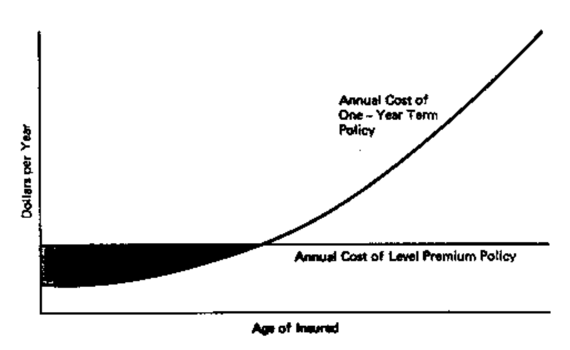


Figure 5-1 The Level Premium Concept

Not Contracts of Indemnity

According to time indemnity principle, as outlined in Chapter 4, the function of most kinds of insurance is to repay the policyholder for time actual amount of an insured loss. The principle is supported by law and by insurance policy provisions and practices that prevent the insured from profiting from the occurrence of a loss. We should realize that life insurance policies are not contracts of indemnity. Chapter 4 pointed out that several of the measures that support the indemnity principle do not apply to life insurance: (a) no attempt is made to measure the actual loss resulting from the policyholders death; (b) a life insurance company cannot subrogate against another party responsible for an insured's death: and (c) life insurance policies do not contain other insurance" provisions. (The fourth measure which supports tile indemnity principle, the insurable interest requirement, does apply to life insurance, however.)

The fact that life insurance policies are not indemnity contracts is important in another way: It makes it possible for a person to use life insurance for estate creation. An estate is the property that a person owns. One reason for acquiring and estate is to furnish financial security for one's dependents. By means of a will or other legal arrangement, the estate can be transferred to others upon the death of the owner. By purchasing a life insurance policy, the buyer immediately creates an estate in the amount of the policy.

The Newlin family can illustrate this point. Charlie's income is supporting Ruth and 2-year-old Julie. One evening Charlie begins to think, "What would happen to Ruth and Julie if I should die? Would they have decently? Would they be able to keep time house and the car?" He talks it over with Ruth. A lift insurance agent named Bill Scott has been wanting to talk with them, and they decide it might be a good idea to

learn what he bias to say. They contact Mr. Scott, and he comes to their home the next evening.

Charlie and Ruth at first a re rather surprised by their conversation with Mr. Scott. They had expected him to try to sell them a policy right away. Instead, he talks with them about themselves, their plans, and their reasons or needing insurance. They discuss Charlie's job and his future prospects with Metro Industries. They talk about Ruth's part-time job and her qualifications for full-time work. Bill Scott inquires about their income and their present life insurance. Finally, they discuss the family's need for protection. The most important need, all agree, is to provide adequate income while Julie is growing up. Charlie and Ruth hope that Julie will go to college, but that goal is a long way away. The first need is for income between now and her graduation from high school. If she graduates at age 18, that will be 16 years from now. "Let's concentrate on that income need first," says Bill.

Charlie does some quick calculations, lie is earning about $2,000 a month. Sixteen years is 192 months, and 192 times $2,000 is $384000. 'There's no way we can insure that," he thinks But Bill's approach is different, He suggests that if Ruth had $1,200 a month, that amount plus her part-time earnings might do the job. It wouldn't be easy for her, but it would be possible to lead a decent sort of lift with that much income. Then Bill points out that Ruth would receive about $700 a month in social security benefits, In addition, through Metro Industries Charlie has $15,000 of group life insurance. If the proceeds of that policy were paid to Ruth in monthly installments for the 16 years, they would provide about $100 a month, That plus the social security benefits would total $800 a month, leaving $400 to be supplied in order to reach their $1,200 goal. A $60,000 policy, the agent says, would guarantee the needed $400 a month for 16 years.

Returning to the point about estate creation, if Charlie buys the $60,000 policy, he will immediately add that much to his estate. There is no other way (short of robbing a bank!) that he could do this. Of course, he could set out to save the $60,000 from his current earnings. If he saved enough over a long enough period of time and invested it successfully, he would thereby create time $60,000 addition to his estate. But such saving would take a great deal of self-discipline and perseverance. Most of all, it would take time. Amid Charlie and Ruth want security for their family now, not ten years from now,

If Charlie buys the $60,000 policy and dies while it is in force, Ruth will receive that amount, even if Charlie lives to pay only the first month's premium. If that happens, the policy will have created something-an estate-that did not previously exist. Estate creation is quite different from the basic insurance function of reimbursing an insured for something like a hospital bill or a damaged car. It can be done only because life insurance is not fully subject to the indemnity principle and therefore is not confined to indemnify-unique aspect of life insurance.

TYPES OF LIFE INSURANCE

To a person not familiar with the field, there would appear to be many different types of life insurance. Like auto manufacturers, life insurance companies put a variety of names on their products. But the fact is that there are only three basic types: All policies provide either term insurance, endowment insurance, whole life insurance, or some combination of these three. In this section, we will examine the three types, and in the next section, we will look at some of the combination policies.

Term Insurance

Term lire insurance covers lie mortality risk for a stated length of time. It is the simplest type of life insurance. In the case of a one-year term policy, for Instance, the company promises to pay the face amount, time amount stated in the contract, if the insured dies during the year. If the insured survives the one-year term, the policy expires without payment. Like auto or fire insurance, term life insurance provides no savings element, no cash value. In straight tern, insurance, there is no guarantee that the company will renew the policy. If it is renewed, a higher premium is charged.

FORMS

Most term policies are written for terms longer than a single year. Five-year and ten-year term policies are common, They have level premiums during their terms; if they are renewed, the premiums for the next term are at a higher level, Term-to-65 policies provide death protection at a level, premium until the insured reaches age 65.

People who are in poor health must pay higher rates for life insurance. If their condition is serious, they may not be able to secure coverage at any price. Because any policyholder can develop poor health, straight term insurance us not very desirable. Most term policies therefore are either renewable, convertible, or both.

A renewable term policy can be renewed at the option of the insured without showing evidence of insurability or, in other words, regardless of the state of the insured’s health. Thus, a five-year term policy might be renewable for three additional terms, guaranteeing a total of twenty years of protection.

Convertible term policies give the insured the option of converting the coverage to a whole life or endowment policy, again without evidence of insurability. Usually, the conversion must take place prior to the last year or two of time term,

Another common forum of term insurance is decreasing term. This policy

decreases in amount each month or year. Decreasing term often is written in combination with Whole life, as will be explained later.

USES

Term insurance is appropriate when (a) the need for protection is temporary, or (b) the objective is to secure the greatest possible amount of protection for the cost of the insurance. An example of a temporary need is that in connection with a loan. A person might borrow $100,000 to start a business, the loan to be repaid in five years. A five-year term policy in the same amount would pay off the loan if the borrower died and thus would prevent the borrower's family from having to shoulder the debt.

In the case of mortgage debt, decreasing term policies frequently are used. For instance, an $80,000 thirty-year home mortgage can be protected by an $80,000 thirty-year decreasing term policy. The amount of insurance initially is the same as the amount of the mortgage. As the point during the thirty-year period, the amount of protection is roughly equal to the amount needed to pay off the debt.

Term insurance costs much less than types of life insurance, as Table 5-3 shows. As a result, it frequently is used by people who need maximum protection at a minimum outlay in premium dollars. Charlie and Ruth Newlin may decide that they need $60,000 of insurance, but that

they cannot afford a permanent form of protection (that is, a whole life or long-term endowment policy) at this time. A $60,000 five-year convertible term policy would cost them about $337 a year. Five years from now, when Charlie is 30, they may be able to convert to a straight Life policy at a cost of about $965 a year.

If their sole objective is income protection while Julie is growing up, an even less expensive policy for the Newlins would be decreasing term. They need $60,000 of protection now to furnish $400 a month for the lb years between Julie's present age of 2 and the point at which she will be IS. But, as the years go by. the $400 a month will be needed for fewer years. If Charlie dies when Julie is 10, only 8 years will remain until she is 18, and only about half as much in policy proceeds would be needed for the income payments. A $60,000 twenty-year decreasing term policy would cost only about $230 a year.

Table 5-3 Annual Cost of $100,000 Policies

Type of Policy	Age of Insured at date of Purchase				
	20	25	30	35	40
Five-year convertible term	$ 555	$ 562	$ 585	$ 648	$ 794
Straight life	1,170	1,362	1,607	1,918	2,315
Whole life paid up at 65	1,309	1,549	1,861	2,287	2,901
Twenty-payment whole life	2,031	2,284	2,593	2,952	3,362
Endowment at 65	1,530	1,830	2,235	2,799	3,613
Twenty-year endowment	4,359	4,372	4,403	4,466	4,574

[a] Costs shown are those charged by one company for policies covering men. Rates for women are lower. For simplicity, rates for nonparticipating policies are shown; participating policies would have higher rates but would pay annual dividends.

LIMITATIONS

Term insurance has three important limitations. Each of the three should be clearly understood and considered before a decision is made to purchase term protection.

First, term policies expire at the end of their terms, but the need for protection may continue. The need for continuing protection is especially pressing for policyholders who have developed poor health. As explained earlier, a person who is in poor health may be unable to secure another policy. This problem can be overcome by purchasing convertible term and by exercising the option to convert to a permanent form of protection. Buyers of convertible policies should be aware, however, that many insureds fail to convert their policies while they have a chance to do so. Most of us seem to be inclined to put off until later things that we don't have to do today, particularly if they cost us money.

The second limitation of term insurance is its increasing cost. Table 5-3 shows the annual cost of' renewing $100,000 five-year term policy at successive five-year intervals. The cost would be $1,415 at age 50 arid 52,940 at age 60. Somewhere along the line those who continued to renew this: policy amid who had a continuing need for the protection night wish that they mad purchased a level premium permanent policy instead. A $100,000 straight life policy purchased at age 25 costs $1,362 a year, but the annual premium remains $1,362 for as long as the insured keeps the policy. Some people prefer term insurance in spite of its increasing cost. Their reasoning is that increases in their annual income will permit them to handle the rising cost without undue difficulty. Also, they may figure that their need for protection will decline as their children grow older and that they therefore will be able to reduce the amount of their coverage gradually and perhaps be able to hold its cost relatively constant.

The third limitation of term insurance is the absence of cash values. Other types of life insurance combine savings with protection, furnishing cash values for emergencies and for retirement income. Many people do not regard this as a shortcoming of term insurance, because they prefer to separate their savings programs from their life insurance coverage. But Table 5-4 makes it clear that many other life insurance purchasers prefer policies that provide cash values. Less than 40% of the ordinary insurance in force is term insurance. If the comparison were on the basis of premium volume, term insurance would comprise an even smaller part of the total, because of its lower cost per $1,000.

It sometimes is argued that many of those who buy cash value insurance do so because they fail to consider the alternative of separating their insurance and savings programs or because they are talked into buying the more expensive policies by agents who thereby earn higher commissions. There undoubtedly is some element of truth in this viewpoint. However, the majority of cash value insurance purchasers probably know what they are doing and prefer to achieve at least part of their savings through life insurance, largely because of the "forced savings" aspect of this system. Most of us are all too human when it comes to saving money. There always seems to be some excellent reason why we can't save as much out of the current paycheck as we had planned, and it is difficult to avoid drawing on long-range savings when the car breaks down or the washing machine has to be replaced. But life insurance premiums almost always get paid, because they tend to be regarded as an essential part of a family budget, much like the rent or mortgage payments. If the premiums are not paid when due, the policy may lapse, something that would threaten the future welfare of the family. Therefore, many families go to great lengths to pay their insurance premiums regularly. In doing so, they force themselves to save money in the form of cash values, Such people regard the absence of cash values in term insurance as a shortcoming of that type of coverage.

Table 5-4 Type of Ordinary Life Insurance in Force

Type of Policy	Number of Policies (millions)	Amount of Insurance (billions)	Percentage Of Amount
Term	20.8	$ 760.1	38.5
Endowment and retirement income	10.9	$ 70.2	3.5
Whole life	116.9	$ 1,147.8	58.0
Total	148.6	$1,978.1	100.0

Source: Life Insurance Fact Book 2005 (New York: American Council of Life Insurance, 2006), p. 19.

Endowment Insurance

Endowment insurance pays a stated amount if the insured dies during a specified period of time or pays the same amount if the insured is living at the end of the period. Unlike term policies, endowment policies have cash value. The cash value increases gradually until at the end of the endowment period it equals the face of the policy. At that point the policy endows; that is, it pays the face amount to time policyholder. If Charlie Newlin buys a $60,000 twenty-year endowment policy, it will pay Ruth $60,000 if Charlie dies during the twenty-year period. If he is still living at the end of the period, he will receive $60,000.

An endowment policy can be thought of as a combination of an increasing savings fund and decreasing term insurance. At the start of the policy period, the term insurance equals the full amount of the policy. As he years pass and time savings part grows, the term Insurance part declines. At any point, the total of the amount of term insurance and the amount of the savings fund equals the face of the policy. The policy endows at time end of its term when the savings fund has reached the face amount and the tent insurance has declined to zero. Because endowment policies have high cash value (equaling the face amount of the policy at the end of the endowment period), their cost is high. The shorter the term of the policy, the higher the premium.

FORMS

Endowment policies are written for various periods, such as 20, 25, or 30 years. They also can be written to endow at a stated age, perhaps 65 or 70. An endowment-at-age-65 policy issued at age 25, of course, would be the same thing as a 40-year endowment.

The retirement income policy is a special form of endowment that provides even more savings and less decreasing term protection than regular endowment contracts. The policy provides a retirement income of $10 per month (starting at age 65) for each $1,000 at face amount. A $100,000 retirement income policy would pay the insured $1,000 per month during retirement. This policy is even more expensive than other endowment contracts. A $100,000 retirement income policy purchased at age 25 from the company whose rates are shown in Table 5-3 would cost $2,514 per year.

USES

Endowment insurance really is a form of savings protected by life insurance. In effect, insureds who live until their policies endow complete their savings goals by means of their premium payments, while the protection aspect of the policy assures them that their savings goal will be completed, whether or not they live to complete it themselves.

Endowment policies sometimes are used as a means of saving money for college expenses or for other purposes. A more appropriate use is as a systematic way of accumulating a fund for retirement.

LIMITATIONS

The chief limitation of endowment insurance-particularly of short-term policies is its very high cost. In spite of the high cost, endowment contracts sometimes are bought because of the idea that I don't have to die to win. In other words, purchasers are attracted by the idea that the policy will pay off in full whether they live or die; they also like the thought that they probably will be alive to receive the proceeds themselves. The real difficulty this creates is that the purchase of an endowment is unlikely to fill the need for income protection. Charlie and Ruth Newlin may be tempted by the idea of buying an endowment policy so that its cash value can be used for Julie's college expenses. But if they spend the same amount for an endowment as they otherwise would spend for term insurance, the policy will be much smaller and its proceeds won't go as far toward filling Julie's need for income if Charlie dies.

A second limitation of endowment insurance is that a low rate of return is earned on its very important savings element. Because such a large part of the premium is devoted to this aspect of the policy, potential buyers should compare the rate of return on endowment with the return offered by other financial instruments, such as bonds and money funds. Such comparisons must be made with caution, because of the many differences between insurance policies and other types of investment. However, if the objective is to maximize the return on one's funds, the conclusion must be that other forms of investment are preferable.

Whole Life Insurance

Whole life insurance provides lifetime protection by means of level-premium, cash value policies. In terms of both cost and amount of cash value, whole life occupies the middle ground between term and endowment. That is, whole life policies are generally more expensive

than term but not as expensive as endowment, and they provide some cash value but not as much as endowment contracts do. The intermediate nature of whole life is shown in Table 5-5. For an example of a whole life policy, see Appendix D.

A whole life policy can provide lifetime protection. It can be kept in force until the policyholder either cashes it in or dies. The cash value continues to Increase as long as the policy remains in force. If the policy is still in force when the insured attains age 100, the cash value reaches the face amount and a check for that amount (along with a letter of congratulations from the company president) is sent to the insured. Note that, in effect, whole life policies endow at age 100. In one sense, therefore, whole life policies are endowments at age 100, and there are only two different types of life insurance, term and endowment, In a more practical sense, however, whole life is a separate type, because the people who buy whole life policies don’t expect them to endow; whole life is neither thought of nor used as endowment Insurance.

FORMS

There are three forms of whole life insurance: single premium, limited premium, and straight life. The difference between the three forms is the length of the period during which premiums are payable.

Single premium whole life policies are paid for in one payment. A $100,000 policy can be purchased for a single premium of about $30,000 at age 25. The policy will remain in force with no further payment until it either is cashed in or pays $ 00,000 to tine beneficiary. Single premium policies ale rarely purchased; they are mentioned primarily to help explain the nature of whole life insurance.

Table 5-5 Comparison of Three $100,000 Policies Issued at Age 25

Policy	Annual Cost	Cash Value		
		Age 35	Age 45	Age 65
Five-year convertible term	$ 562	$ 0	$ 0	$ 0
Whole life (straight life)	$ 1,362	$ 9,300	$24,700	$ 57,700
Endowment at 65	$ 1,830	$ 14,200	$ 36,500	$ 100,000

[a] See note, Table 5-3.

Limited payment whole life policies provide for premiums to be paid either for a stated number of years or until the insured reaches a stated age. On a 20-payment whole life policy, premiums are payable for 20 years. (Payments could be made semiannually, quarterly, or monthly rather than annually, so there actually could be more than 20 payments; all would be made within a 20-year period, however). A whole life paid up at 65 is also a limited payment policy. If it were issued at age 25, it would have a 40-year premium payment period and would be the same as a 40-payment whole life policy. The longer the premium payment period, the lower are each year's premiums. Limited payment whole life policies should not be confused with endowments. At the end of the premium paying period, a limited payment policy is fully paid for, but it does not endow at that point as an endowment policy would. The coverage continues titer the policy is paid tip; he cash value is still less than the face amount and continues to increase as long as the policy remains in force (up to age 100).

Straight life policies are the thud and by far the most popular form of whole life insurance. The name of this policy is a source of confusion because it sometimes is called the whole life policy or the ordinary life policy. The most descriptive name would be continuous-premium whole life policy, but that term seldom is used. The premiums for straight life

policies are payable for the lifetime of the insured, Extending the premium payment period beyond a limited number of years lowers the annual cost of the policy and makes it the least expensive form of permanent life insurance. Because the premiums are lower, the cash value of straight Life policies increases more slowly than any other form of cash value insurance.

USES

The single-premium whole life policy has very limited use except in certain tax situations or as a gift to a young person. Short-term limited payment whole life policies also have rather united use. It usually makes little sense, for instance, for a 25-year-old to buy a 20-payment or a 30-payment policy and thereby compress the premium payments into the period when the family's expenses are higher and its income is lower than they will be in later years.

The straight life policy and the whole life paid up at 65, on the other hand, are well suited for the needs of many people. They offer a compromise between term and endowment insurance, combining reasonable amounts of income protection and savings at a reasonable cost. Charlie and Ruth Newlin the purchase of a straight life policy. It will cost about $817 a year and will have a cash value of $5,580 in 10 years. $14,820 in 20 years, and $34,620 in 40 years when Charlie is 65. The cash value can be borrowed for emergencies, and when Charlie retires he can cash in the policy to supplement his retirement income if he wishes. In the meantime, the policy will provide the protection that Ruth and Julie need.

LIMITATIONS

Four of the policies we have been considering are seen from a different viewpoint in Table 5-6. Instead of comparing policies of equal size, this table compares policies of equal cost. Again, we see term and endowment at the extremes. For a given cost, much more term insurance can be purchased, but it, of course, provides no cash value. Endowment offers the greatest amount of cash, value, but at a sacrifice of the amount of protection which a given premium will buy. The two whole life policies that are shown in the table provide amounts of both protection and savings that are between the extremes furnished by term and endowment.

Table 5-6 also illustrates the limitations of whole life insurance: Compared with term, whole lift is expensive; compared with endowment, ii provides less cash value. These of course, arc limitations only in a relative sense. Whole life avoids the more serious limitations of term (expiring protection, increasing cost, and absence of cash value) and of endowment (high cost).

It sometimes is suggested that a limitation or shortcoming of the straight life policy is the fact that its premiums are never paid up. This is not a valid criticism and is based on a misunderstanding of the purposes of the policy and of the reasons why it is purchased. People who buy straight life policies rarely Intend to continue paying for them indefinitely, and there is no reason why they should. The company is not "cheated" if a policy is cashed in when the protection no longer is needed. In fact, this and other cash value policies offer several options that are designed to serve the best interests of policyholders if they decide to stop paying premiums and use the accumulated cash value. These options are described in the next chapter.

COMBINATION POLICIES

Although there are only three basic types of life insurance, two or more of the three can be combined in a single contract. A wide variety of combination policies is available. The most important are the family income, family maintenance, and family plan policies.

Table 5-6 What $800 a Year Will Buy at Age 25[a]

Policy	Amount	Cash Value at 65
Five-year convertible term	$ 142,000	$ 0
Straight life	59,000	34,000
Whole life paid up at 65	52,000	36,000
Endowment at 65	44,000	44,000

[a] Approximate amounts. See note, Table 5-3.

Family Income Policy

The family income policy is a combination of straight life and decreasing term insurance. It is designed for young families who need added income protection during the child-raising period. The use of decreasing term insurance for this purpose was discussed earlier.

The decreasing term portion of the family income policy usually covers 20 years and is sufficient to pay in income of $10 per month for each $1,000 of straight life insurance. Thus, a $60,000 family income policy would pay $600 per month starting from the date of the policyholder's death within the first 20 years of the policy and ending 20 years after the policy was purchased. If a policy had been purchased at age 25, the income payments to the beneficiary would continue until the insured would have reached age 45. The $60,000 proceeds from the straight life part are paid at the end of the 20-year period. If the insured is still living when the 20-year term portion expires, the straight life part continues in the regular manner, but at a lower premium level.

Benefits similar to those provided by the family income policy can be provided by adding a family income rider to a regular whole life policy. The family income rider is simply decreasing term coverage. In this case, the proceeds of the whole life policy are payable at death instead of at the end of the family income period. This arrangement generally is preferable, because the need for funds is likely to be greatest at the time of the insured's death.

Family Maintenance Policy

As Figure 5-2 shows, the family maintenance policy is similar to the family Income policy. Both combine term and straight life insurance. However, the term portion of the family maintenance policy is level rather than a decreasing amount.

The amount of term insurance (about $15,000) again is sufficient to pay $10 per month for each $1,000 of straight life. But, because the family maintenance policy includes a level amount of term insurance, its income payments continue for a full 20 years if the insured dies at any time during the first 20 years of the policy. As explained earlier, if a family income policy is purchased at age 25 and the insured dies at age 35, the decreasing term portion pays until the insured would have been 45. But, if a family maintenance policy is purchased at the same age and the insured dies at age 35, the level term portion pays until the insured would have been 55. Again, the straight life portion continues at a lower premium level if the insured lies beyond the initial 20-year period. Because of the additional protection furnished by the family maintenance policy, it naturally is somewhat more expensive than the family income policy.

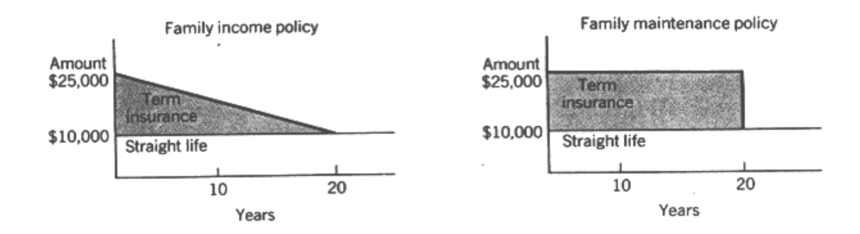


Figure 5-2 Family Income Policy Family Maintenance Policy

Family Plan Policy

The family plan policy, like other life insurance contracts, is sold under a variety of names. It also combines term and whole life insurance but does so by providing straight life coverage on the father and term insurance on the other members of the insured family. A typical family plan policy furnishes units of $5,000 of straight life on the father, $1,000 of term-to-age-65 on the mother, and $1,000 of term-to-age-21 on each child. Additional children born to or adopted by the family are covered with no additional charge. In the event of the father's death, the coverage's on the mother and children automatically become paid up for their remaining terms.

The family plan policy is attractive to many families as a way of securing relatively low-cost protection on the mother and children. To some, it appears less "selfish" than a policy insuring only the father. However, if a family relies on the income produced by the father, insuring against its interruption should always be regarded as the main objective of life insurance. In other words, the family should consider the alternative of securing a more adequate amount of income protection before diverting premium dollars for other purposes. This approach, of course, is consistent with the rules of personal risk management.

Other Combination Policies

It should be clear to you by now that there is no limit to the ways in which the three basic types of life insurance can be combined and modified, Anyone who has a firm understanding of the basic types usually can deduce the ingredients of combination contracts. Descriptions of three other combination policies follow.

MODIFIED LIFE POLICY

This is a form of straight lift policy with reduced premiums for the fist few years. It is designed for young purchasers with limited incomes who expect higher earnings several years later. An example is a $100,000 straight life policy with a premium oh about $851) for each of the first live years a rid $1,600 per year thereafter. After the period of reduced cost, the cost increases to a level somewhat higher than that charged for a regular level premium straight life policy. The modified life policy differs from regular whole life contracts only with regard to the premium arrangement.

MULTIPLE PROTECTION POLICY

The multiple protection policy pays a multiple of the face amount if the insured dies within a specified period after the policy is purchased. An example is an endowment at age 70 offering double protection during its first 20 years. Rather obviously, this contract is simply a combination of endowment insurance and an equal amount of 20-year term insurance.

RETURN OF PREMIUM POLICY

Some combination policies are primarily marketing devices. The return of premium policy is such a contract; it seems to offer the purchaser “something for nothing”. It is a policy in which the company promises that if the insured dies during the first 20 years it will pay the face amount and will also return all of the premiums that have been paid until that point. Before reading on, see if you can figure out how it can do this. How, for instance, can a company that sells a $100,000 straight life policy with an annual premium of $2,000 guarantee that it will pay $140,000 for death during the first year, $104,000 if the insured dies the second year, and so on up to the twentieth year, the coverage drops back to $100,000 and remains at that level.

If you have deduced that the return of premium policy is a combination of straight life and some form of term insurance, you are right. If you have decided that it is increasing term insurance, you are exactly right. The term insurance portion of the contract simply equals the cumulative premiums for the first 20 years, increasing from $2,000 the first year to $40,000 the twentieth year. The cost of the policy is sufficiently higher than regular straight life to pay the added cost of the increasing term insurance. Therefore, the purchaser doesn't really get something for nothing. As with any other kind of insurance, one pays for what he or she gets. The trouble is that (a) those who buy this policy probably don't understand what they are paying for, and (b) they usually would be better off with decreasing term rather than increasing term, because the need for income protection is generally a decreasing need rather than an increasing one.

NEW LIFE INSURANCE POLICIES

Inflation is a great enemy of life insurance and of most other arrangements that are intended to provide long-range financial security. To illustrate, due to rising price levels, the proceeds of a $100,000 life insurance policy purchased in 1970 would supply only about $40,000 worth of purchasing power in 1983. Recognizing this problem, many consumers have been reluctant to buy traditional forms of life insurance. Responding to the need for innovative products, these include the variable, adjustable, and universal life insurance policies.

Variable life

A variable life insurance policy's face amount and cash value fluctuate with the yield on "rue or more securities hinds selected by the purchaser from among the company's investment plans. Investment plans available usually include common stock funds, bond funds, and money market funds. If the chosen investment plan prospers, the policy's yield

should surpass that of conventional whole life insurance because the funds backing variable life policies are separated from the insurer's general accounts (which include older low-rate bonds and mortgages). If the particular investment plan does poorly, the policy's death benefit and cash values will decline. A minimum death benefit is stipulated, but there is no guaranteed minimum cash value.

In a modified version of the variable lift policy, the benefits actually are tied to a cost-of-living index, varying automatically as the index rises or falls.

In Europe, which has hived with serious inflation longer than the United States, a significant percentage of the life insurance now being sold is some form of variable life. The policy has not had wide acceptance in this country.

Adjustable Life

The unique feature of the adjustable life policy is its flexibility. It is a cash value policy that gives its owner the option of changing the size of its premium, the face amount, the length of the premium-paying period, or the length of the protection period. For instance, a policyholder who gets a raise or has a baby and who wishes to increase the policy's face amount can do so, either by increasing the premium payments, agreeing to pay premiums for a greater number of years, or cutting back the length of time for which the policy will provide protection.

The emphasis of adjustable life is on flexibility to meet the changing needs of its owners. Because the contract can be altered in so many ways, people who have adjustable life policies do not need to purchase additional policies as their needs or ability to pay change. The contract therefore is advertised as "the only life insurance policy you'll ever need,"

Universal Life

Universal life insurance policies are flexible contracts whose cash value accumulates on the basis of current market rates of interest, The policies are designed to combine life insurance protection with a form of investment paying a greater yield than conventional whole lift or endowment policies. Some people believe that universal life eventually will replace the current forms of cash value insurance.

From each premium (usually ,aid monthly) the company deducts a charge for expenses and an amount for term insurance protection. The remainder accumulates interest, as the policy’s ‘cash value,” but unlike the cash value of a traditional policy it grows at a variable rate rather than a predetermined rate. In other words, the policy does not have a fixed table of cash values. Policyholders receive monthly statements indicating the current level of their cash values.

After the first premium is paid, the owners may pay subsequent premiums whenever and in whatever amounts they choose. They may even skip one or more payments if they wish to. If a premium is skipped, the charge for term insurance is simply taken out of tile accumulated cash value. A staled minimum interest rate, commonly 4%, is guaranteed on the savings portion; the amount actually credited is determined periodically (monthly in many cases). Depending on the company selling the policy, the interest rate may vary with he company’s current investment earnings or with a specified standard, such as the rate being paid on short-term U.S. Treasury bills. Crediting current market rates of interest makes it possible for life insurers selling universal life policies to compete with money funds and other financial institutions which in recent years have offered high “new money” interest rates.

Another feature attractive to investors is the fact that the universal life policy-like other life insurance policies-accumulates interest earnings tax-free until the funds are withdrawn. This point is especially important to purchasers in high tax brackets. The universal life policy is claimed by some of its advocates to be ‘the policy of the future-a major breakthrough in consumer-oriented life insurance. Whether or not that turns out to be the case, the development of this and other new policies demonstrates the desire and the ability of the life Insurance industry to respond in imaginative ways to the needs of consumers and the challenges of the changing business environment.

THE BEST POLICY

Our discussion of the kinds, uses, and limitations of life insurance policies leads to a very important conclusion: The best policy is the one that best serves the needs of the purchaser. There is no single best” policy, because one that serves the needs of one family very well will not necessarily suit the needs of another. Well-informed life insurance buyers understand that the primary differences among various policies are the relative amounts of protection and savings which they provide. They also know that the costs of the policies reflect these differences. Knowing these facts, each purchaser must decide what balance of protection, savings, amid cost is proper for his or her particular situation.

IMPORTANT TERMS

cash value
mortality risk
mortality table
actuary
endowment
whole life
single premium whole life
limited payment whole life
straight life
mortality rate
level premium
estate creation
term life
family income policy
family maintenance policy
family plan policy
modified life policy
multiple protection policy
face amount
renewable term
convertible term
decreasing term
return of premium policy
variable life
adjustable life
universal lire

KEY POINTS TO REMEMBER

1. Life insurance covers the mortality risk. Rates are based on data compiled in the form of mortality tables.
2. With the exception of the insurable interest requirement, the indemnity principle does not apply to life insurance. Because it does not, life insurance can be used to create or add to an estate.
3. There are three basic types of life insurance: term, endowment, and whole life.
4. Term policies provide pure protection for a stated length of time. Most term policies provide options to renew for additional terms or to convert to a permanent form of life insurance.
5. Term insurance is appropriate when the need for protection is temporary. Because it is the least expensive type of life insurance, it also can be used to secure the greatest possible amount of protection for a given cost.

6. Term policies have three limitations: They may expire while protection is still needed, their cost increases at each renewal, and they do not provide cash values.
7. Endowment insurance combines income protection with a relatively large element of savings. An endowment policy pays the face amount if the insured dies during the endowment period or pays the same amount if the insured is still living at the end of that period.
8. Whole life insurance consists of several forms of level premium, cash value policies that provide lifetime protection.
9. The premiums for whole life policies are payable either all at once (single premium In policy), for a stated period (limited payment policies), or for the lifetime of the insured (straight life policy).
10. The family income policy combines straight. life with decreasing term insurance to furnish additional income protection during the first 20 years. The family maintenance policy is similar nit uses a level amount of 20-year term insurance.
11. The family plan policy covers all members of the insured family, providing straight life insurance on the father and term insurance on the mother and children.
12. Several new policies, including variable, adjustable, and universal life, are designed to meet the needs of an inflationary environment.
13. There is no one “best” policy. The policy that is best for a particular family depends upon the balance of protection, savings, and cost that is most appropriate for their needs.

REVIEW QUESTIONS

1. Why do people buy life insurance.
2. List brim ways in which life insurance differs from most other kinds of insurance.
3. What are mortality tables? What. are they user' for?
4. How do term, endowment, and whole life 'insurance differ from one another?
5. Why ate the renewability and convertibility options desirable ort term policies?
6. For what special uses can decreasing term insurance he used?
7. How does the concept of forced savings" apply to life insurance?
8. Explain the nature of a $100,000 endowment at (15.
9. Why is an endowment at 65 more expensive titan a whole life policy paid up at 65?
10. What is the difference between a limited payment whole life policy and a straight life policy? In what ways are these two policies similar?
11. How does the protection furnished by a family maintenance policy differ from the protection furnished by a family income policy?
12. What is a modified life policy?
13. What is a multiple protection policy?
14. What are the key features of the adjustable life policy? Of the universal life policy?

DISCUSSION QUESTIONS

1. The chapter indicates that the only legitimate way in which one can immediately create an estate is that through the purchase of life insurance. Couldn't this also be done by borrowing the money?
2. Term policies that are either renewable or convertible are more expensive than those that are not. Why?
3. In view of the fact that endowment and whole life policies accumulate cash values, is the absence of cash values in term insurance unfair?
4. Why are straight lie policies, for which premiums are payable for the insured's lifetime, more popular than other whole life policies for which premiums are payable for only a limited number if years?
5. Which would be more appropriate for a family like the Newlins, a family income policy or a family maintenance policy?
6. If you were visited by a life insurance agent who immediately told you that he had a policy that was just the thing for you, what would your reaction be?
7. Many people purchase cash value life insurance because of its forces savings aspect. Do you agree with limit that this is a good reason for preferring cash value insurance to term insurance?

CHAPTER 6

LIFE INSURANCE FUNDAMENTALS

Part Two

CHAPTER OBJETIVE5

After studying this chapter, you should be able to:

1. Define and compare the four classes of life insurance.
2. Distinguish between participating and nonparticipating policies and list the common dividend options.
3. Describe the three non-forfeiture options.
4. Identify the four usual settlement options.
5. Summarize the other important life insurance policy provisions.
6. Discuss the nature of life annuities and outline their principal features.

The preceding chapter described the basic nature ~if life insurance and identified the “a rim’s kinds of policies. This chapter deals with several other aspects of life insurance, beginning with the ways that it is marketed. I’. then describes lie important policy provisions acid options and’ concludes with an explanation of annuity contracts.

CLASSES OF LIFE INSURANCE

Life insurance is classified on lie basis of how it is marketed. Basically, there are two different marketing systems, individual and group. Both are divided into, two classes, making four classes in all. The individual marketing system involves the sale of policies to individual persons and families. The two classes using the individual marketing system are ordinary insurance acid industrial insurance. The group marketing system uses the technique of insuring a number of people under a single policy. The two classes that use this system are called group insurance and credit insurance.

Individual Marketing

ORDINARY

Ordinary is the oldest and largest class of life insurance. The policies generally cover multiples of $1,000. Only the first premium is collected by the agent: later premiums are sent directly to the company and are paid either annually, semiannually, quarterly, or monthly. The policies may be any of the types described in Chapter 12. This is now the principal way of marketing individual policies, and, unless otherwise indicated, the policies discussed later in this chapter are those marketed as ordinary insurance.

INDUSTRIAL

Industrial life insurance was developed to serve the needs of low-income families who could not afford large policies and who were unable

to save money for larger, infrequent premium payments. The policies are generally in amounts of $1,000 or less. They may be either whole life or endowment insurance. The premiums (perhaps 25 cents or 50 cents) are collected by the agent each week or month at the insured's home. For this reason, industrial is sometimes called "home service" life insurance. During the first 60 years of this century, more Americans were covered by industrial than by any other class of life insurance. As people have become better able to afford ordinary insurance and as group insurance and social security have grown, the volume of industrial has declined relative to the other classes. But, as Table 6-1 shows, many people still rely on industrial life insurance.

Table 1 Classes of Life Insurance in Force, United States, 1991 and 2001

Class	*1991*		*2001*	
	Number of Persons (millions)	*Amount of Insurance (billions)*	*Number of Persons (millions)*	*Amount of Insurance (billions)*
Ordinary	123	$ 792	149	$1,978
Industrial	76	39	55	35
Group	82	590	123	1,889
Credit	76	82	73	162
Total	357	$1,503	400	$4,064

Group Marketing

GROUP

Most insurance is sold to employers to cover groups of employees. The coverage is paid for either by employers alone or by both employers and their employees. Other group policies cover members of unions nr of associations. Most group life coverage is term insurance. As Table 1

shows, group is the fastest growing class. The nature of group insurance is described more fully.

CREDIT

Credit life insurance covers the lives of people who owe installment debt. The insurance pays off the debt if the borrower dies before completing the payments. Some credit life insurance is sold on the individual basis, but most us provided by group contracts arranged by banks, finance companies; credit unions, and retail stores. 1he coverage is term insurance, usually decreasing term,

Credit life insurance frequently is encountered in connection with the insurance of an automobile. The hank or other financing institution may require the borrower to furnish life insurance in the amount of the loan, with the lender named as beneficiary. In fact, the batik loan officer may simply quote installment loan payments that include the insurance, It is worth knowing that in most states the financing institution cannot legally require that it be the source of the insurance. That is, borrowers may assign existing insurance to the lender or may purchase additional coverage in their own; they do not have to buy it from the lender. Buying the insurance elsewhere may be more economical. In many cases, credit life insurance is expensive because the insurers pay large commissions to the lenders. Some states recently have adopted consumer protection legislation in this field.

POLICY PROVISIONS

There are no standard life insurance policies, but all policies contain certain standard provisions, some of which are required by law. A knowledge of the more important policy provisions gives one a fuller understanding of how life insurance operates.

Beneficiary and Assignment

BENEFICIARY CLAUSE

The beneficiary is the person (or other entity) named by the policyholder to receive the proceeds of the policy after the death of the insured. Note that the policyholder (who owns the policy) is not necessarily the person whose life is insured. The policyholder might, for instance, be a parent who had purchased a policy insuring the life of a son or daughter. More typically, however, the policyholder owns a policy covering his or her own life. The beneficiary's name us stated in the application, which later is attached to and made a part of; the policy.

The insurable interest requirement does not apply to the beneficiary. The policyholder must have an insurable interest in the life of the person whom the policy insures, but the policyholder can select whomever he or she chooses to be the beneficiary.

The policyholder also elects whether or not to retain the right to change the beneficiary. If this right is retained, the beneficiary is said to be revocable. If the policyholder does not retain the right to make a change, the beneficiary us irrevocable. The distinction can be an important one. A revocable beneficiary can be changed by the policyholder at any time without the beneficiary's permission. But a policyholder who names an irrevocable beneficiary gives up the right to make a change later, even if there is a divorce or some other reason for change. In this case the policyholder also may not assign the policy, surrender it for its cash value, or borrow on it without the beneficiary's consent.

A contingent beneficiary Frequently is named, This person receives the policy proceeds if the primary beneficiary is no longer living when the insured dies, In many cases the spouse of the policyholder is the primary beneficiary, and the couple's children (or a guardian or trustee named to act on their behalf) are named as contingent beneficiaries.

The naming of life insurance beneficiaries is similar to writing a will and should be done with equal car-c and precision. At due time when policies are written, the policyholders know whom they intend to receive the benefits, but flatly years later this may be subject to dispute. Misunderstandings are particularly likely to develop when there has been a divorce or when additional children have been born or adopted. For this reason, policies should be reviewed occasionally to make sure that beneficiary designations are still appropriate.

ASSIGNMENT PROVISION

If an irrevocable beneficiary has not been named, the policyholder retains not only the right to change the beneficiary but also the right to assign (that is, transfer) the policy to another party. The usual purpose of assigning a policy is to provide collateral for a bank loan. When this is done, the owner of the policy instructs the insurer to pay death claims to the bank to the extent of the balance of the loan.

Life insurance policies can be assigned without the insurer's consent. However, the assignment provision states that the company is not legally bound by an assignment unless it has received a copy of the assignment document. This wording protects the company from being notified of an assignment after it has paid the policy proceeds to the named beneficiary.

Grace Period end Reinstatement

GRACE PERIOD

The grace period is one of the provisions required by law. The policy states the amount of the premium payments and the dates when they are due. The grace period allows a number of days, usually 31, to elapse after the due date. if the premium is paid during this period, the policy remains in force. If the premium is not paid by the last day of the grace period, the policy lapses.

REINSTATEMENT

The reinstatement provision provides that a lapsed policy may be put back in force within a stated period, usually three or five years. Before reinstating the policy, the company can require evidence that the insured still is in good health. Also, all overdue premiums must be paid plus interest. Reinstating a lapsed policy may be preferable to replacing it with a new one if the old policy has favorable provisions that are not otherwise available.

Incontestability. Suicide, and Misstatement of Age

INCONTESTABLE CLAUSE

In Chapter 4, we learned that insurance policies are contracts of utmost good faith. If a person misrepresents important facts when applying for a policy, the contract can later be voided by the company. This would suggest that if a man lied about the state of his health when applying for life insurance and the misrepresentation was discovered after his death ten years later, the beneficiary would not be entitled to receive the policy proceeds. Actually, that would not happen. The incontestable clause gives the insurer a period of time, two years on most policies, to check the information supplied by the applicant. Only during that period can be company contest a death claim or seek to void the contract. After the contract has been in force Fur two years, it cannot he contested by the insurer, even if it discovers that the applicant made material misrepresentations.

The incontestable clause is a rather surprising provision in that it appears to condone fraud. Nothing like it is contained in other insurance policies. It is required by law in life insurance because of the long-term nature of most policies and because of the difficulty of proving or disproving statements many years after they are made. The clause forces insurers to investigate applications carefully. It also enhances the value of life insurance by assuring beneficiaries that they will receive the benefits stated in the policy.

SUICIDE CLAUSE

If an insured dies by suicide during the first two years of policy coverage, the beneficiary receives only the premiums that had been paid until that time. After the policy has been in force for two years, it applies to suicide just as it dues to death resulting front other causes.

The purpose of the suicide clause is to prevent the insuring of people who plan to kill themselves. According to some estimates, nearly 2% of the deaths in this country result from suicide. Psychologists tell us that many of those who commit suicide threaten to do so for a period of time beforehand. If such people were able to insure their lives, life insurance rates would have to be raised; other insureds would be forced to help pay the death benefits of those who had purchased policies with the intent of killing themselves. This clearly would be a case of adverse selection, with a disproportionate number of policies being purchased by persons who would die shortly thereafter. Interestingly, the suicide clause apparently assumes that suicide-prone people will not buy policies with the intention of taking their lives more than two years later.

MISSTATEMENT OF AGE

A 30-year-old applicant for life insurance who stated that he was only 25 would be charged a lower rate. Such misstatement of age is treated as a special form of misrepresentation to which the incontestable clause does not apply. Instead, the policy provides that the amount of insurance will be reduced to the amount that would have been purchased if the age had been stated correctly. If a 30-year-old man who said he was only 25 had purchased a $100,000 straight life policy, the amount would he reduced to $84,750 when the truth was discovered (based upon the rates used in Fable 2.3). It is quite likely that the insured’s true age would be discovered when his death certificate was submitted to the company, if not before.

Dividend Provisions

PARTICIPATING AND NONPARTICIPATING INSURANCE

Life insurance policies are either participating or nonparticipating. A participating policy is one on which annual dividends are payable to the policyholder. Holders of nonparticipating policies do not receive dividends. Although the percentage of nonparticipating insurance is rising, the majority (about 55%) of the ordinary life insurance now in force is participating.

The participating basis is occasionally used in other lines, but it is especially well suited to life insurance because of the long-term nature of the policies. Rates for life insurance are based on mortality, interest earnings, and company expenses. At the time a policy is issued, there is no way of knowing what any of these three elements actually will be during the years or decades ahead. In making the rates for a participating policy, actuaries make conservative estimates. They assume rather high mortality rates, low company interest earnings, and high company expenses. These assumptions cause the policy to be priced higher than it would be if more realistic estimates were used, and it gives the company a margin of safety if things don't turn as well as expected. During the life of the participating policy, dividends are computed and paid to the policyholder. The dividends are based on the company's actual experience in mortality, interest, and expenses. The result is that the net premiums (after the dividends are paid) reflect the company's actual cost of providing the insurance.

Rates for nonparticipating policies are based on more realistic assumptions. They also include an allowance for the possibility that mortality or expenses will be higher or that interest earnings will be lower than expected. If any of these things happens with regard to participating policies, the company can reduce the dividend payments. When the insurance is nonparticipating, however, the price is fixed and cannot be adjusted to reflect either adverse or favorable developments.

Table 2 illustrates the differences between typical participating and nonparticipating policies. The rate for the participating policy is higher; that is, for a given amount of premium the purchaser of a nonparticipating policy will get more coverage. However, the annual dividends reduce the net premium for the participating policy. The dividends can only be estimated (projected); they cannot be guaranteed. But, on the basis of the dividends that this company estimates it will pay during the first 20 years, the net premium for that period will be less for the participating than for the nonparticipating policy. You should bear in mind, however, that the nonparticipating policy is larger.

The two systems are actuarially equivalent, meaning that a participating policy costing $500 theoretically should be worth exactly as much as a nonparticipating policy costing $500 (if they otherwise identical policies). Choosing between the two is similar to choosing between a straight life policy and a family income policy. That is, purchasers shouldn’t base their decisions upon which of the two they think is the better “bargain”; instead, they should decide which is more suitable for their particular needs.

Table-2 Comparison of Participating and Nonparticipating Straight Life Policies Purchased from One Company for $500 by a 25-Year-Old Man

	Participating Policy	Nonparticipating Policy
Rate per $1,000	$ 13.66	$ 10.03
Size of policy for $500 premium	36,600.00	49,850.00
Gross premium for 20 years	10,000.00	10,000.00
Dividends for 20 years (projected)	3,570.00	0
Net premium for 20 years (gross premium minus projected dividends)	6,430.00	10,000.00
Cash value at the end of 20 years	8,009.00	10,155.00

IDEND OPTIONS

Participating policies allow the policyholder to choose from among at least different dividend options. First, dividends may be taken in cash; the patsy will mail the policyholder a dividend check each year. Second, the policyholder may choose to use the dividends to reduce the premium payments. If this option is selected, the premium notice from the company will the dividend subtracted from the premium and will bill the policyholder he difference.

The third option permits dividends to be left on deposit with the company. Using option is similar to putting the dividends into a savings account. company pays interest on the accumulated amount. A guaranteed interest rate is stated in the policy; a higher rate is paid if, as normally is the case, company’s interest earnings permit. Dividends accumulated at interest by g this option can be withdrawn at the policyholder’s request. If not withdrawn, they aye added to the amount paid when the policy endows or when insured dies. Over an extended period of time dividend accumulations add up to a surprising amount. For instance, based on the recent dividend and interest rate paid by one company, dividend accumulations would I $88,360 at age 65 on a $100,000 straight life policy issued at age 25. he fourth dividend option uses each year’s dividend as a single premium to a paid-up addition to the policy. For example, a $250 dividend at age 35 [d be used to add about $600 to the amount of a straight life policy. If the dividends were used this way every year on a $100,000 straight life policy chased at age 25 (again, using the present dividend scale of the company in previous example), the paid-tip additions at age 65 would total $121,660, ~ than doubling the amount of the policy. The paid-up additions themes have a cash value that always is at least as large as the amount of the dividends that were used to purchase the additions. The use of this option thus increase the cash and loan value as well as the death protection that the policy provides.

Some companies offer other dividend options. These may include paving a limited payment policy in fewer years, shortening the period of an endowment policy, or purchasing one-year term insurance additions.

Non-forfeiture Provisions

Cash value policies that have been in force for at least a year or two cannot expire without value. In other words, a policyholder who stops paying the premium does not thereby forfeit the policy's cash value. For this reason, the cash value is sometimes called the non-hsrfeiture value.

Cash value policies provide the non-forfeiture options. If the policyholder decides to stop paying the premiums, he or she can take the cash value as cash, use it to buy a reduced amount of paid-up insurance, or use it to buy extended term insurance. The policyholder may also borrow the cash value while keeping the policy in force. The cash value and the amounts of the other non-forfeiture options are specified in a table of guaranteed values in each policy. Table 3 is an example.

Table-3 Table of Guaranteed Values from One Straight Life Policy, Issued to Male, Age 25

End of Policy Year	*Cash or Loan Value per $1,000 Face Amount*	*Paid-up Life Insurance per $1,000 Face Amount*	*Extended Term Insurance*	
			Years	*Days*
1	$ 0.00	$ 0	0	0
2	0.00	0	0	0
3	8.24	23	3	26
4	21.20	58	7	298
5	34.47	92	11	248
6	48.05	125	14	167
7	61.94	158	16	189
8	76.16	190	18	27
9	90.70	221	19	91
10	105.57	252	20	58
11	120.74	282	20	313
12	136.23	311	21	143
13	152.01	340	21	291
14	168.05	368	22	33
15	184.34	395	22	106
16	200.88	422	22	153
17	217.64	447	22	175
18	234.64	473	22	177
19	251.86	497	22	161
20	269.21	521	22	129
Age 62	545.43	780	16	145
Age 65	592.35	811	15	58

CASH OR LOAN VALUE

There is little or no cash value for the first year or two because the company’s expenses are concentrated in that period. The agent may be paid half or more of the first year’s premiums as sales commission, and there are other initial expenses, such as underwriting the application, preparing the policy, and setting up necessary accounting records. After

the first year or so. the cash value increases at a rate determined by the company anti stated its the contract.

Cash values are an important element of competition among companies. Because of this, policies that at-c alike In other respects may have different cash value at different Lilacs. One company may increase the values vapidly in the early years; another may increase the values more slowly that they cats reach a higher point at age 65. Also, some companies may furnish higher cash values but provide lower policy dividends or less generous settlement options than others.

The policyholder has a contractual right to the cash value. Assume Ken Harris owns a $100,000 straight life policy that provides the non-forfeiture values shown in Table 3. If he decides to cash in the policy at the end of the fifth year, the company will send him a check for $3,447 (34.47 × 100, the policy being for $100,000). If Ken waits until he is 65, the cash value will be $59,235.

If a policyholder needs to use some or all of the cash value but wants to continue the policy, a policy loan can be made. That is, the company will lend the policyholder part or all of the cash value. This also is a contractual right; the company will send the amount requested without asking the reason for the loan or when it will be repaid. Until quite recently, cash value policies always specified the interest rate that would be charged on policy loans. The rate stated in most policies issued before 1976 is 6% or less. Later, as prevailing interest rates rose, insurers increased the loan rate on newly-issued policies. The standard then became an 8% rate. During 1982 and 1983 most of the states (which formerly had required fixed rates) changed their laws to permit flexible rates. Thus, many policies now being issued do not specify a fixed percentage, but instead provide that the policy loan interest rate will be determined periodically on the basis of an index of the current returns on long-term corporate bonds.

The amount of any unpaid loan and any unpaid interest is deducted front the proceeds otherwise payable at the death of the insured. If Ken Harris should die while a $20,000 loan is outstanding on his $100,000 policy, his beneficiary would receive $80,000 instead of the full face amount.

Policyholders frequently ask why they have to pay interest when they borrow their own cash value. The reason is that in computing premium rates company actuaries assume that the funds held by the company will be invested and will earn interest. You may recall that interest earnings are one (If three components of life insurance rates. (Mortality rates and company expenses are the oilier two.) Thus, from the company’s viewpoint, lending cash values to policyholders is not really different from lending the funds to an industrial firm or some other borrower; the company must earn interest oil the funds however t hey are used.

The availability of policy loans is a valuable aspect of cash value life insurance. Cash values can be borrowed for emergencies, for making the down payment on a house, or for any purpose that the policyholder wishes. There is no repayment schedule. In fact, the loan never has to lie repaid unless the policyholder wishes to do so. This possibility creates a potential danger, however. Although most people intend to repay their policy loans, the absence of any repayment requirement makes it very easy to put off or avoid repaying. And, when loans are not repaid, the protection provided by the policy is reduced. If the interest on the loan is paid each year, the amount of the loan will remain constant, but interest that is not aid when due is added to the principal of the loan. In the case of a $20,000 loan on a policy with an 8% loan interest rate, the interest is $1,600 the first year. If no payment is made, the amount of the loan becomes $2 1,600 the second year and the interest payable for that year is $1,728 (8 pet-cent of $21,600). By the end of only the sixth year, still

assuming no payment of either interest or principal, the loan would be $31,737.40. The point is that, although policy loans are valuable and convenient, those who utilize them will drain away their protection if they don’t discipline themselves to follow a repayment schedule.

A policyholder, of course, has the right to borrow from the cash value to pay premiums. When the automatic premium loan provision is its force, this borrowing is done automatically whenever the premium is not paid by the end of the grace period. That is, if a premium is not paid, the company will deduct the amount of the premium from the cash value (as a policy loan) in order to prevent the policy from lapsing.

Some policies provide automatic premium loan payments without request. In other cases, the provision will be included (without extra charge), if it is requested when the policy is applied for. Because it prevents unintended policy lapses, the provision is a desirable one. Like regular policy loans, the automatic premium loan can be abused, however. Life insurance purchasers might do well to make sure the provision is included and then forget that it exists.

PAID-UP INSURANCE OPTION

The second non-forfeiture option permits the policyholder to use the cash value to purchase a reduced amount of paid-up insurance. The amount is indicated in the policy’s table of non-forfeiture values.

To illustrate, if Ken Harris elects to stop paying for his $100,000 straight life policy at the end of its tenth year (and again using the values in Table 3), he cats convert the policy to a fully paid-up straight life policy in the amount of $25,200. If he chooses this option at age 65, the amount of the paid-up insurance will be $81,100. The paid-up insurance has cash and loan values, and it pays dividends if the insurance was originally provided by a participating policy.

The option is particularly useful at retirement age. For the typical policyholder, the need for life insurance has diminished by then, but a limited amount to pay for funeral costs and other final expenses is desirable. The retired person therefore may elect to stop paying premiums, put some of the insurance on the reduced amount paid-up option, and use the remaining cash value to supplement retirement income.

EXTENDED TERM INSURANCE OPTION

When the third non-forfeiture option is used, the cash value buys extended term insurance. The amount is not reduced; it is the full face amount of the policy (assuming there is no policy loan outstanding). The length of time the term insurance covers is shown in the table of non-forfeiture values.

Again referring to Table 3, at the end of the tenth year Ken Harris can convert his $100,000 straight life policy to $100,000 of term insurance that will remain in force with no further payment for the next 20 years and 58 days. If he exercises this option at age 65, the term insurance will run for 15 years and 58 days.

Settlement Options

The main purpose of life insurance is to replace income that is cut off by retirement or death. Accordingly, life insurance benefits can be paid in the form of monthly incomes. The payments are made to either the policyholder or the beneficiary, depending upon whether they represent cash value or endowment proceeds in the one case or death proceeds in the other.

The policy proceeds can also be taken in a single lump sum cash payment. In fact, the great majority of proceeds are paid that way.

However, because what the recipient usually needs is more income, use of one of the settlement options may be preferable. The options that may be chosen are: interest income, installments for a fixed period, installments in a fixed amount, or life income.

INTEREST INCOME

Under this option the company holds the proceeds, and the payee (the policyholder or beneficiary) receives interest earnings. A minimum rate of interest is guaranteed in the policy a higher rate is paid if the company's earnings permit.

This option can be used temporarily until a beneficiary decides upon a more permanent arrangement. It also may be used if the proceeds are to be paid to children when they reach a certain age and the primary beneficiary is to receive interest income until that time.

INSTALLMENTS FOR A FIXED PERIOD

This settlement option pays out the principal and interest in a fixed number of monthly or yearly payments. An example was given in Chapter 12 when Charlie Newlin considered buying a $60,000 policy in order to provide income for Ruth while Julie is growing up. The $60,000 would pay at least $400 a month if payments were spread over a 16-year period. Depending upon the company's interest earnings, the monthly payment could be larger.

If this option is chosen, payments continue for the stated period whether or not the initial payee survives. If that person dies before the end of the period, the remaining installments will be paid to his or her estate or to a contingent beneficiary if one was named.

INSTALLMENTS IN A FIXED AMOUNT

This option is similar to the fixed period option. The difference is that in this case a fixed amount per payment is specified, rather than the length of the payment period. For instance, the company might be instructed to make payments in the exact amount of $1,000 per month for as long as the money lasted. Again, the payments will continue after the death of the initial recipient if that person dies before receiving the full amount.

LIFE INCOME

This settlement option guarantees an income for the remaining lifetime of the recipient. In effect, the proceeds of the policy are used to purchase a life annuity.

Life Annuities. A life annuity is a series of payments that continue for as long as the payee (the annuitant) fives. It is the opposite of life insurance. A life insurance policy guarantees that a fund equaling the face amount of the policy will be created, even if the person who buys the policy lives only a short period of time. In contrast, a life annuity guarantees that an existing fund (such as the proceeds of a life insurance policy) will provide a lifetime income, even if the person who buys the annuity lives for a very long time. Both the life insurance policy and the annuity transfer a risk to the insurance company. In the case of the life insurance policy, the risk is dying too soon (before a sum of money can be saved). In the case of the annuity, the risk is living too long; that is, the risk is living beyond the time when the fund would have been used up. Both are based on the law of large numbers, both are insurance, and the rates for both are based on average mortality rates. People who live to pay life insurance premiums for a longer than average length of time help pay for the policies of those who die sooner. In contrast, annuity purchasers who die sooner help pay for the income of those annuitants who live longer than average lives. See Figure 1.

Life Income Options. Most life insurance policies offer three life income options: life only, life annuity with period certain, amid refund annuity. Each guarantees a lifetime income to the annuitant.

The life only option pays a stated amount per month as long as the annuitant lives. The payments end with the death of the annuitant, regardless of how many (or how few) payments the annuitant has received at that time. If the annuitant dies after only two years, the payments stop at that time.

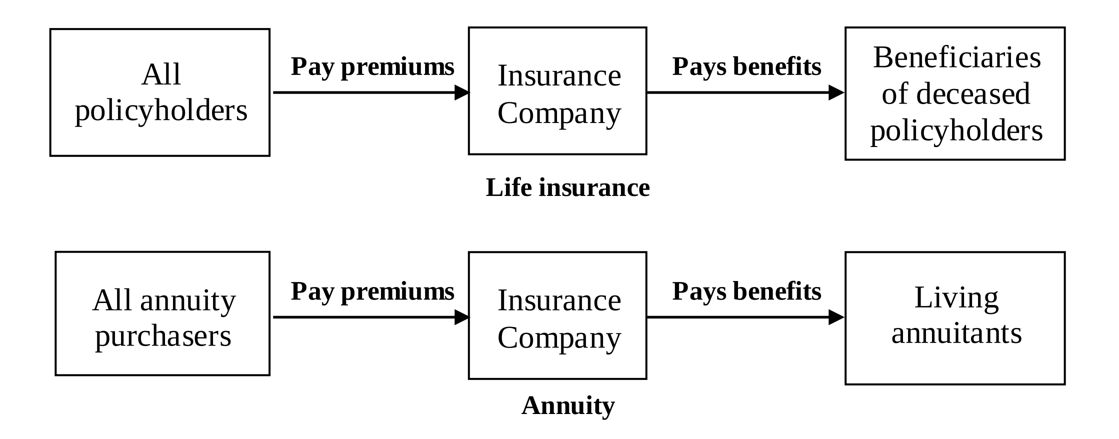


Figure 1 life Insurance and Annuities

A life annuity with a period certain pays as long as the annuitant lives and also guarantees to pay or a certain minimum length of time. For instance, a life annuity with five years certain will pay as long as the annuitant lives, but will pay foe a minimum of five years. If the annuitant dies at the end of the second year, payments will be made to the contingent beneficiary for three more years. Life annuities with 10 or 20 years certain are also available. Naturally, the longer the guaranteed period, the less each payment will be This seen in Table 4.

The other life income option, the refund annuity, also includes a minimum guarantee. In this case, the company promises to pay out in installments at least the full amount of the purchase price. Table 4 shows that a widow, age 70, who is the beneficiary of a $50,000 life insurance policy and who chooses this option will receive a lifetime monthly income of $425.15. If she dies before receiving $50,000 in monthly installments, the company will continue the $425.15 payments to the contingent beneficiary until the full $50,000 has been paid out.

All policies contain tables showing the amounts payable under each of the settlement options. These usually are minimums, the amounts that the company will pay even if it has extremely low interest earnings. Normally, interest earnings are sufficient to pay substantially more. The payments shown in Table 4 are amounts that were actually being paid by a particular company during a recent year; they are much higher than the payments guaranteed in the policies issued by this company.

Table-4 Monthly Life Income Payments offered by One Company to Females, Age 70

Life Income Option	Payment per $10,000 of Proceeds	Payment per $50,000 of Proceeds
Life only	$88.81	$444.05
Life, 10 years certain	84.65	423.25
Life, 20 years certain	76.03	380.15
Installment refund	85.03	425.15

Guaranteed Insurability

Guaranteed insurability is an optional provision that many companies will add for an additional premium. It permits the policyholder to purchase specified additional amounts of coverage at stated times. The value of the provision lies in the fact that the additional insurance is available whether or not the insured is in good health when the option is

exercised. Thus, it literally does guarantee the insured person's continued insurability.

A typical guaranteed insurability rider on a $50,000 policy issued at age 25 permits the purchase of additional $50,000 amounts at ages 28, 31, 34, 37, and 40. If the policyholder elects to) acid the coverage, standard rates at the attained ages apply. Because people who are in poor health are very likely to take advantage of the option to buy more coverage, insurers anticipate adverse selection and price the rider accordingly.

Health Insurance Benefits

Three types of health insurance benefits can be added to life insurance policies: accidental death benefit, waiver of premium, and disability income. These are optional benefits for which additional charges are made.

ACCIDENTAL DEATH BENEFIT

The accidental death benefit is an agreement to pay the beneficiary a multiple of the face amount of the policy if the insured is killed by accident before reaching age 65. In effect, it is the same as a separate accidental death policy. The amount paid is usually the same as the face amount, the policy thus providing "double indemnity" in the event of accidental death. Triple indemnity is offered by some companies. Death caused by suicide, war, or mental infirmity is commonly excluded.

Most life insurance experts frown upon the accidental death benefit feature. They believe that it gives policyholders an exaggerated idea of their life insurance protection, causing them, for instance, to think of a $50,000 policy as providing $100,000 of protection. Also, the logic of the accidental death benefit is questionable. After all, survivors need

the same amount of income replacement whether an insured's death is caused by cancer or by an automobile accident. It probably would be-better to use the money that could be spent for double indemnity to buy a larger face amount of insurance instead.

WAIVER OF PREMIUM

This provision states that future premiums will not have to be paid if the insured becomes totally and permanently disabled before a stated age, usually 60 or 65. All benefits and values provided by the policy, including dividends if the policy is participating, continue just as if premiums were being paid.

This is a desirable benefit. Its cost is quite low, amid it prevents a disabled insured front having to let a policy lapse at a time when it may be especially needed.

DISABILITY INCOME

This provision is not as commonly used as the preceding two, because of the availability of disability income protection under separate health insurance policies. When it is added to a life insurance policy, the disability income rider usually provides $10 per month for each $1,000 of face amount. The benefit is paid if the insured becomes totally and permanently disabled before age 55 or 60. If the insured is still disabled at age 65, the disability income payments stop and the face amount of the policy is paid.

ANNUITIES

We have seen that life annuities are a form of insurance providing a lifetime income arid that they are available as life insurance settlement options. Annuities are more frequently purchased as separate contracts.

They are the basis of either individual or group pension plans. In this section we consider annuities with regard to the various features shown in Figure 2.

Ways of Buying Annuities

Annuities can be purchased in any of four ways. First, they can be obtained with the proceeds of a life insurance policy. One of the life income settlement options can be used to liquidate either the cash value (the annuitant in this case being the insured) or the face amount of the policy after the death of the insured (the annuitant in this case being the beneficiary). Annuities can also be purchased with a cash hump-sum payment the source of which is not life insurance. A person who has accumulated investments in securities or real estate may choose to sell some of the investment at retirement and use the proceeds to buy an annuity. The third method of buying an annuity is through a group pension plan. Many workers are covered by group annuities that provide income after retirement. These arrangements are described in Chapter 13. Fourth. annuities can be purchased by individuals, with premiums being paid in installments during the person’s working years.

Time When Payments to the Annuitant Begin

Annuities are either immediate or deferred, depending on when payment to the annuitant begin. Immediate annuities begin making income payments the first month after they are purchased. An annuity purchased at age 70 with the proceeds of a life insurance policy probably would provide immediate payments. Most annuities are deferred annuities, paid for in installments and with income payments postponed until the purchaser reaches retirement age. Pension plans and individually purchased retirement annuities are usually written on the deferred payment basis.

1. How purchased?
Lump sum: From life insurance proceeds
Lump sum: From savings
Installments: Group pension
Installments: Individual contract

2. When will income payments begin?
Immediately
Deferred

3. How long are payments guaranteed?
Life income only
Life income with minimum number of years guaranteed
Life income with refund of purchase price guaranteed

4. How many lives are covered?
One
Two or more

5. How many dollars will be paid?
Fixed amount
Variable amount

Figure 2 Features of Various Life Annuities

Guarantee of Payments

Life annuities guarantee income payments to the annuitant either (a) for life only, (b) for life with at least a minimum number of payments, or (c) for life with at least the full amount being refunded. These alternatives were described in the earlier discussion of life insurance settlement options.

It thus be emphasized1 that all life annuities guarantee a lifetime income. Refund annuities and those with a period certain also provide a guaranteed minimum pay-out; there is no maximum pay-out other than the fact that no one lives forever. Annuitants who live a very long time receive back as income a total sum that far exceeds the purchase price, even when interest is taken into account.

Number of Lives Covered

Most annuities arc single life annuities, covering the lifetime of one person. Subject to the minimum guarantees furnished by period certain annuities and refund annuities, payments end with the death of the single annuitant.

The joint and survivor annuity covers the lives of more than one person. This arrangement is frequently chosen by retired couples who need income payments as long as either spouse lives. The joint and survivor annuity will pay the surviving spouse the same amount that was paid while both persons for a longer period than it would under a single life annuity. Each monthly form of a joint and survivor annuity that pays a reduced benefit to the survivor. For example, the joint-and-two-thirds-survivor annuity reduces the payments by one-third upon the death of the first spouse. If payments had been $900 a month, they will Continue in the amount of $600 a month for the rest of the survivor's life.

For people who are about to retire, choosing among these options can be tough decision. On the one hand, the single life annuity will provide the highest income. On the other hand, the joint and survivor annuity will provide greater security, but with lower income. The joint and survivor annuity with two-thirds benefit to the survivor frequently is selected as a reasonable compromise.

Fixed or Variable Payments

Most annuities are designed to provide a fixed amount of monthly income, $1,000 a month, for instance (subject to dividend additions, if the contract is participating). During periods when general price levels are rising, this creates a major problem. People living on fixed incomes find that their purchasing power is declining, because for the same number of dollars they can buy less and less in goods amid services. Inflation is especially hard on purchasers of deferred annuities, due to the length of the period that such contracts cover. There may be 50 or 60 years between the time when dollars are first paid toward the purchase of a deferred annuity and the time when the last monthly retirement income check is received. Even a small rate of inflation will destroy much of the buying power of a fixed number (If dollars over that lengths of tulle. In an attempt to cape with this problems, the variable annuity Was devised.

A variable annuity is one that does not guarantee to pay a fixed number of dollars of monthly retirement income. Instead, it pays an income that is based upon the fluctuating value of the common stocks in which the insurance company has invested. The theory is that over a long period of time, common stock prices will move in the same direction as the cost of living. If living costs rise, says the theory, common stock prices will rise also; annuitants will then receive more dollars to pay for the higher costs of food, housing, energy, and so forth.

An example of a variable annuity is the one that is available to the employees of most colleges and universities through the College Retirement Equities Fund (CREF). Employees may divide their premium payments (and their employer’s payments) between CREF and a regular fixed-dollar annuity plan. The CREF variable annuity is divided into an accumulation period and a liquidation period. During the employee’s working years, a number of “accumulation units” are purchased. At retirement, these are converted into “annuity units” that then become the basis of the individual’s retirement income.

Lets assume that Professor Long and this college put $100 into CREF each month. Each payment buys a number of accumulation units. The number of units credited to Professor Long each month depends on the current value of the accumulation unit. The value of the accumulation unit is determined each 0)010th by dividing the current market value of CREF's diversified common Stock holdings by the total number of accumulation units. Each unit thus represents a small fraction of CREF's investment portfolio. If the current value of the stock is $60 million and there are 3 million accumulation units, then the value of each unit is $20. In this case, Professor Long's $100 will buy 5 accumulation units. He will buy fewer units for his $100 when the accumulation unit value is higher and more units when it is lower. As the months and years go by, the number of his accumulation units grows. The total value of these units continues to fluctuate with the monthly calculation of the accumulation unit's value.

When he reaches retirement age, Professor Long will have acquired a certain number of accumulation units. If he has, say, 2,000 units and they are then valued at $40 each, their total value will be $80,000. As we know, the $80,000 could be used to purchase a regular, fixed-dollar annuity at that point. Instead of doing that, however, CREF will convert the 2,000 accumulation units into annuity units. Using a mortality table, CREF will then calculate the number of annuity units that can be allocated each month for Professor Long's retirement income. For instance, this might be 20 units. The dollar value of the annuity units, like the dollar value of the accumulation units, depends of the value of CREF's common stock investments. It is computed each year to reflect the changing value of the investments. The year when Professor Long retires the annuity units might be worth S30 each, giving him an income of 20 x $30, or $600 a month for that year. If stock prices advance, the annuity units may be valued at $31 the next year. If so, Professor Long will receive 20 x $31, or $620 per month that year.

As we have seen, both the accumulation unit and the annuity unit are variable. The units are different because they perform different functions. The accumulation unit measure the current market value of Professor Long’s share in CREF’s investments during the years before he retires. The annuity unit is used to spread out the payments to Professor Long during his retirement years.

The variable annuity is designed to deal with the risk of inflation, but it is not without risks of its own. It is based upon a number of studies that appear to show a correlation between long-run changes in stock prices and long-run changes in the cost of living. Some financial experts say that this correlation does not exist; many say that, although it may exist over a long number of years, it should not be expected to hold for shorter periods. During the 1970s, for instance, stock prices declined while living costs continued to rise. If Professor Long had retired in 1972, his income would have dropped for several years thereafter, while the prices of the things he needed to buy rose each year. The future will reveal whether the variable annuity is based on sound principles or not. In the meantime, many people are following a middle course and are dividing their retirement dollars between fixed and variable annuities.

IMPORTANT TERMS

ordinary life insurance
industrial life insurance
group life insurance
credit life insurance
beneficiary
assignment
grace period
reinstatement provision
incontestable clause
suicide clause
misstatement of age clause
participating policy
nonparticipating policy
dividend options non-forfeiture options
policy loan
automatic premium loan
paid-up insurance option
extended term insurance option
settlement options life annuity
life income options
life only annuity
refund annuity
life annuity with a period certain
guaranteed insurability
accidental death benefit
waiver of premium
disability income benefit
immediate annuity
deferred annuity
single life annuity
joint and survivor annuity
variable annuity

KEY POINTS TO REMEMBER

1. Life insurance is classified as either ordinary, industrial, group, or credit. The first two classes are individually marketed; the last two use the group marketing system.
2. The life insurance policyholder names the beneficiary, who receives the policy proceeds after the insured's death. Beneficiaries may be either revocable or irrevocable. Contingent beneficiaries are usually named also.
3. Life insurance policies can be assigned as collateral for loans, but assignments are not binding on the company unless it receives a copy of the assignment.
4. If the premium is not paid by the end of the grace period (usually 31 days after the due date), a policy will laps.
5. The reinstatement provision permits a lapsed policy to be put back in force within a stated number of years. The person insured must be in good health and overdue premiums must be paid.
6. A life insurance company can void a policy if it finds that the applicant made a false statement that was important to the acceptance of the application. The incontestability clause imposes a time limit of two years for the company to do this. After two years the policy cannot be contested by the company.
7. Suicide is not covered during the first two years of a policy. The purpose of the suicide clause is to avoid adverse selection.
8. If the insured's age is misstated, the amount of insurance becomes the amount that the premiums paid would have purchased if the proper rate had been used.
9. Participating policies pay policyholder dividends; non participating policies do not.
10. Participating policies provide at least four dividend options, permitting dividends to be (a) taken in cash, (b) used to reduce premiums, (c) left with the company to earn Interest, or (d) used to buy paid-up additions to the policy.

11. Non-forfeiture options state three ways in which the cash value of a policy can be used if the policyholder stops paying premiums. The three options are: (a) cash payment, (b) purchase of a reduced amount of paid-up Insurance, or (c) purchase of extended term insurance.
12. The policyholder can borrow the cash value at an interest rate stated in the policy. Unpaid loans and interest are deducted from the policy proceeds that otherwise would be paid.
13. If the automatic premium loan provision is included, unpaid premiums are automatically paid by means of policy' loans.
14. Life insurance policies offer four settlement options in addition to lump sum cash payment; (a) interest income, (b) installments for a fixed period, (c) installments us a fixed amount, or (d) life income.
15. The life income settlement option is a life annuity, a form of insurance that guarantees an income for the lifetime of the annuitant.
16. Most policies offer three life income options: (a) life only, (b) life annuity with period certain, and (c) refund annuity.
17. The guaranteed insurability rider permits the purchase of additional amounts of insurance at stated times regardless of the condition of the insured's health.
18. Accidental deaths benefit, waiver of premium, and disability income are three kinds of health insurance benefits that cats be added to) life insurance contracts.
19. Life annuities often are purchased separately, rather than as a life insurance settlement option.
20. The variable annuity is designed to reduce the impact of inflation. The annuitant's income depends on the value of the insurer's common stock investments.

REVIEW QUESTIONS

1. Distinguish among the four classes of life insurance.
2. What is the difference between a revocable and an irrevocable beneficiary?
3. In what circumstances can a life insurance company contest the payment of a policy’s proceeds?
4. Are policy proceeds paid if the insured commits suicide?
5. Can an insurer void a policy if it discovers that the applicant’s age was misstated?
6. Explain the ‘paid-up additions” dividend option.
7. Why do policyholders have to pay interest if they borrow their cash value?
8. Explain the difference between the paid-up insurance and the extended term non-forfeiture options.
9. What is the difference between the fixed period and the fixed amount settlement options?
10. How do the three life income options differ from one another?
11. Upon what grounds is the accidental death benefit criticized?
12. What is the waiver of premium rider?
13. In what ways can life annuities be purchased?
14. What is the difference between immediate and deferred annuities?
15. Why n-night an annuity be written to cover the lives of more than one person?
16. Upon what theory is the variable annuity based?

DISCUSSION QUESTIONS

1. It sometimes is said that cash value life insurance policies are "flexible in filling the needs of their owners. "In what ways is this true"?

2. Why does the length of time covered by the extended term option (see Table 3) at first increase and then decrease?

3. Why is it suggested that people make sure their life insurance policies include the automatic premium loan provision and then forget that the provision exists?

4. Which is more expensive, participating or nonparticipating life insurance?

5. Does the two-year suicide clause seem to you to be a reasonable policy provision?

6. (a) Applicants for life insurance often must pass a medical examination. Why? (b) Applicants for annuities don't have to take a medical examination. Why not?

7. Why would anyone in their right mind buy an immediate life annuity, when they could instead invest the money, use the interest income, and leave the principal to their heirs?

8. On refund annuities, the insurance company promises to pay out at least the full purchase price, and if the annuitant lives long enough, it will pay out much more than that. Isn't this a money-losing proposition for the insurance company? How can it afford to do this?

9. Does it seem to you that the variable annuity is based on sound principles? Would you advise your parents to buy one?

CHAPTER 7

SOCIAL INSURANCE

CHAPTER OBJECTIVES

After studying this chapter, you should be able to:

1. Outline the ways in which social insurance differs from private insurance.
2. List the principal provisions of the social security program.
3. Describe the role of private Medicare supplement insurance.
4. Explain how social security is financed and describe the funding method that is used.
5. Discuss the difficulties that social security will confront during the next half-century.
6. State the purpose of unemployment insurance and indicate what persons qualify for its benefits.

Our study of insurance emphasizes pri9vate insurance, that which is written by private insuring organizations. Insurance provided by the government is another major source of financial security. We saw an indication of its importance when we looked at Charlie and Ruth Newlin's life insurance program. Their largest single source of income protection was social security. In addition to income for survivors of deceased workers, social insurance furnishes disability income protection, retirement pensions, health care cover ages, and unemployment compensation.

In this chapter, we examine the nature and development of social insurance, with emphasis on the U.S. social security program.

THE NATURE OF SOCIAL INSURANCE

Social insurance is provided by government on a compulsory basis. A more precise definition is difficult because governments can use their power and resources to apply the insurance method or modifications of it in a wide variety of ways. In other words, social insurance can take whatever form a particular country wants it to take. In the United States, there has been a tendency to look first to the private insurance industry for coverage of the risks that society deems important. When such risks have not been handled adequately by private insurance, social insurance plans have been established.

Characteristics of Social Insurance

A closer look at social insurance reveals several ways in which it differs from private insurance:

1. Social insurance is based on law rather than on contract. Both the cost and the benefits are established by and can be changed by governmental units.

2. Participation is compulsory for all persons to whom the law applies they cannot chose to decline to participate, nor can they select the coverage or the amounts of the benefits.

3. Social objectives are primary. The purpose is to provide some minimum level of economic security for large portions of the population. This is called the **floor of protection concept.** The philosophy is that in an economic system that stresses free enterprise and individual initiative, people should not rely entirely on governmental programs. Social insurance is designed to guarantee economic security at minimal levels; those who want and can afford more adequate benefits obtain them through personal savings and private insurance.

4. The benefits of social insurance are weighted in favor of certain groups, usually those with low incomes. This approach is necessary in order to carry out the social objectives. Unless low-income groups are subsidized by high-income groups, the total revenues of the system will not be large enough to furnish the minimum levels of protection that are desired. At the same time. However, each person's benefits generally are somewhat related to his or her contributions.

5. Social insurance usually covers only those who are or who have been employed. Most social insurance plans concern the interruption of in come (by death, disability, unemployment, or retirement) earned through employment. Covering employed persons also facilitates the collection of premiums, often by means of payroll taxes.

Social Insurance and Public Assistance

Social insurance should not be confused with public assistance programs. The latter include federal and state aid to families with dependent children; the federal Supplementary Security Income program for the aged, blind, and disabled; and the state Medicaid programs that pay medical and hospital charges that Medicare does not cover.

The difference between social insurance and public assistance is that public assistance payments are given to those who show they need them, whereas social insurance payments go to al who are eligible for them, regardless of need. A further distinction is that public assistance programs are financed by the general revenues of the government, whereas most social insurance programs are financed by specific payroll taxes paid by participating individuals and their employers.

THE DEVELOPMENT OF SOCIAL INSURANCE

The United States has not been a leader in the development of social insurance programs. Several European countries established programs before 1900; ours did not get underway until 1935. Today, numerous countries have more extensive programs than we do. The U.S. program is expanding rapidly, however. Aggregate payments to benefit caries have increased from 6% of total U.S. personal income in 1960 to over 11% today.

The development of our social insurance program is closely associated with several changes in U.S. society, including industrialization and urbanization, a rising standard of living, an increasing percentage of elderly persons, and growing concern for the well-being of others.

Until this century, U.S. society was chiefly agricultural. Most people were members of large, virtually self-sufficient farm families. Economic security was provided by the family and by the farm itself. With growing industrialization, population gradually shifted to the cities,

and employment changed from farm to factory. This movement continued until the mid-1900s. For the bulk of the population, self-sufficiency became a thing of the past; almost everyone now depended on wages earned from outside employment. If the family wage earner was laid off, died, or became disabled, the family was threatened with destitution. Although some families were very well off indeed, many found it difficult or impossible to provide for their financial security through savings or insurance. The demands on private charity, religious and ethnic organizations, and local poor relief programs son exceeded their ability to respond. Increasingly, people turned to the state and federal governments. Until this time, most people had believed that the government's role was to provide a stable and secure environment within which individuals could work to achieve their own well-being. But many Americans adopted the view that government was directly responsible for guaranteeing at least a minimum level of welfare for all.

As the nation prospered and living standards rose, attitudes about the minimum acceptable level of well-being changed. The floor of protection concept developed. The economy seemed capable of furnishing subsistence for all, and social insurance was regarded as an appropriate method of providing it.

Another factor accounting for the growth of social insurance is the increasing percentage of older persons in the U.S. population. Today over 10% of al Americans (more than 25 million persons) are age 65 or above. This percentage has about doubled during the last two generations. Not only are there many more elderly persons, but thanks to better health care and improved living conditions, they are living longer than ever before. Thus, the elderly have more years of retirement, a longer period during which they cannot depend upon their current earning to supply the necessities of life.

Finally, there has been growing concern for the welfare of others, an increase in the feeling that society as a whole has some responsibility for all its members. To some extent, this philosophy seems to be replacing the concept that responsibility rests with the individual, the family, and

the local community. We should realize, however, the social insurance is only one part of our system; it is but one of the things that contributes to our economic security. Government programs have by no means replaced individual responsibility. Although social insurance can be relied on to provide a minimum floor of protection, really satisfactory levels of protection require that it be supplemented by group insurance and by individual savings and insurance.

OLD AGE, SURVIVORS, DISABILITY, AND HEALTH INSURANCE

Almost all of the U.S. social insurance program is based on the Social Security Act of 1935. This Act originated during the administration of President Franklin Roosevelt and was part of what came to be known as the New Deal program. Since then, it has been amended and expanded in a variety of ways. Today 9 out of 10 Americans are protected by social security; nearly 1out of 6 receive monthly social security income checks.

Originally, social security provided only retirement benefits. In 1939, the Act was amended to pay income to the survivors of deceased workers. Disability income coverage was added in 1954, providing benefits to workers who become totally disabled. In 1965, the program was expanded to include Medicare, furnishing hospital and medical protection for persons age 65 and over.

Although the Social Security Act includes programs of public assistance as well, our concern is its social insurance cover ages. These constitute the Old Age, Survivors, Disability, and Health Insurance **(OASDHI)** program, which is commonly referred to as **social security.** We will review each of the four cover ages and see who is protected by them. we also will consider how the program is financed. Because OASDHI rules and benefits frequently are changed by Congress, detailed descriptions are impractical. Local offices of the Social security Administration can supply the latest detailed information.

Retirement Benefits

The basic retirement benefit is paid to those who retire at age 65. Workers can retire as early as age 62, but their payments are permanently reduced if they do. The reduction is 20% at age 62, 131/3 % at age 63, and 62/3% at age 64. Higher benefits are paid to those who delay their retirement beyond age 65. And additional 3% is added for each year that a person continues working from age 65 to age 72. The retirement benefit is increased for those with dependents. The spouses and the dependent children (under age 18) of retired workers also receive benefits under this section of the program.

The size of the monthly retirement income checks is also related to the amount of the retiree's covered wages (the amount on which social security taxes were paid). Table 7-1 indicates the maximum covered wages and the tax rate paid by employees and matched by their employers since 1970. In computing an individual's benefit amount, his or her covered wages are adjusted to take account of year-by-year changes in the country's average wage levels. These adjusted earnings are averaged together, and a formula is applied to determine the benefit amount. This method, called wage-indexing, is designed to keep benefit amounts in line with the recipient's real pre-retirement earnings in spite of inflationary increases in wage levels during his or her career. The benefit formula is weighted so that low-wage workers receive a greater percentage of their pre-retirement incomes than high-wage earners. It is estimated that in 1986, social security retirement benefits will replace about 5% of pre-retirement earnings for low-income earners, about 42% for those with average earnings, and about 25% for high-income employees.

Since 1972, social security benefit payments have been tied to the cost of living. The law provides that, if living costs increase by 3% or

more in any year, payments will be increased by the same amount starting with the checks issued for the following July.

To be eligible for retirement benefits, a person must have credit for a certain amount of employment under social security. The length of employment required depends on the worker's age and the date of his or her retirement. For instance, persons reaching age 62 in 1983 need eight years of employment. Those who reach 62 in 1991 or later will need ten years.

Table 7-1 Basis of Social Security Taxes

Year	*Taxable Annual Wages*	*Rate Paid by Employers and Employees (percentage)*
1970	$ 7.800	4.80
1971	7.800	5.20
1972	9.000	5.20
1973	10.800	5.85
1974	13.200	5.85
1975	14.100	5.85
1976	15.300	5.85
1977	16.500	5.85
1978	17.700	6.05
1979	22.900	6.13
1980	25.900	6.13
1981	29.700	6.65
1982	32.700	6.70
1983	35.700	6.70
1984	A	7.00
1985	A	7.05
1986	A	7.15
1988	A	7.51
1990	A	7.65

a Adjusted annually.

b For 1984, employees receive 0.3 percent tax credit, making their net rate 6.70.

Survivor Benefits

Survivor benefits are payable to the families of deceased workers. These benefits, which are comparable to life insurance, include:

1. Income for widows or widowers with children under age 16.
2. Income for children under 18.
3. Income for widows or widowers age 60 or above.
4. Income for dependent parent or parents.
5. A small lump-sum payment (usually $255).

To illustrate, if a man dies at age 40h, leaving a widow and a ten-year-old child, monthly income payments will be made to the widow and child. The payments to the widow stop (during the so-called blackout period) from the time the child becomes 16 until the widow reaches age 60. At that time she becomes eligible for a lifetime pension. Her pension payments will be larger if she delays receiving them until she is older than 60. The child's payments continue until he or she reaches age 18.

To be eligible for survivor benefits, a certain amount covered employment is required, as is for retirement benefits. However. In this case a shorter period may be sufficient, depending on the age at which the worker dies. A worker who dies at age 28 or younger needs to have been working only $1^{1/2}$ years. Those who are older at the time of their death need longer periods of employment, the maximum being 10 years.

Both survivor benefits and retirement benefits are subject to an earnings test. Survivors and retirees who earn no more than a certain limit per year from current wages receive full benefits. Those who earn more than that amount lose $1 in benefits for each $2 they earn above the limit. The limit in 1983 for people under 65 was $9.920; for those 65 to 70, the

limit was $6.600. The test does not apply to persons 70 or above. To illustrate, assume that a young widow with children qualifies for survivor benefits of $600 a month, or $7.200 a year. If she works and earns $10.920 a year, her earnings exceed the $4.920 limit by $6.000. As result, her benefits will be reduced by half of the $6.000, or $3.000, and will be $4.200 rather than $7.200. Thus, her total income will be $10.920 from her employment plus $4.200 from social security, or a total of $15.120.

The earnings test applies only to current wages. Income from savings, investments, or other pensions is not counted because to do so would discourage people from saving and investing for their retirement years.

Disability Benefits

Workers who become totally disabled before they retire and who are expected to remain disabled for at least 12 months receive **disability benefits.** Disability is defined as a physical or mental condition that prevents the person from engaging in any substantial gainful work. In interpreting this definition, the word "2ny" is not applied literally. In other words, people are judged to be disabled if they cannot do work that is reasonable in view of their age, education training, and experience.

The monthly benefit amount is the same as the person would receive for retirement; it is increased when there are dependents. Payments begin after a five-month waiting period and continue until age 65, at which time they become pension payments in the same amount. Notice that the disability benefits provide monthly income only; they do not specifically pay for the costs of medical treatment.

The eligibility requirements for disability benefits are rather similar to those for survivor benefits, again ranging from $1^{1/2}$ to 10 years, depending on the worker's age.

Medicare

Medicare is a health insurance program for people aged 65 and above. It also covers certain disabled persons below that age. The program has two parts. Art A is a basic hospital insurance program; Part B is a voluntary program of medical insurance. The hospital insurance helps pay for in-patient hospital care and for follow up care after a patient leaves the hospital. The medical insurance pay for doctors' services, outpatient hospital services, and certain other medical items and services not covered by the hospital insurance.

It is difficult to summarize Medicare benefits because they are determined by a tangle of complex rules and regulations. Furthermore, the deductible amounts are changed rather frequently. The brief summaries that follow indicate the benefits effective in 1983.

MEDICARE HOSPITAL INSURANCE

The hospital insurance part (Part A) of Medicare provides the following benefits:

1. Up to 90 days of in-patient hospital care for each benefit period. (A benefit period begins when a patient enters a hospital; it ends when the patient has been out of the hospital or nursing home for 60 days.) The patient pays an initial deductible amount ($304 in 1983) for the first 60-day period of hospitalization. After that, there is a $76 daily deductible for the 61^{st} through the 90^{th} day.
2. An extra 60 days of in-patient hospital care, subject to a $152 daily deductible. These are called "reserve days." They can be used anything during a person's lifetime when he or she is hospitalized for more than 90 days in a single benefit period.

3. Up to 100 days of in-patient care in a skilled nursing facility. After the first 20 days, the patient pays $38.00 per day for the remaining 80 days. The nursing home care must begin within 30 days after being hospitalized for at least 3 days.
4. Home health care services, covering part-time nursing care, physical therapy, and speech care. An unlimited number of visits are covered, but there is no coverage for full-time nursing care at home, drugs, meals, or homemaking services. The care must be medically necessary, and the patient must be home-confined.

Medicare hospital insurance pays for semiprivate hospital and nursing home rooms, regular nursing services, and special care units such as a hospital's intensive care facility. It also pays for drugs furnished to in-patients of a hospital or nursing home. It does not cover doctor's bills or the cost of private nurses.

This part of Medicare is compulsory, being financed by a payroll tax that is paid by employers and employees. Practically everyone at least 65 years old is eligible, whether or not they have retired.

MEDICARE MEDICAL INSURANCE

Part B of the Medicare program is available on a voluntary basis to anyone age 65 or above. Those who want **Medicare medical benefits** pay a monthly premium ($13.50 through June 1984): the government supplements those funds out of its general revenues. Coverage must be applied for at a local social security office. The insurance provides the following benefits:

1. Physicians' services (including surgery) at home, in a doctor's office, in a hospital, or elsewhere. Drugs that cannot be self-administered are included.

2. Home health visits. Medically necessary visits by nurses and other health professionals are covered.
3. Physical therapy and speech pathology services.
4. Other medical services and supplies, such as outpatient hospital services, diagnostic services, X-ray and radiation treatment, certain ambulance service, and rental or purchase of durable medical equipment including wheelchairs and hospital beds.

There is an annual $75 deductible, after which the medical insurance pays 80% of the patient's expenses. The benefits do not include routine physical checkups, prescription drugs, glasses, hearing aids, or full-time nursing care in the patient's home.

ADMINISTATION OF MEDICARE

Although Medicare is a part of the social security program, much of its administration is handled by private organizations. These organizations, including Blue Cross, Blue Shield, and commercial insurance companies, operate under contracts with the social security administration. They determine the reasonableness of the charges for services and disburse the funds to health agencies and physicians.

PRIVATE MEDICARE SUPPLEMENT INSURANCE

Although Medicare provides important protection, its benefits are limited. It is estimated that the program pays only about 40% of the total health car costs of the nation's elderly. It is essential, therefore, for most people over 65 to supplement Medicare's protection with private health insurance. **Private Medicare supplement insurance,** sometime called "medigap" coverage, is designed to make up at least part of the difference between actual medical bills and the amounts paid by Medicare.

Many group health policies can be converted to some form of Medicare supplement when covered employees reach age 65 and can be continued after retirement. Such group protection usually is less expensive and often provides more coverage than policies purchased individually.

Unfortunately, elderly persons seeking to buy individual policies to supplement Medicare face an array of contracts ranging from man whose coverage is generous to others that are nearly worthless. Worse yet, they may encounter agents who use high-pressure, scare tactics to induce them to buy overpriced and redundant policies. The Federal Trade Commission found that 25% of the people with supplement policies had overlapping cover ages. An investigation in Illinois identified one woman, age 80, who had been conned into spending $50.000 in three years for thirty-one policies! In response to revelations like these, the states (under pressure from the federal government) have established minimum standards for policies that are sold to supplement Medicare. Only the ones that meet these standards can legally be called Medicare supplement policies. Table 7-2 illustrates one such policy.

Table 7-2 Example of Medicare Supplement Insurance

Service	*Benefit period*	*Medicare Pays in 1983*	*One Medicare Supplement Policy Pays*
Medicare Part A:			
Hospitalization	1st 60 days 61st-90 days	All but $304	$304
		All but $76 a day	$76 a day
	91st –150th days	All but $152 a day	$152 day
	Beyond 150 days	Nothing	90% of actual costs for 365 days
Post-hospital	1st 20 days	100%	
Skilled nursing care	21st-100th days	All but $38.00 a day	$38.00 a day
	Beyond 100 day	Nothing	50% for 265 days
Medicare Part B:			
Medical expense		80% of eligible expenses after $75 annual deductible	Remaining 20%, subject to $7.500 annual maximum
Self-administered prescription drugs		Nothing	50% of cost

The new regulations will improve matters in this area, but the problems will not disappear. Human nature being as it is, some agents and some insurers will continue to prey upon this particularly susceptible market. Limited policies, including cancer insurance and hospital indemnity policies, will continue to be sold. Fortunately, satisfactory Medicare supplement policies such as the one outlined in Table 7-2 now are available. Many of the state insurance departments offer printed materials designed to help the elderly shop successfully for this very important protection.

FINZNCING THE OASDHI PROGRAM

The OASDHI program (except Medicare medical insurance) is financed by a special payroll tax paid by employers and employees and an earnings tax paid by self-employed people. When the program began in 1935, the payroll tax was 1% on the first $3.000 of a worker's annual

income. As the benefits have grown, both the tax rate and the amount of annual income subject to the tax

In 1983, the tax for employed persons was 6.7%. As of then, the law called for increases that would bring the rate to 7.65% in 1990. The maximum amount of income subject to the tax in 1983 was $35,700. This amount is increased automatically as average income levels rise. The full tax is paid by both employee and employer. Workers earning $35.700 or more in 1983h paid $2.392 in social security taxes, with an equal amount being paid by their employers.

The taxes are collected by the Internal Revenue Service and deposited in social security trust funds. Both benefit payments and administrative costs come from the trust funds. Amounts not currently needed are invested in U.S. government securities.

Funding Method. The funding of OASDHI differs from that of life insurance, a point that sometimes causes confusion. A life insurance company must be "fully funded"; that is, the funds on hand always must be enough so that they, together with future income from premiums and investments, will be sufficient to meet all of the company's future obligations. Because of this requirement, life insurance companies set aside a large part of their revenues each year to pay policy benefits during the years ahead. Over a period of many years the amounts set aside in this manner reach enormous proportions. (One company alone has over $60 billion.)

The social security program is funded differently. Since 1939, it has been on a "pay-as-you-go" basis; the benefits it pays each year come almost entirely from that year's revenues. Because benefits are paid from current revenues rather than from those of previous years, the social

security system does not accumulate vast funds. In a recent year its trust funds totaled “only” $23 billion.

Confessor arises when OASDHI funding is compared with life insurance funding. If they are judged on the same basis, it may appear that the social security program is practically broke because its funds are so limited. But such a comparison is incorrect and the conclusion is unjustified. Life insurance companies are private organization; they are required to be fully funded to protect policyholders and beneficiaries. Social security, on the other hand, is based on the taxing owner of the federal government. If it is assumed that the program will be continued indefinitely, there is no need for it to be funded the way that private insurance companies are or to have the enormous funds that they do.

The “Brink of Bankruptcy”? The method of funding aside, many people fear that future OASDHI tax revenues will not be sufficient to pay the benefits that have been promised. For instance, President Reagan once said that the “Social Security system is teetering on the brink of bankruptcy.”

Actually, the usual concept of bankruptcy is misleading when applied to social security. To most people bankruptcy means going out of business, and the term often gives rise to the image of a boarded-up restaurant or filling station. When the concept of bankruptcy is applied to OASDHI, one may envision millions of destitute elderly or disabled people who no longer are receiving the monthly checks they had relied upon. But unless our economic system suffers an utter and complete collapse, bankruptcy in that sense will not happen to the social security system. The reason it will not happen is its pay-as-you-go funding. For, as long as people in this country work and earn incomes, they will pay social security taxes. And as long as tax dollars flow to the Social Security Administration, benefits will be paid.

But, one may ask, couldn't the amount of payroll tax revenues turn out to be inadequate to pay the promised OASDHI benefits? The answer is yes, they could. The relationship between income and outgo can get out of balance; if that happens, monthly checks may have to be delayed or even reduced. But the system will not grind to a halt and go out of business; the windows will not be boarded up. When that point is understood, one then is in a position to consider social security's problems more realistically.

The real problems of social security, as they relate to the system's financial stability, concern the possibility that revenues will fall short of the total amount needed to pay scheduled benefits. Two sets of causes lie behind this potential imbalance between income and outgo: economic causes and demographic causes.

Economic factors have caused OASDHI benefits to increase more rapidly than the system's revenues during recent years. When benefits were tied to the Consumer Price Index in 1972, it was assumed that prices would continue to rise at about 3% a year and that wages would rise at a somewhat faster rate. Those assumptions seemed reasonable at that time. However, during the late 1970s and early 1980s, not only did prices soar, but wages lagged behind prices, as a prolonged recession gripped the nation. Consequently, benefit payments increased due to automatic cost-of-living increases, while at the same time revenues from payroll taxes fell below projected levels. During that period, the Social Security Administration paid out considerably more each day than it collected, causing a severe drain on the system's trust funds.

On a longer range basis, several demographic factors present more fundamental problems for OASDHI. The factors, which relate to such things as birth rates, mortality rates, and average retirement age, are crucial because of their impact on the balance between tax revenues and

benefit payments in future decades. In addition, demographic factors determine the relative size of (a) the working, tax-paying population and (b) the retired, pension-receiving population. Three demographic factors are especially important. First, the total population of the United States increased rapidly for a period of time following World War II; since then, the population growth rate has slowed. The first contingent from the postwar "baby boom" will reach age 65, presently the normal retirement age, in 2010. Second, people are retiring earlier. In 1960, 4 out of 5 men aged 60 to 65 were in the work force. By 1979, only 3 out of 5 men in that age group were employed. The third demographic factor is a significant improvement in the mortality rate of the elderly; old people are living longer. In 1900, 29% o the retired population was age 75 or order, by the year 2000, 43% of retired Americans will be 75 or above.

A foreseeable result of these three developments is a change in the ratio of workers to retirees. There now are three workers providing old-age benefits for each retired person. By 2025, the ratio is expected to be two-to-one. As the number of workers available to support each retiree drops, it appears that tax rates will have to be increased drastically if scheduled benefit levels are to be maintained.

One alternative, of course, would be to reduce benefits; that is, the size of the monthly checks being sent to retirees could be cut. Another alternative would be to raise the retirement age. Many people believe that the increased longevity of our population justifies changing the age at which full retirement benefits are payable from the present 65 to age 68 or 70. Whether or not these actions are taken, additional increases in the payroll tax rates seem inevitable.

Is a large tax increase feasible? Or, as some people assert, have taxes already become so burdensome that the general public simply will not put up with more increases? Table 7-3 indicates that some countries

have much higher social security tax rates than the United States. This suggests that our

Table 7-3 Cost of Social Security, Selected Countries, 1979

Country	*Tax Rate as Percentage of Payroll*		
	Paid by Employee	*Paid by Employer*	*Total*
Canada	3.15	4.69	7.84
France	10.04	37.41	47.45
Germany, West	16.40	18.15	34.55
India	9.50	12.00	21.50
Japan	8.80	10.15	18.95
Mexico	3.75	12.22	15.97
Netherlands	22.75	24.01	46.76
Norway	9.40	16.50	25.90
Panama	6.75	13.22	19.97
Philippines	4.55	7.25	11.80
Sweden	-	31.25	31.25
Switzerland	4.60	7.60	12.20
USSR	-	6.00	6.00
United Kingdom	6.50	13.50	20.00
United States	6.13	10.63	16.76

Source: Robert J. Myers, Social Security (Homewood, II. :R. D. Irwin, 1981), p. 837.
Note: Includes cost of benefits for work injuries.

Rates *can* go much higher if we decide they should; whether the rates *should* or *will* go much higher are unanswered questions. In the opening paragraphs of this chapter, it was noted that social insurance can take whatever form a particular country wants it to take. Some nations have built far more expensive-and costly-systems than others. In other words, those nations allocate relatively more of their resources to public economic security programs than others do. The form that our social insurance system will take in the future, and the cost of that system, remain to be determined.

UNEMPLOYMENT INSURANCE

Unemployment insurance is designed to provide short-term protection *for* regularly employed persons who lose their jobs and who are willing and able to work. Each state has its own unemployment insurance program. The state programs came about because of one of the provisions of the federal Social Security Act. That Act provided for a federal unemployment insurance program to be financed by a special payroll tax. Because the intent really was to encourage the states to establish programs of their own, the federal law provided that most of the tax revenues would be returned to any state that set up an acceptable unemployment insurance plan. Not surprisingly, all of the states established such plans.

The federal tax, which is paid by employers, is 3.5% of the first $7.000 of annual wages paid to each employee. The federal government's portion of the revenues is used to pay federal and state administrative costs and to help finance additional benefit payments during periods of high unemployment. To strengthen their programs, many states have elected to set higher tax rates and higher taxable wage bases. Many employers actually pay considerably less than even the stated 3.5% rate, however. This is due to an experience rating system which is designed to encourage stable employment practices. Experience rating reduces the tax rate of firms for which few unemployment claims have been paid.

Eligibility and Benefits

Federal law sets minimum standards for eligibility and benefits. As long as they meet the federal standards, the states can set their own eligibility rules and benefit patterns.

Eligibility rules normally require that the worker (a) has been regularly employed in the past, (b) is unemployed through no fault of his

or her own, (c) is able to work, and (d) is available for work and is willing to accept suitable employment. Al of the states require persons receiving unemployment compensation to register for work with the state employment service, and most require further evidence that they are actively seeking jobs.

Weekly benefit payments are based on the worker's past earnings, subject to stated minimum and maximum amounts. The payments average about $115 a week but vary considerably from one state to another; in some states, they are twice as high as in others. Recipients usually must have been out of work for a week before payments begin.

In most states, the normal maximum benefit period is 26 weeks, During periods of high general unemployment, the Congress frequently has adopted legislation permitting the states to set somewhat longer benefit periods, with part or all of the extra cost being financed by the federal government. The relatively short maximum benefit periods are consistent with the intent of the laws, which is to provide a basic level of economic protection to workers who are involuntarily unemployed for short periods of time. Unemployment insurance is not designed to deal with joblessness in chronically depressed industries or localities, nor is it intended to handle the layoffs associated with lengthy and widespread economic depressions. Such problems must be dealt with in other ways, such as with area redevelopment programs or monetary and fiscal policies.

Disabled workers are not eligible for unemployment insurance benefits in most states. If their disability was caused by an occupational injury or illness, they, of course, receive workers' compensation benefits. If their disability is due to other causes, they may be eligible for payments from group or individual non-occupational disability income policies. Those who are totally disabled for a year or more may be eligible for OASDHI benefits. Five states (California, Hawaii, New Jersey, New

York , and Rhode Island) and Puerto Rico have separate **temporary disability insurance laws.** These laws furnish income for workers who are disabled due to a non-occupational injury or illness. The benefits are similar to those provided by unemployment insurance.

Problems

Scarcely anyone is satisfied with the unemployment insurance programs. Workers and labor unions complain that the payments are to small, the benefit periods to short, and administrative standards too strict. At the same time, employers say that the costs are to high because benefits are paid to many people who should not receive them. they point to cases when strikers and seasonal workers have refused to accept jobs that appeared to be suitable.

The use of experience rating to lower the cost of unemployment insurance for some employers is especially controversial. This device is intended to discourage unnecessary layoffs, but its critics say that it really has little or no influence upon employment practices. Employment in some firms is stable by the nature of the industry they are in, while the seasonal or cyclical nature of other firms is beyond the control of their managers. Furthermore, it is alleged that experience rating induces some employers to resist the payment of valid unemployment claims.

IMORTANT TERMS

social insurance
floor of protection
Concept
OASDHI
Social security
retirement benefits
survivor benefits

earnings test
disability benefits
Medicare
Medicare hospital
Insurance
Medicare medical
insurance

Private Medicare
supplement insurance
Unemployment
Insurance
Temporary disability
insurance laws

KEY POINTS TO REMEMBER

1. Social insurance is provided by the government on a compulsory basis. It has social objectives, including the provision of a minimum level of economic security for a large part of the working population. Benefits usually are weighted in favor of low-income groups.
2. Social insurance differs from public assistance in that payments are made to all eligible persons as a matter of right, rather than only to those who are in need. Also, social insurance is financed by special payroll taxes instead of from the government's general revenues.
3. After a slow beginning, the social insurance program of the United States is expanding rapidly. It is designed to furnish only a floor of protection; more adequate protection requires group or individual insurance.
4. The U.S. social security program primarily consists of the cover ages provided by OASDHI: retirement pensions, survivor benefits, disability income benefits, and health insurance for persons over 65.
5. Medicare, the health insurance portion of social security, consists of two parts: a compulsory hospital insurance program and a voluntary medical insurance program.
6. Private Medicare supplement insurance is available to help fill the gas not covered by Medicare.
7. Social security (except medical insurance) is financed by special payroll taxes paid by employers and employees. It is funded on a "pay-as-you-go" basis.
8. The cost of social security to the current working population is affected by several economic and demographic factors. After the turn of the century. The cost will rise considerably if scheduled benefit levels are maintained.
9. State unemployment insurance programs provide short-term income protection for people who lose their jobs and who are willing and able to work.

REVIEW QUESTION

1. In what ways does social insurance differ from private insurance?
2. What changes in U.S. Society were associated with the development of social insurance?
3. List the types of protection provided by the U.S. social insurance program.
4. What factors determine the amounts of social security retirement benefits that a person receives?
5. What is the blackout period?
6. What is the earnings test?
7. What major benefits are provided by the two parts of Medicare?
8. Why is there a need for private Medicare supplement insurance?
9. How is social security paid for?
10. What are some of the economic demographic factors that affect social security income and payment levels?
11. What are the usual eligibility requirements for unemployment insurance?

DISCUSSION QUESTIONS

1. Certain changes in U.S. society have been associated with the development and growth of social insurance. Do you believe that the changes mentioned in the text justify the social insurance system that now exists?
2. Social insurance benefits are weighted in favor of low-income groups; that is, to some extent their benefits are subsidized by higher income workers. Is this fair?
3. Social insurance programs are designed to furnish only a floor of protection. Is this the right objective, or should the goal be to provide a comfortable level of living?
4. How can it be logical to fund social security on a “pay-as-you-go” basis when life insurance companies must be fully funded?
5. Would it be desirable to expand social insurance enough to eliminate the need for private insurance?
6. Some people say that social security (OASDHI) actually is not insurance at all; others say that it is. What arguments can be presented pro and con?
7. What changes do you believe should be made in the social security system?

CHAPTER 8

MORTALITY TABLES

PROBABILITY

In the life insurance business, the subject of *probability* has its application in calculating and using the *probabilities of living and dying*. The concern is with predicting future deaths, based on past experience. In so doing, it is usually expedient to use "mortality tables". This chapter deals with the subject of *probability* and its use in connection with "mortality tables". Succeeding chapters will show how *compound interest* and *probability* are combined in life insurance calculations.

In general, the probability, or likelihood, of some event occurring is expressed mathematically as a fraction or a decimal. This indicates how many times the event may be expected to occur out of a certain number of *opportunities* for it to occur. For example, the probability in one attempt of drawing an ace at random out of a deck of 52 playing cards (four of which are aces) is $\frac{4}{52}$, or simply $\frac{1}{13}$. This means that an ace can be expected to be drawn one time out of every 13 attempts.

If a very large number of such attempts were made, it would actually happen that very nearly $\frac{1}{13}$ of the attempts produce aces. This statement can be made with certainty, because of the statistical concept known as the *law of large numbers*. In this particular example, the probability given (namely $\frac{1}{13}$) could be derived either by:

1) Exact mathematical calculation, since it is known that $\frac{1}{13}$ of the cards in the deck are aces; or

2) By observation of a large number of attempts, and calculation of the ratio of aces drawn to total draws.

A second example might be the probability of a person now age 75 dying within the next year. This probability might be 0.07337. This decimal may be expressed as the fraction $\frac{7.337}{100.000}$. This means that persons now age 75 can be expected to die within the next year out of every 100,000 such persons now alive. In this particular example, the probability can be derived only by the observation of a large number of persons age 75. It cannot be calculated exactly by prior knowledge, as was the case with the aces in a deck of cards.

To Illustrate - A doctor has attended 5,000 births in his lifetime and observed that on 57 of these 5,000 occasions the birth was a multiple one (twins, triplets, etc.). Calculate the probability of a birth being a multiple one.

Solution

If the event occurs 57 times out of 5,000 opportunities for it to occur, the probability can be expressed as the fraction $\boxed{\frac{57}{5000}}$

If 57 is divided by 5,000, the above probability can be expressed as a decimal: $\boxed{.0114}$

Note that 5,000 - 57 = 4,943 of the observed births were not multiple. Hence, the probability that a birth will not be multiple can be calculated as: $\boxed{\frac{4943}{5000}}$

When an event is certain to occur, the probability of its occurrence is 1. This is evident because, if the event were to occur upon every opportunity, then the numerator of the fraction would always be the same as the denominator. Any number divided by it self is 1.

It is often necessary to consider probabilities involving the happening of more than one event. One of the two following rules Will usually be applicable:

Rule 1. *If only one out of several events can occur, the probability that one such event will occur is the sum of the probabilities of each individual event happening.*

For example, if the "several events" are

1- Drawing an ace when one card is drawn from a deck, or

2- Drawing a king when the above drawing takes place,

It will be seen that only one can occur at any one time. Hence, the probability that one of the events will occur (that the draw will produce either an ace or a king) is the total of the probabilities of each event happening individually:

$$\boxed{\text{Total Probability} = \begin{pmatrix}\text{Probability of} \\ \text{Drawing an Ace}\end{pmatrix} + \begin{pmatrix}\text{Probability of} \\ \text{Drawing a King}\end{pmatrix}}$$

$$= \frac{1}{13} + \frac{1}{13}$$

$$= \frac{2}{13}$$

If the total two probabilities equal 1, the probabilities are said to be *complementary*. In that instance, it is a certainty that either one or the other event will happen. The most common application of this situation in life insurance is the probabilities of either living or dying. In calculating the probability that a person will either live or die within a certain period, Rule 1 stated above is applicable (because only one of the two events can occur). When the two separate probabilities of living or dying are added, the total equals 1, because it is a certainty that one or the other of the events will occur.

To Illustrate - If the probability that a certain person will die within the next year is given as .0648, calculate the probability that the person will live at least to the end of the year.

Solution

Since only one of the two events can occur, Rule 1 is applicable. Therefore, the probabilities of the individual events are added.

$$\begin{pmatrix}\text{Probability}\\ \text{of Dying}\end{pmatrix} + \begin{pmatrix}\text{Probability}\\ \text{of Living}\end{pmatrix} = \begin{pmatrix}\text{Probability of Either}\\ \text{Living or Dying}\end{pmatrix}$$

In this illustration, the probability of dying is given. Also, it is known that the probability of either living or dying equals 1 because it is a

certainty that either one or the other will happen. Hence, the above equation can be solved for the probability of living:

$$\begin{pmatrix}\text{Probability}\\ \text{of Dying}\end{pmatrix}+\begin{pmatrix}\text{Probability}\\ \text{of Living}\end{pmatrix}=\begin{pmatrix}\text{Probability of Either}\\ \text{Living or Dying}\end{pmatrix}$$

Substituting .0648 for probability of dying, 1 for probability of either living or dying

$$0.0648+\begin{pmatrix}\text{Probability}\\ \text{of Living}\end{pmatrix}=1$$

Subtracting .0648 from each. side

$$\begin{pmatrix}\text{Probability}\\ \text{of Living}\end{pmatrix}=0.9352$$

Rule 2: *If several events are* independent *(i.e., the happening of any one has no effect on the happening of the others),* the *probability that all of the events wilt happen is the product of the probabilities of the individual* events *multiplied together.*

For example, if the "several events" are

1- Drawing an ace when one card is drawn from a yellow deck, and

2- Drawing a king when one card is drawn from a blue deck,

The happening of either one has no effect on the happening of the other. The probability that both of the events will occur is the product of the two individual probabilities multiplied together:

$$\begin{pmatrix}\text{Probability of}\\\text{Drawing Both}\end{pmatrix}=\begin{pmatrix}\text{Probability of}\\\text{Drawing an Ace}\\\text{from Yellow Deck}\end{pmatrix}\begin{pmatrix}\text{Probability of}\\\text{Drawing a King}\\\text{from Blue Deck}\end{pmatrix}$$

$$=\left(\frac{1}{13}\right)\left(\frac{1}{13}\right)$$

$$=\frac{1}{169}$$

To Illustrate - Calculate the probability that a newly married couple will live to celebrate their 50th wedding anniversary, if the probability that the husband will live 50 years is .277033, and the probability that the wife will live 50 years is .521894.

Solution

Since the fact the one person stays alive has no effect upon another person's probability of living, Rule 2 is applicable.

$$\begin{pmatrix}\text{Probability of}\\\text{Both Living}\\\text{50 Years}\end{pmatrix}=\begin{pmatrix}\text{Probability of}\\\text{Husband Living}\\\text{50 Years}\end{pmatrix}\begin{pmatrix}\text{Probability of}\\\text{Wife Living}\\\text{50 Years}\end{pmatrix}$$

Substituting the given values for the probabilities

= (.277033)(.521894)

Multiplying; rounding to 6 decimal places

= .144582

DERIVING PROBABILITIES

The most important probability in life insurance mathematics is the probability that a particular person will die within one year. It has been found that many characteristics of the particular person affect that probability, but the most important such characteristic is the person's age. Therefore, a different probability exists for each age. Chart 7-1 shows examples of such probabilities.

CHART-1

Person's Age	*Probability That the Person Will Die within One Year*
20	0.00179
21	0.00183
22	0.00186
.	.
.	.
85	0.16114

It has been pointed out that probabilities of dying can be derived only by observing a large number of persons. In actual practice, insurance companies usually observe all of the persons they are currently insuring, but over a very limited period of time. By this means, persons of all ages are observed within this period, and probabilities of dying within one year are calculated for each age.

Basically, the probability of dying within one year at each age, called the *rate of mortality,* is equal to the ratio of the number dying at that age to the number who are exposed to the risk of dying at that age. (The number dying at a certain age includes those persons who die within the year

starting at that exact age and before their next birthday.) The rate of mortality for a certain age is calculated by dividing the number dying at that age by the number so exposed. For example, if 910 persons are being observed who are all age 54, and 12 of them die during the year of observation, the probability that a person age 54 will die within a year (the rate of mortality at age 54) may be calculated as

$$\frac{12}{910}, \text{ or } .01319 \text{ (rounded off)}$$

To Illustrate- A certain group of insured persons all the same age has been observed over a period of years. The group contains 4,112 persons celebrating their 64th birthday. Calculate the rate of mortality at age 63, if it was observed that 87 out of this same age group had died the previous year.

Solution- Since 87 of the group died the year before, there were actually 4,112 + 87 = 4,199 persons attaining age 63 (one year previous). Hence, the rate of mortality at age 63 may be calculated as

$$\text{Rate of Mortality at Age 63} = \frac{\text{Number Age 63 Dying within the Year}}{\text{Number Exposed at Age 63}}$$

Substituting 87 for number *dying,* 4199 for number exposed

$$= \frac{87}{4{,}199}$$

$$= .02072$$

Most of the practical problems faced by an insurance company in deriving these probabilities are associated with the calculation of the number

so exposed. For example, while the period of time being observed may be a calender year (from January 1 to December 31) birth dates occur over the entire year. If a particular person attained the age of 45 on July 1 that year, he would be exposed to death as a person age 44 for half the year and as a person age 45 for half the year Adjustments also have to be made for those persons who enter the group or terminate at some time during the observation period It would be wrong to exclude them completely from the number exposed, because had they died while under observation, they would certainly have been included in the number dying.

The actual rates of mortality for each age experienced by a single insurance company will show considerable fluctuation from year to year. To produce valid and reliable estimates of the mortality rates and to minimize accidental fluctuations, it is usual to base the rates of mortality on the experience of a number of years, rather than on that of a single year. However, the number of years used is small enough to reflect current experience. In addition, it is common to combine the experience of a number of companies.

Even with a large volume of experience, the actual rates of mortality will not vary from age to age exactly in the manner that would be theoretically expected, because of accidental fluctuations in the experience. These rates of mortality are then adjusted slightly to correct for accidental fluctuations, in order to obtain the theoretically proper relationship between mortality rates for the various ages. The mathematical process of adjusting the experience rates to produce a theoretically consistent mortality table is called *graduation.*

In addition to age, there are other important characteristics which affect the probability of dying. For a life insurance company, the most important are

1- The person’s sex;

2- The health status of the person at the tine he became insured; and

3- The length of time since he became insured.

Therefore, separate probabilities of dying within one year (age by age) are often derived for each of these characteristics.

In practice, safety margins are usually added to the computed rates of mortality before the rates are used for certain insurance calculations, such as setting premiums. These provide for unpredicted increases in mortality or for temporary adverse mortality fluctuations.

EXERCISES

1) What is the probability of getting a 3 with a throw of 1 ordinary 6-faced dice?

2) If a red dice and a green dice are thrown, what is the probability that the red dice will show a 2 and the green dice will show a 5?

3) If the probability that an expectant woman will bear twins is .0114, triplets is .00 17, quadruplets is .0003, and quintuplets is virtually 0, what is the probability that an expectant woman will have a multiple birth?

4) If the probability that a man age 35 will live to age 36 is .9994, and the probability that a man age 36 will die before reaching age 37 is .0008, what is the probability that a man age 35 will live to age 36 and then die before reaching age 37?

5) If the probability that a newborn baby will be a boy is .5039, what is the probability that a newborn baby will be a girl?

6) If a group of 1,000 persons age 25is observed for 1 year and 2of them die during that year, compute the rate of mortality at age 25.

7) If a group of 1,000 persons age 25 is observed for 1 year and 2 of them die during that year, compute the probability that a person age 25 will live at least 1 year.

8) If an observed group has 100 persons living 1 year after they all became age 88, and 22 persons were observed to have died out of that age group in the past year, compute the rate of mortality at age 88.

TYPES OF MORTALITY TABELS

A mortality table, as the name implies, is a tabulation of the probabilities of dying during the year at each age, i.e., the rates of mortality. Generally, it also includes related information which can be derived from these rates.

There are two principal types of modality tables, depending on the origin of the data used in deriving. The rates of mortality:

1) Tables derived from population statistics. These are generally prepared by the National Office of Vital Statistics, based upon data collected during a regular census and registered deaths. An example is the "1960 U.S. Life fables".

2) Tables derived from data on insured lives. These generally represent the pooled experience of a number of life insurance companies. These tables are classified into two types:

   a) Annuity mortality tables, for use with anntuity contracts (benefits payable only if the contract-holder is alive). An example is the "Annuity Table for 1949" which is printed in the Appendix of this text in Table II.

   b) Insurance mortality tables, for use with life insurance contracts (benefits payable when the contract-holder dies). An example is the "1958 C.S.O. Table" which is printed in the Appendix of this text in Table III.

Experience has shown that the rates of mortality for persons buying annuity contracts are lower, age by age, than for those buying life insurance

contracts. This apparently results from the fact that some persons base their selection of one or the other type of contract upon the knowledge that their own probabilities of dying are belier or worse than the average. Therefore, life insurance companies use different mortality tables for life insurance and for annuities.

In both annuity and insurance operations, it is important that the rates of mortality assumed be conservative. In life insurance, this requires that the table used should exhibit higher rates of mortality than will probably be experienced. This is needed so that the company will not be required to pay death benefits sooner than was anticipated. The converse is true in the selection of a conservative table to use for annuity contracts. Here the rates of mortality assumed should be lower than the expected rates, so that the company will not be required to pay annuity benefits for a longer period of time than was assumed in the calculations.

In general, there has been an observed trend over a period of many years toward lower mortality rates. This has been a result of our economic and medical advances. Therefore, conservative insurance modality tables have tended to become more conservative. On the other hand, conservative annuity mortality tables have tended to become less conservative (because more people are living longer).

It was pointed out in Section 7.2 that separate probabilities of dying are often derived for each sex. Experience has shown that the rates of mortality for females are lower, .age by age, than for males. At many ages this difference is very substantial. This accounts for the fact that the Annuity Table for 1949 is actually two separate tables: one for males and one for females. Only the male Table is printed in Table II.

The 1958 Commissioners' Standard Ordinary Table, which is used in connection with life insurance, was developed from experience for both male and female lives. However, it is customary (and permitted by law) to assume that the rates of mortality contained in this table apply only to males. When using this table for females, the usual procedure is to subtract three years from the true age of the female. For example, a woman age 25 is considered to be subject to the rate of mortality applicable to a man age 22. This is known as "using a 3-year setback," or a "3-year rating down in age." In Table III, the label "male" is used, as a reminder that it is customary to make an adjustment where a female life is involved. At the very young and very old ages, a 3-year setback is not appropriate, however, and other types of adjustments are customary.

The Annuity Table for 1949 (Table II) and the 1958 C.S.O. Table (Table III) will be used for all calculations in this text. However, the principles discussed are general and can be applied to any mortality table.

STRUCTIURE OF A MORTALITY TABELS

A mortality table is generally shown with four basic columns. The beginning and ending portions of the 1958 C.S.O. Table, which is printed in full in Table III, appear in Chart 7-2. This shows the columns to be described, and will be used later to explain the interrelationships among them.

CHART-2

Portions of the 1958 C.S.O. Table

Age *x*	*Number Living at Age x* l_x	*Number Dying Between Age x and* Age *(x+1)* d_x	*Rate of Mortality* q_x
0	10,000,000	70,800	0.00708
1	9,929,200	17,475	0.00167
2	9,911,725	15,066	0.00152
.	.	.	.
.	.	.	.
.	.	.	.
97	39,787	18,456	0.48842
98	19,331	12,916	0.66815
99	6,415	6,415	1.00000

COLUMNS FOR AGE AND OF MORTALITY. The first column represents the age. Ages are very often shown starting with age zero (a person's first year of life).

Another column contains the rates of mortality. The rate of mortality shown opposite each age represents the assumed probability of dying within one year for a person who is that particular age. It is customary to assume that the probability of dying is a certainty (equals 1) at some very high age, such as 99. This age, then, is the highest age shown in the first column.

The rate of mortality, or the probability of dying, at age *x* is represented by the symbol. q_x

In this symbol, the letter *x*, shown to the right and slightly lower than the q, is a "subscript." Hence, it is a pad of the whole symbol, and q, does not mean "*q* multiplied by *x*". The symbol is read "*q* sub *x*" or simply "*q x*". An example would be q_{27}

Which is read "*q* sub 27" or "*q* 27". It means the rate of mortality at age 27, or the probability that a person age 27 will die within a year (before he reaches age 28).

This symbol q_x appears at the top of the rate of mortality column in a mortality table. Since the letter *x* is used therein as a general representation for age, the letter *x* appears at the top of the age column.

GRAPHIC PRESENTATION* OF *q Relative values of q_x can be seen on the graph, Figure-1. The two curved lines represent values of q_x. (for males) from the Annuity Table for 1949 and the 1958 C.S.O. Table. Ages from age 0 to age 70 are shown along the bottom of the graph. For each age, the distance up to one of the lines indicates the value of q_x at that age, as set forth in the particular mortality table. Space limitations make it impossible to show the graph to the highest age in the tables. The value of q_x is 1.000 at age 99 in the 1958 C.S.O. Table and at age 109 in the Annuity Table for 1949. This gives an indication of how rapidly the death rates increase after age 70. This graph would have to be 20 times as tall as it is to show values for all ages up through the highest age.

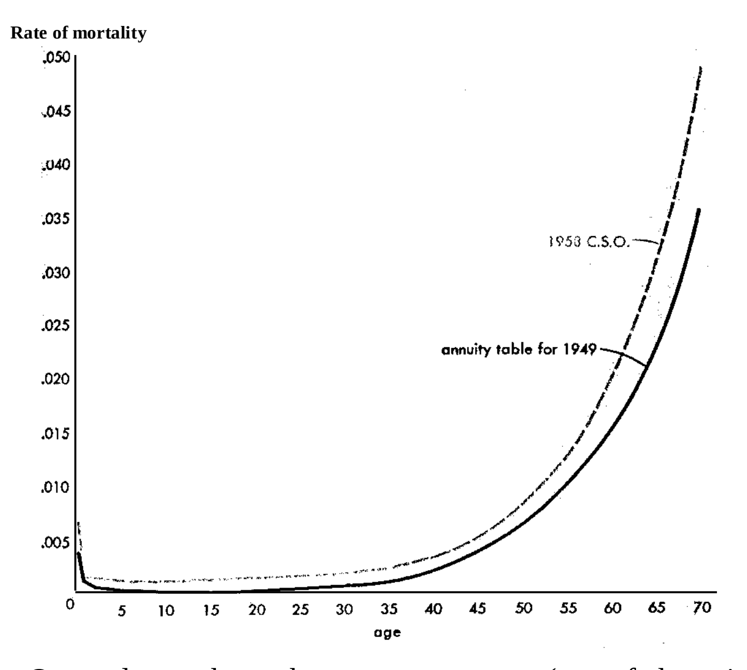
It is interesting to note that the rates of mortality actually decrease age by age from age 0 to approximately age 10. From this point, the rates increase very slowly until about age 40. It should also be noted that the graph clearly portrays that rates of mortality for those buying annuity contracts are lower, age by age, than for those buying life insurance contacts.

COLUMNS FOR NUMBER LIVING AND DYING. The other two basic columns of a mortality table are set up to show what happens each year to a large group of people all the same age, starting when they are all a certain low age, such as age zero. An arbitrary number of people, such as 10,000,000, are assumed to be alive at this time. During the first year a certain number will die, leaving the remaining persons to begin the second year. Then a certain number of these will die before reaching the end of the second year, etc.

FIGURE-1

Values of q_x

(black line is Annuity Table for 1949-colored line Is 1958 C.S.O. Table)



One column shows how many persons (out of the original group) are assumed to still be alive at each age. The number shown opposite each age represents the assumed number out of the original group who are still living at that particular age.

The number of this group who are still alive at age *x* is represented by the symbol. l_x

That is, *l* with a subscript *x*. It is read “ι sub *x*” or simply “*lx*.” An example would be l_{20}

Which is read “ι sub 20” or”*l*20”. It means the assumed number out of the original group who are still living at age ‘20.

The other column shows how many persons out of this group are expected to die at each age. The number shown opposite each age represents the number of persons who are expected to die while they are that particular age, that is, in the year after reaching that age but before reaching the next age.

The number out of this group who die in the year they are age *x* is represented by the symbol d_x**.**

That is, *d* with a subscript *x*. It is read “*d* sub *x*" or simply “*dx*.” An example would be d_{74}

Which is read “*d* sub 74” or “*d* 74.” It means the number of the group who are expected to die while age 74 (in the year after reaching age 74 but before reaching age 75).

Referring to the basic columns shown above in the portion of the 1958 C.S.O. Table, the $ι_x$, column shows that 10,000,000 persons (in this hypothetical group) start out life together at age zero. This is the same as saying that $ι_0$ = 10,000,000. The same column shows that it is assumed that 9,929,200 of them will still be alive at age one, and 9,911,725 of them will still be alive at age two. That is, the table shows

l_1 = 9,929,200

l_2 = 9,911,725

The d_x column shows that 70,800 of the persons in this group die while they are age zero (during the year after birth but before reaching age one). This is the same as saying that d_0 = 70,800. The same column shows that it is expected that 17,475 of them will die during the year they are age one, and 15,066 will die during the year they are age two. That is, the table shows

d_1= 17,475

d_2= 15,066

The probability of dying while age x, that is, during the year between age x and age (x + 1), is sometimes called the probability of dying at age x. Likewise, the number so dying is sometimes called the number dying at age x.

EQUATIONS FOR INTERRELATIONSHIP. In general terms, it may be said that if the number dying in a given year (d_x) is subtracted from the number living at the beginning of that year (l_x), the result will represent the number still alive at the next higher age (l_{x+1}). In equation form, this is written:

$$l_{x+1} = l_x - d_x$$

To Illustrate- Calculate the value of l_{98} for the 1958 C.S.O. Table using the above equation.

Solution

Basic equation

$$l_{x+1} = l_x - d_x$$

Substituting 97 for x (the age)

$$l_{98} = l_{97} - d_{97}$$

Substituting the values for l_{97} and d_{97} from the table

=37,787 - 18,456

= 19,331

This value of l_{98} agrees with that given in the table.

In general terms, it may be said that if the number living at a certain age (l_x) is multiplied by the rate of mortality at that age (q_x), the result will represent the number expected to die during the year they are that age (d_x). In equation form, this is written

$$d_x = l_x q_x$$

To Illustrate- Calculate the value of & for the 1958 C.S.O. Table using the above equation a

Solution

Basic equation

$$d_x = l_x . q_x$$

Substituting 97 for x (the age)

$$d_{97} = l_{97} q_{97}$$

Substituting values for l_{97} and q_{97} from the table

= (37,787)(.48842)

Multiplying; rounding to nearest whole number

= 18,456

This value of d_{97} agrees with that given in the table.

In general terms, it may be said that if the number expected to die during the year in which they are a certain age (d_x) is divided by the number living at that age (l_x), the result will represent the rate of mortality at that age (q_x).

The equation, $d_x = l_x . q_x$, may be solved for q_x by dividing both sides by l_x, giving this relationship in equation form:

$$\frac{d_x}{l_x} = q_x$$

To Illustrate- Calculate the value of q_2 for the 1958 C.S.O. Table using the above equation.

Solution

Basic equation

$$q_x = \frac{d_x}{l_x}$$

Substituting 2 for x (the age)

$$q_2 = \frac{d_2}{l_2}$$

Substituting values for d_2 and l_2 from the table

$$= \frac{15,066}{9,911,725}$$

$= 0.00152$

This value of q_2 agrees with that given in the table.

It will be observed that the value for q_{99} shown in the 1958 C.S.O. Table is *certainty*, i.e., 1. This is done arbitrarily for the purpose of conveniently ending the table, and not because it was observed that everybody who reaches age 99 dies before reaching 100.

CONSTRUCTION OF A MORTALITY TABLE

After the rate of mortality, q_x, is established for each age, the other columns can be constructed. The youngest age in the table should be the youngest age for which it is expected the table will be used. In most cases, tables begin with age zero. The entire mortality table can be constructed by the following steps:

1- Assume an initial value for l_x (at the youngest age in the table). This is usually some large round number, such as 1,000,000 or 10,000,000.

2- Calculate the number of deaths between this age, x, and the next age, $(x + 1)$. This is done using the equation

$$d_X = l_x \, q_x$$

3- Calculate the number living at the second age $(x + 1)$, using the equation

$$l_{x+1} = l_x - d_x$$

4- Repeat steps 2 and 3, successively, for each higher age.

By way of example, it can be seen how the columns of the 1958 C.S.O. Table were constructed by this process after the rates of mortality were known. The portions of the table appearing in Section 7.4 indicate that an initial value of l_x was chosen to be 10,000,000 at age zero. In other words,

L_0 = 10,000,000

The number of deaths between age zero and age one was calculated by applying the basic equation:

$$d_x = l_x q_x,$$

Substituting 0 for x

$$d_0 = l_0 q_0$$

Substituting 10,000,000 for l_0, and the value for q_0 from the table

= (10,000,000)(.00708)

Multiplying; rounding to nearest whole number

= 70,800

Next, the number living at age one was calculated as follows:

Basic equation

$$l_{x+1} = l_x - d_x$$

Substituting 0 for x

$$l_1 = l_0 - d_0$$

Substituting 10,000,000 for l_0, and the value of d_0 calculated above

= 10,000,000- 70,800

= 9,929,200

Repeating this process, the number of deaths at age one (between age one and age two) was calculated.

Basic equation

$$d_x = l_x q_x$$

Substituting 1 for x

$$d_1 = l_1 q_1$$

Substituting the value of l_1 calculated above, and the value for q_1

= (9,929,200)(.00 176)
= 17,475

The number living at age two was then calculated as follows:

Basic equation

$$l_{x+1} = l_x - d_x$$

Substituting 1 for x

$$l_2 = l_1 - d_1$$

Substituting the values for l_1 and d_1 calculated above
= 9,929,200 - 17,475
= 9,911,725

All these values agree with those in the portion of the table shown earlier in this chapter. This process was repeated successively until the entire 1958 C.S.O. Table was constructed.

In constructing the Annuity Table for 1949, age 10 was chosen as the lowest age. As shown in Table II, 10,000,000 was chosen as the value of '10 However, this table was later extended to begin at age 0. The values of l_x and d_x for ages under age 10 were then calculated in such a way as to produce this same value for l_{10}, namely 10,000,000. As a result, the initial figure, l_0, is 10,104,755.

EXERCISES

1- According to the Annuity Table for 1949 (see Table 11 in the Appendix of this book), what is the probability that a man age 60 will die before reaching age 61?

2- According to the 1958 C.S.O. Table (see Table III in the Appendix of this book), what is the rate of mortality at age 60? Express the answer as a fraction.

3- If the rate of mortality at a certain age is.00742, and the number of persons living at that age is 107,412, how many of them may be expected to die within a year?

4- Using Table III, calculate the probability that a man age 79 will live to age 80. (Hint: It is a certainty that he will either live or die that year.)

5- Using a "3-year setback" for females, what is the female rate of mortality at age 29, according to Table 111? (Hint: Consider that a female is subject to the rate of mortality for a male 3 years younger.)

6- If a mortality table shows l_{18} = 994,831 and d_{18} = 1,094, calculate the value of l_{19}.

7- If a mortality table shows l_{42} = 9,408,108 and l_{43} = 9,374,239, calculate the value of d_{42}.

8- If a mortality table shows l_{36} = 951,003 and q_{36} = .0022, calculate the value of d_{75}.

9- Using the figures in Exercise 8, calculate the value of l_{37}.

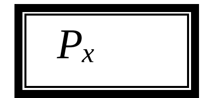
10- If a mortality table shows l_{75} = 4,940,810 and d_{75} = 361,498, calculate the value for q_{75}.

11- Calculate the missing items in the following portion of a mortality table:

Age x	l_x	d_x	q_x
20	92,637	…	…
21	91,914	…	…
22	91,192	…	…
23	90,471		

PROBABILITIES OF LIVING AND DYING

PROBABILITIES OF LIVING OR DYING IN* ONE *YEAR. The probability that a person age x will live to reach (x+1) is represented by the symbol



That is, p with a subscript x. It is read “p sub x’’ or simply ‘‘p x’’ An example would be

$$p_{43}$$

Which is read “p sub 43” or “p 43”. It means the probability that a person age 43 will live to reach age 44, that is, will be alive for at least one whole year.

In general terms, it may be said that if the number living at age (x+1) is divided by the number living at age x, the result will he the probability that a person age x will live to reach age (x+1). In equation form, this is written

$$p_x = \frac{l_{x+1}}{l_x}$$

To Illustrate- Using the Annuity Table for 1949 (Table II), and the above equation, calculate the value of p_{95}.

Solution

Basic equation

$$p_x = \frac{l_{x+1}}{l_x}$$

Substituting 95 for x

$$p_{95} = \frac{l_{96}}{l_{95}}$$

Substituting the values for l_{96} and l_{95}, from Table II

$$= \frac{150{,}429}{220{,}194}$$

$$= .683166$$

The result shows that, according to this particular table, the probability that a person age 95 will live for at least one whole year is .683166.

It is a certainty that a person will either live for one year or die within that year. Since only one of those two events can occur, Rule 1 in Section-1 is applicable: the probability that one of the events will happen is the *total* of the probabilities of each individual event happening.

$$\begin{pmatrix}\text{Probability of}\\ \text{Living 1 Year}\end{pmatrix} + \begin{pmatrix}\text{Probability of}\\ \text{Dying within}\\ \text{1 Year}\end{pmatrix} = \begin{pmatrix}\text{Probability of}\\ \text{Either Living or}\\ \text{Dying That Year}\end{pmatrix}$$

Symbols can be substituted for each of the above expressions, as follows:

Substitute $\boxed{p_x}$ For $\begin{pmatrix}\text{Probability of} \\ \text{Living 1 Year}\end{pmatrix}$

Substitute $\boxed{q_x}$ For $\begin{pmatrix}\text{Probability of} \\ \text{Dying Within 1 Year}\end{pmatrix}$

$\text{Substitue 1 (certainty) for} \begin{pmatrix}\text{Probability of Either} \\ \text{Living or Dying That Year}\end{pmatrix}$

Consequently, the equation is

$$\boxed{P_x + q_x = 1}$$

As an example, according to the 1958 C.S.O. Table, q_{21}=.00183.

Expressed as a fraction, this is $\frac{183}{100,000}$ This means that, out of 100,000 persons all age 21, there will be 183 deaths during the year. If 183 persons out of 100.000 can be expected to die between the ages of 21 and 22, then 100,000-183=99,817 will survive to age 22. Therefore, the probability that a person age 21 will be living at age 22 is

$$\frac{99,817}{100,000} \quad \text{or} \quad .99817$$

In other words:

p_{21} = .99817

Using these figures, it can he shown that the equation $p_x + q_x = 1$ is applicable:

Basic equation

$$P_x + q_x = 1$$

Substituting 21 for x

$$p_{21} + q_{21} = 1$$

Substituting the values given above for p_{21}, and q_{21}

.99817 + .00183 = 1

Adding; result verifies the equation

1.00000 = 1

The equation $p_x + q_x$, = 1 can be used to calculate either p_x or q_x when the value of only one of these probabilities is known.

To Illustrate- Given that p_{46} = .995138, how many persons age 46 can be expected to die before reaching age 47 out of a group of 1,000,000? Out of a group of 100,000? Out of a group of 10,000?

Solution

Basic equation

$$p_x + q_x = 1$$

Substituting 46 for x

$P_{46} + q_{46} = 1$

Substituting the given value for p_{46}

$.995\ 138 + p_{46} = 1$

Subtracting .995138 from both sides

$q_{46} = .004862$

This result can also be written:

$$q_{46} = \frac{4,852}{1,000,000}$$

Which means that out of 1,000,000 persons age 46, 4,862 can be expected to die in the succeeding year, before reaching age 47.

To find the number out of a group of 100,000, the numerator and denominator are each divided by 10. This is done by moving the decimal points one place to the left:

$$q_{46} = \frac{486.2}{100,000.0}$$

The numerator would seem to imply a number of persons dying which is not a whole number. This need not be confusing if it is remembered that such numbers are approximations for the exact number predicted to die, or averages based on observations of more than one year. Hence, this expression means that, out of 100,000 persons age 46, approximately 486 can be expected to die in the succeeding year.

To find the number out of a group of 10,000, the numerator and denominator are each divided by 10 again by the method of moving the decimal points one place to the left:

$$q_{46} = \frac{48.62}{10{,}000.00}$$

This expression means that, out of 10,000 persons age 46, about 48 or 49 can be expected to die in the succeeding year.

PROBABILITIES OF LIVING OR* DYING *IN* n *YEARS, The concepts presented above can be extended to include the probabilities of a person living for any number of years, or dying within any number of years. The probability that a person age x will live at least *ii* more years, or that he will reach age $(x+n)$, is represented by the symbol ${}_np_x$.

That is, p with subscripts of n preceding and x following. It is read *"n p x"*. An example would be ${}_{15}p_{20}$

Which is read "15 p 20". It represents the probability that a person age 20 will live at least 15 more years, that is that he will reach age 35.

In general terms, the probability that a person age x will live at least it more years (${}_nP_x$) is found by dividing the number living at age $(x+n)$ by the number living at age x. In equation form this is written

$$ {}_n p_x = \frac{l_{x+n}}{l_x} $$

To Illustrate- Using Table II, calculate the probability that a man age 48 will live at least 6 more years. Show the answer to 5 decimal places.

Solution

Basic equation

$$_n p_x = \frac{l_{x+n}}{l_x}$$

Substituting 48 for x, 6 for *n*

$$_6 p_{48} = \frac{l_{48+6}}{l_{48}}$$

$$= \frac{l_{54}}{l_{48}}$$

Substituting the values for *15]* and b~ from Table II

$$= \frac{9{,}103{,}034}{9{,}493{,}401}$$

$= .95888$

To Illustrate Again- Using Table Ill with a “3-year setback” for females, calculate the probability that a woman age 36 will live to reach age 46. Show the answer to 5 decimal places.

Solution

Three years must be subtracted from the ages before using the table. This means using the table as if calculating the probability of a male age 33 reaching age 43. The number of years involved, *n,* is 10.

Basic equation

$$_n p_x = \frac{l_{x+n}}{l_x}$$

Substituting values 33 for x, 10 for n

$$_{10}p_{33} = \frac{l_{33+10}}{l_{33}}$$

$$= \frac{l_{43}}{l_{33}}$$

Substituting values for l_{43} and l_{33} from Table III

$$= \frac{9{,}135{,}122}{9{,}418{,}208}$$

$$= .96994$$

<u>The probability that a person age *x* will die within *n*</u> years, or will die before reaching age $(x+n)$, is represented by the symbol $_nq_x$.

That is, q with subscripts of *it* preceding and x following. It is read "$n\ q\ x$" An example would be $_{12}q_{65}$.

Which is read "12 q 65". t means the probability that a person age 65 will die within the next 12 years, that is, that he will die before reaching age 77.

In general terms, the probability that a person age x will die within n years ($_nq_x$) found by dividing the difference between the number living at ages x and $(x+n)$ by the number living at age x. This is expressed in equation form as

$$_nq_x = \frac{l_x - l_{x+n}}{l_x}$$

The numerator equals the number who *die* between ages x and $(x+n)$ because the number living at age x is reduced by all those who die in the interval in order to arrive at the number still living at age $(x+n)$.

To Illustrate- Using Table II, calculate the probability that a man age 30 will die within the next 20 years. Show the answer to five decimal places.

Solution

Basic equation

$$ {}_{n}q_{x} = \frac{l_{x} - l_{x+n}}{l_{x}} $$

Substituting 30 for x, 20 for n

$$ {}_{20}q_{30} = \frac{l_{30} - l_{30+20}}{l_{30}} $$

$$ = \frac{l_{30} - l_{50}}{l_{30}} $$

Substituting values from Table II

$$ = \frac{9,870,777 - 9,388,071}{9,870,777} $$

$$ = \frac{482,706}{9,870,777} $$

$$ = .04890 $$

The number in the numerator, namely 482,706, is the number who die between ages 30 and 50. It is equal to the total of the numbers in the d_x column, beginning with d_{30} and ending with d_{49}.

It is a certainty that a person age x will either live at least n years or else die within n years. Therefore, the total of these two individual probabilities is equal to 1. In equation form, this is written as

$${}_np_x + {}_nq_x = 1$$

This equation is similar to that discussed above for the relationship between the probabilities of living and/or dying for one year.

***SOLVING FOR* OTHER UNKNOWNS.** The above equations for ${}_np_x$ or ${}_nq_x$, namely,

$${}_np_x = \frac{l_{x+n}}{l_x}$$

$${}_nq_x = \frac{l_x - l_{x+n}}{l_x}$$

Can be solved for any desired unknown value which appears therein.

To Illustrate- Using Table III, to what age does a man age 24 have a 50-50 chance of living?

Solution

The question may be stated in another way by asking, "For what value of n is ${}_np_{24}$ equal to .50?"

Basic equation

$${}_np_x = \frac{l_{x+n}}{l_x}$$

Substituting .50 for $_np_x$, 24 for x

$$0.50 = \frac{l_{24+n}}{l_{24}}$$

Substituting the value for l_{24} from Table III

$$0.50 = \frac{l_{24+n}}{9{,}593{,}960}$$

Solving for l_{24+n} by multiplying both sides by 9,593,960

4,796,980 = l_{24}+n

The problem asks for the *age* for which l_x 4,796,980; this age will equal (24+*n)*. Reference to Table III shows that l_{73} 4,731,089 is the nearest to the desired value. Hence, age 73 is the sought-after age. Since 73-24 = 49, the sought-after number of years, *n* equals 49.

PROBABILITIES INVOLVING MORE THAN ONE EVENT. Probabilities involving the happening of more than one event may be calculated using the rules given in Section 7.1.

To Illustrate- Using Table II, calculate the probability that a man age 30 will die either at age 50 or at age 51.

Solution

Since only one of the events can occur, Rule I given in Section 7.1 is applicable: the two individual probabilities are added, The probability that a man age 30 will die during the year he is age 50 is equal to the number so dying divided by the number living at age 30:

Basic equation

$$\begin{pmatrix}\text{Poebability of} \\ \text{Dying at 50}\end{pmatrix} = \frac{d_{50}}{l_{30}}$$

Substituting the values for d_{50} and l_{30} from Table II

$$= \frac{61,558}{9,870,777}$$

= .00624

Similarly, the probability that a man age 30 will die during the year he is age 51 is

Basic equation

$$\begin{pmatrix}\text{Poebability of} \\ \text{Dying at 51}\end{pmatrix} = \frac{d_{51}}{l_{30}}$$

Substituting the values for d_{51} and l_{30} from Table II

$$= \frac{67,869}{9,870,777}$$

= .00688

The desired probability equals the total of the two individual probabilities:

Basic equation

$$\begin{pmatrix}\text{Poebability of} \\ \text{Dying at 50 or 51}\end{pmatrix} = \begin{pmatrix}\text{Probability of} \\ \text{Dying at 50}\end{pmatrix} + \begin{pmatrix}\text{Probability of} \\ \text{Dying at 51}\end{pmatrix}$$

= .00426 + .00688

= .01312

To Illustrate Again- Using Table III, calculate the probability that a man, age 50, and his son, age 20, will both live at least 15 more years.

Solution

Since the happening of one event has no effect upon the happening of the other, Rule 2 in Section 7.1 is applicable: the two individual probabilities are multiplied.

For the man:

Basic equation

$$ {}_n p_x = \frac{l_{x+n}}{l_x} $$

Substituting .50 for x, 15 for n

$$ {}_{15} p_{50} = \frac{l_{50+15}}{l_{50}} $$

$$ = \frac{l_{65}}{l_{50}} $$

Substituting the value for l_{65} from Table III

$$ = \frac{6{,}800{,}531}{8{,}762{,}306} $$

= .77611

For the son:

Basic equation

$$ {}_n p_x = \frac{l_{x+n}}{l_x} $$

Substituting 20 for x, 15 for n

$$ {}_{15} p_{20} = \frac{l_{20+15}}{l_{20}} $$

$$= \frac{l_{35}}{l_{20}}$$

Substituting the value for l_{35} from Table III

$$= \frac{9,373,807}{9,664,994}$$

= .96987

The desired probability that both will live at least 15 years equals the product of two individual probabilities multiplied together:

Basic equation

$$\begin{pmatrix} \text{Poebability of} \\ \text{Both Live} \end{pmatrix} = \begin{pmatrix} \text{Probability} \\ \text{Man Lives} \end{pmatrix} \begin{pmatrix} \text{Probability} \\ \text{Son Lives} \end{pmatrix}$$

Substituting the probabilities calculated above

= (.77611)(.96987)

= .75273

SELECT AND ULTIMATE MORTALITY TABLES

It was stated in Section 7.2 that the length of time which has elapsed since a person became insured affects his probability of dying. At the time a person becomes insured, it is established whether he is in good health (by a medical examination or otherwise). Those whose health is impaired may pay a higher insurance premium. It is customary for mortality tables to be based on the experience arising from those persons who were found to be in good health.

It has been observed that the rates of modality for persons whose good health has just been established are lower than for other person of the same age whose good health was established in the past. However, as the years go by, differences in rates of modality between these two groups

gradually disappear. The period of years during which there is a significant discernible difference in the rates of mortality is known as the *select period*. A mortality table which records the values of basic mortality functions during the select period is known as a *select mortality table*.

To indicate that a group consists of persons whose good health has just been established, it is customary to enclose the age in square brackets. Thus, instead of writing l_x, the number living at age x when all have just been established as being in good health is written $l_{(x)}$.

That is, l with a subscript $[x]$. The x in brackets is pad of the whole symbol, and $l_{[x]}$ does not mean "l multiplied by x". An example would be $l_{[x]}$.

Which means the assumed number of persons in a group, all of whom are age 35 and all of whom have just been established as being in good health.

The number of survivors of the $l_{[x]}$ group at the end of one, two, and three years is expressed by

$$l_{[x]+1} \qquad l_{[x]+2} \qquad l_{[x]+3}$$

Respectively. Thus, the age at which good health was established remains a part of the symbol, enclosed in brackets. An example would be $l_{[24]+4}$

Which means the number still living four years after good health was established, which took place at age 24. In other words, there were $l_{[24]}$ persons in the group at age 24 when good health was just established. Now four years later (at age 28), $l_{[24]+4}$ of that original group are still living.

Beyond the select period, when the effects of the selection have worn off, the number of survivors of the original select group constitute the

ultimate mortality table. The combination of these two tables is called a *select and ultimate mortality table.* An example of portions of such a table appears in Chart-3.

CHART-3

Portion of a Select and Ultimate Mortality Table
(5-year select period)

Select						*Ultimate*	
Age [*x*]	$l_{(x)}$	$l_{(x)+1}$	$l_{(x)+2}$	$l_{(x)+3}$	$l_{(x)+4}$	$l_{(x)+5}$	*Age* *x*+5
30	950,875	949,734	948,471	947,039	945,439	943,624	35
31	949,221	948,044	946,717	945,212	943,501	941,576	36
32	947,447	946,215	944,834	943,237	941,435	939,354	37
33	945,589	944,303	942,830	941,142	939,194	936,959	38
34	943,623	942,255	940,700	938,875	936,781	934,373	SO
35	941,488	940,048	938,375	936,423	934,176	931,570	40
36	939,211	937,661	9.35,870	933,774	931,346	928,477	41
37	936,729	935,071	933,145	930,887	928,225	925,088	42
38	934,023	932,248	930,178	927,704	924,800	921,341	43
39	931,088	929,170	926,903	924,215	921,017	917,195	44

The first column records the various ages at which it may be assumed good health has just been established. The second column records the number assumed living in such a group for each such age, namely $l_{(x)}$. Starting at any age recorded in the first column, the number of survivors at successive ages is found by reading the table *horizontally to the right.* It will be seen that these columns to the right are labeled: $l_{(x)+1}$, $l_{(x)+2}$, $l_{(x)+3}$, $l_{(x)+4}$ and $l_{(x)+5}$. There are no brackets on the *x* in the $l_{(x)+5}$ column. This is because persons still living five years after their good health was established are considered to be subject to the same rates of mortality as any other person of the same age. That is, the *select period* only lasts for five years (in this particular table). Hence, the $l_{(x)+5}$ column may be considered by itself to be an *ultimate*

mortality table. Accordingly, the ages represented by (x + 5) are recorded (in the right-hand column) beside the respective values of $l_{(x)+5}$.

For example, the $l_{(x)}$ column records 947,447 persons at age 32 whose good health has just been established. That is,

$L_{[32]}$ = 947,447

Reading across the table horizontally to the right from there, the table records that 946,215 of them are still living one year later (at age 33). That is,

$l_{[32]+1}$ = 946,215

Similarly, 944,834 of them are still living two years later, 943,237 three years later, and 941,435 four years later. Finally, the l_{x+5} column records that there are 939,354 of the original group still living five years later (at age 37). At this point it makes no difference at what age good health was established. It is only the current age which is used, and therefore the survivors in succeeding years are found by *reading' down* the l_{x+5} column. The number still surviving at age 38 is 936,959, for example.

A select and ultimate mortality table may be used to calculate certain probabilities of living and dying.

To Illustrate- Calculate the probability that a person age 35 will be living at the end of one year, using the above table. First assume good health was established at age 35; then assume it was established at age 34; then at age 33; then at age 32; then at age 31; and finally assume it was established at age 30.

Solution

The equation $p_x = \frac{l_{x+1}}{l_x}$ may be used. The various ages will be expressed with the age at which good health was established enclosed in brackets.

Assuming Good Health Was Established at 35

Basic equation

$$p_x = \frac{l_{x+1}}{l_x}$$

Substituting [35] for x (person is age 35 and good health was just established)

$$p_{[35]} = \frac{l_{[35]+1}}{l_{[35]}}$$

Substituting the values for $l_{[35]+1}$, and $l_{[35]}$ from the table

$$= \frac{940{,}048}{941{,}488}$$

= .99847

Assuming Good Health Was Established at 34

Basic equation

$$p_x = \frac{l_{x+1}}{l_x}$$

Substituting [34]+1 for x (person is age 35 and good health was established at age 34)

$$p_{[34]+1} = \frac{l_{[34]+2}}{l_{[34]+1}}$$

Substituting the values for $l_{[34]+2}$, and $l_{[34]+1}$ from the table

$$= \frac{940{,}700}{942{,}255}$$

= .99835

***<u>Assuming Good Health Was Established at</u>* <u>33</u>**

Basic equation

$$p_x = \frac{l_{x+1}}{l_x}$$

Substituting [33]+2 for x (person is age 35 and good health was established at age 33)

$$p_{[33]+2} = \frac{l_{[33]+3}}{l_{[33]+2}}$$

Substituting the values for $l_{[33]+3}$, and $l_{[33]+2}$ from the table

$$= \frac{941{,}142}{942{,}830}$$

= .99821

Assuming Good Health Was Established at 32

Basic equation

$$p_x = \frac{l_{x+1}}{l_x}$$

Substituting [32]+3 for x (person is age 35 and good health was established at age 32)

$$p_{[32]+3} = \frac{l_{[32]+4}}{l_{[32]+3}}$$

Substituting the values for $l_{[32]+4}$, and $l_{[32]+3}$ from the table

$$= \frac{941{,}435}{943{,}237}$$

= .99809

Assuming Good Health Was Established at 31

Basic equation

$$p_x = \frac{l_{x+1}}{l_x}$$

Substituting [31]+4 for x (person is age 35 and good health was established at age 31)

In numerator l_{36} is shown instead of $l_{[31]+5}$ since 5 years puts it into the ultimate portion of the table

$$p_{[31]+4} = \frac{l_{36}}{l_{[31]+4}}$$

Substituting the values for l_{36}, and $l_{[31]+4}$ from the table

$$= \frac{941{,}576}{943{,}501}$$

= .99796

Assuming Good Health Was Established* at *30

Basic equation

$$p_x = \frac{l_{x+1}}{l_x}$$

Substituting 35 for x (person is age 35 and good health was established 5 years ago; hence l_x figures come entirely from ultimate portion of the table)

$$p_{35} = \frac{l_{36}}{l_{35}}$$

Substituting the values for l_{36}, and l_{35} from the l_{x+5} column (ultimate portion) of the table

$$= \frac{941{,}576}{943{,}624}$$

= .99783

This illustration demonstrates that the probability of a person age 35 living for one year diminishes as the number of years increases since he was established as being in good health, until the end of the select period. This is as might be expected.

To Illustrate Again- What is the probability that a person age 35, whose good health was established at age 32, will be living at the end of 5 years (at age 40)? Also, what is the probability that this person will die during those 5 years?

Solution

Probability of Living

Basic equation

$$_{n}p_{x} = \frac{l_{x+n}}{l_{x}}$$

Substituting [32]+3 for *x* (person is age 35 and good health was established at age 32)

Substituting 5 for *n;* in numerator l_{40} is shown instead of $l_{[32]+8}$ since 8 years puts it into the ultimate portion of the table

$$_{5}p_{[32]+3} = \frac{l_{40}}{l_{[32]+3}}$$

Substituting the values for l_{40}, and $l_{[32]+3}$ from the table

$$= \frac{931{,}570}{943{,}237}$$

$= .98763$

Probability of Dying

Basic equation

$$_{n}q_{x} = \frac{l_{x} - l_{x+n}}{l_{x}}$$

Substituting [32]+3 for *x* (person is age 35 and good health was established at age 32)

Substituting 5 for *n;* in numerator l_{40} is shown instead of $l_{[32]+8}$ since 8 years puts it into the ultimate portion of the table

$$ {}_5q_{[32]+3} = \frac{l_{[32]+3} - l_{40}}{l_{[32]+3}} $$

Substituting the values for $l_{[32]+3}$, and l_{40} from the table

$$ = \frac{943{,}237 - 931{,}570}{943{,}237} $$

$$ = \frac{11{,}667}{943{,}237} $$

$= .01237$

The total of the two probabilities above is

.98763 + .01237 = 1

This is the expected result, since the person is certain either to survive for 5 years or to die during the 5-year period.

EXERCISES

(Use Table III for Exercises 1 to 11)

1) What is the probability that a man age 20 will live for 1 year?
2) What is the probability that a man age 20 will live for 25 years?
3) What is the probability that a man age 30 will be living at age 50?
4) Using a "3-year setback" for females, calculate the probability that a female age 20 will survive to age 45.
5) How many men out of 10,000 men age 35 can be expected to live to age 65?
6) What is the probability that a man age 30 will die before reaching age 65?
7) Using a "3-year setback" for females, calculate the number of females out of 100,000 females age 27 who can be expected to die within 10 years.
8) To what age does a man age 21 have a $\frac{1}{3}$ chance of living?
9) What is the probability that two men, ages 30 and 40, will both survive 10years?
10) What is the probability that two men ages 30 and 40, will both die in the next 10 years?
11) What is the probability that a man age 20 will die either during the year he is age 70 or during the year he is age 80?

(Use the select and ultimate table in Section 7.7 for Exercises 12 to 16)

12) What is the probability that a person whose good health has just been established at age 38 will survive for 1 year?
13) What is the probability that a person age 38, whose good health was established at age 35, will survive for 1 year?
14) What is the probability that a person whose good health has just been established at age 30 will die during the next 10 years?
15) What is the probability that a person age- 37, who was established as being in good health at age 35, will be living 5 years from now?
16) What is the probability that a person whose good health has just been established at age 32 and a person age 37, who was established as being in good health at age 35, will both die during the next 5 years?

CHAPTER 9

LIFE ANNUITIES

INTRODUCTION

It was pointed out that there are two types of annuities: *annuities certain,* which involve a fixed number of payments, and annuities where the continuation of the payments depends upon the occurrence of some event. A *life annuity* is an example of the second type. Each payment in a life annuity is made only if a designated person is alive to pay or receive it.

Accumulated and present values of life annuities can be calculated in a way similar to the method used for annuities certain in financial math. Accumulated and present values of just one payment will be considered first, as was done for payments certain in financial math. To ask the question:

"How much should a man now age 35 pay for the right to receive $100 at age 60 if he is then alive to receive it?"

Is the same as asking?

"What is the present value to a man now age 35 of $100 payable at age 60, calculated with benefit of survivorship?'

The phrase *with benefit of survivorship* is used to distinguish this situation from one where only rates of interest are involved, as was the case in financial mathematices. If only rates of interest were involved in finding the present value, the answer would be

Basic equation for present value

$$A = S\upsilon^{n}$$

Substituting \$100 for S, 25 for n because 25 years are involved between age 35 and age 60.

$$= \$100 v^{25}$$

But now the element of survivorship is also involved, because the man must survive in order to receive the payment. *With benefit of survivorship,* then, means that payments will be made only if the designated payor or recipient is alive at the time the payment is due.

To begin solving the problem posed above, it is necessary to con-suit a mortality table. If Table II is used, the number shown as living at each of the two ages involved is

$$l_{35} = 9{,}814{,}474$$
$$l_{60} = 8{,}465{,}043$$

This means that if there is a group of 9,814,474 men alive at age 35, then 8,465,043 of this group will still be alive at age 60. In order to solve the problem, it must be assumed that all of these men are individually involved, that is, each one still alive at age 60 will receive \$100. It is desired to know the present value of this money to men when they are age 35 (calculated with benefit of survivorship).

Since \$100 is to be paid to each of the l_{60} men, the total amount that will be paid out altogether is

$$\$100(l_{60}) = (\$100)(8{,}465{,}043)$$
$$= \$846{,}504{,}300$$

Twenty-five years earlier, l_{35} men will pay the money in. The original question now may be stated: “How much will each pay?” The total amount paid in is

$$\begin{pmatrix}\text{Amount Each}\\ \text{Pays In}\end{pmatrix}(l_{35}) = \begin{pmatrix}\text{Amount Each}\\ \text{Pays In}\end{pmatrix} + (9{,}814{,}474)$$

$\begin{pmatrix}\text{Amount Each}\\ \text{Pays In}\end{pmatrix}(l_{35})$ $\qquad\qquad$ $\$100(l_{60})$

*——————————————————|

age 35 $\qquad\qquad$ age 60

The money paid in will earn interest over the 25-year period. For this example, the rate will be assumed to be $2\frac{1}{2}$%. The basic equation for finding present value can be used to show that *all the money paid in equals the present value of all the money to be paid out* 25 *years later.* The amount each pays in can then be found:

$$A = S\upsilon^n$$

substituting $\begin{pmatrix}\text{Amount Each}\\ \text{Pays In}\end{pmatrix}(9{,}814{,}474)$ for *A*, \$846,504,300 for S, and the value of υ^{25} at $2\frac{1}{2}$% from Table I

$$\begin{pmatrix}\text{Amount Each}\\ \text{Pays In}\end{pmatrix}(9{,}814{,}474) = (\$846{,}504{,}300)(.539391)$$

$$\begin{pmatrix}\text{Amount Each}\\ \text{Pays In}\end{pmatrix}(9{,}814{,}474) = \$456{,}596{,}801$$

$$\begin{pmatrix}\text{Amount Each}\\ \text{Pays In}\end{pmatrix} = \$46.52$$

This \$46.52 is less than the present value calculated at interest only. The latter would be

$$A = \$100(\upsilon^{25} \text{ at } 2\tfrac{1}{2}\%)$$

$$= (\$100)(.539391)$$

$$= \$53.94$$

$53.94 is the amount each man would pay in if all were to receive $100 25 years later (dead or alive). The $46.52 payment with benefit of survivorship is smaller because in that case only those who survive are to receive their $100.

It can be proved that $46.52 is the desired present value, with benefit of survivorship, at age 35 of $100 payable at age 60 as follows:

Total amount paid in = $46.52(135)

= ($46.52)(9,814,474)

= $456,569,330.48

Total amount accumulated at $2\frac{1}{2}$% for 25 years

= ($456,569,330.48)($1.025)^{25}$

= ($456,569,330.48)(1.853944)

= $846,453,970.83

Amount payable to each survivor at age 60 (the accumulated fund divided by the number of survivors)

= $846,453,970.83 ÷ l_{60}

= $846,453,970.83 ÷ 8,465,043

= $99.99

(The missing I cent is due to the fact that $46.52 was rounded off to the nearest cent instead of using more decimal places.)

The present value, with benefit of survivorship, at age 35 of $100 payable at age 60 can be written as

$$\$100\left(\frac{l_{60}}{l_{35}}\right)\upsilon^{25}$$

or as

$$\frac{(\$100)(l_{60}\upsilon^{25})}{l_{35}}$$

Both these expressions permit interesting verbal interpretations. In the first, $\left(\frac{l_{60}}{l_{35}}\right)$ equals the probability that a person age 35 will live to age 60. Hence, the first expression says that the $100 is multiplied by the probability of surviving and also by the regular discounting factor for finding present values at interest. The second expression says that the $100 payable to each of l_{60} persons is discounted at interest for 25 years, and this amount is divided among the l_{35} persons to find out how much each must pay in.

In the numerical example, it was shown how compound interest and probability are combined in calculating contingent payments. Using more general terms, the equation is

$$\left(\begin{array}{c}\text{Present Value of \$1 Due in } n\\ \text{Years to a Life Now Age } x\text{, With}\\ \text{Benefit of Survivorship}\end{array}\right) = \$1\left(\frac{l_{x+n}\upsilon^{n}}{l_x}\right)$$

To Illustrate- Using the 1958 C.S.O. Table (Table III) and 3% interest, calculate the present value at age 20 of $400 due in 15 years if the person is still alive; also due in 25 years.

Solution

Due in 15 Years

Basic equation

$$\text{Present Value} = \$400\left(\frac{l_{x+n}\upsilon^{n}}{l_x}\right)$$

Substituting 20 for x (the evaluation age), 15 for n (the number of years)

$$= \$400\left(\frac{l_{35}\upsilon^{15}}{l_{20}}\right)$$

Substituting the values for the l's from Table III, for υ^{15} from Table I (3%)

$$= \$400\left[\frac{(9,373,807)(.641862)}{9,664,994}\right]$$

$$= \$400\left(\frac{6,016,691}{9,664,994}\right)$$

$$= \$249.01$$

Due In 25 Years

Basic equation

$$\text{Present Value} = \$400\left(\frac{l_{x+n}\upsilon^{n}}{l_x}\right)$$

Substituting 20 for x, 25 for n

$$= \$400\left(\frac{l_{45}\upsilon^{25}}{l_{20}}\right)$$

Substituting tin, values for the l's from Table 111, for υ^{25} from Table I (3%)

$$= \$400\left[\frac{(9,048,999)(.477606)}{9,664,994}\right]$$

$$= \$400\left(\frac{4,321,856}{9,664,994}\right)$$

$$= \$178.87$$

As the number of years *increases* before the payment is to be made, the present value *decreases*. This is because there is a smaller probability that it will have to be paid, and because there will be a greater number of years in which to earn interest.

PRESENT VALUE OF LIFE ANNUITY

CALCU*LATION OF PRESENT VALUES.* An annuity is a series of payments. It is not difficult to find the present value of a series of payments where each payment is made only if the designated payor or recipient is alive to pay or receive it, The present value of the annuity is the *total* of the present values of each of the individual payments. The principles explained above can be used to find the present value of each individual payment.

For example, the present value at age 25 of a life annuity of $100 per year for three years, first payment due at age 26, can be represented by the following line diagram:

```
                $100          $100          $100
*───────────────┼─────────────┼─────────────┤
age 25          26            27            28
                1             2              3 years
```

The present value of each of the three payments can be calculated individually, as follows:

The Payment Due at Age 26

Basic equation

$$\text{Present Value} = \$100\left(\frac{l_{x+n}\upsilon^{n}}{l_x}\right)$$

Substituting 25 for *x*, 1 for *n* (Exponent 1 need not be written)

$$= \$100\left(\frac{l_{26}\upsilon}{l_{25}}\right)$$

The Payment Due at Age 27

Basic equation

$$\text{Present Value} = \$100\left(\frac{l_{x+n}\upsilon^{n}}{l_x}\right)$$

Substituting 25 for x, 2 for n

$$= \$100\left(\frac{l_{27}\upsilon^2}{l_{25}}\right)$$

The Payment Due at Age 28

Basic equation

$$\text{Present Value} = \$100\left(\frac{l_{x+n}\upsilon^n}{l_x}\right)$$

Substituting 25 for x, 3 for n

$$= \$100\left(\frac{l_{28}\upsilon^3}{l_{25}}\right)$$

The present value at age 25 of this annuity is the total of these three expressions. The common multiplier ($100) can be factored out. The fractions to be added together have a common denominator (l_{25}). Hence, the present value of the annuity can be expressed as

$$\text{Present Value} = \$100\left(\frac{l_{26}\upsilon + l_{27}\upsilon^2 + l_{28}\upsilon^3}{l_{25}}\right)$$

The *numerator* of this expression represents the total to be paid out to the survivors at each age, with each such amount being discounted at interest to the evaluation date. The *denominator* represents the number of persons alive on the evaluation date, among whom this total present value to be paid in must be allocated.

If, for example, the 1958 C.S.O. Table and 3% interest were being used, the present value of the annuity would be calculated as follows:

From above

$$\text{Present Value} = \$100\left(\frac{l_{26}\upsilon + l_{27}\upsilon^2 + l_{28}\upsilon^3}{l_{25}}\right)$$

Substituting the values for the l's from Table III, for the υ's from Table 1(3%)

$$= \$100 \left(\frac{\begin{array}{c}(9{,}557{,}155)(.970874) \\ +(9{,}538{,}423)(.942596) \\ +(9{,}519{,}442)(.915142)\end{array}}{9{,}575{,}636} \right)$$

$$= \$100 \left(\frac{9{,}278{,}793 + 8{,}990{,}879 + 8{,}711{,}641}{9{,}575{,}636} \right)$$

$$= \$100 \left(\frac{26{,}981{,}313}{9{,}575{,}636} \right)$$

$$= \$281.77$$

It can be verified that the payment of \$281.77 by each of the persons age 25 will provide \$100 to each of the survivors at ages 26, 27, and 28, as follows (using the 1958 C.SO. Table at 3%):

If each of the l_{25}, or 9,575,636, persons contributes \$281.77, a fund is provided of

(\$281.77)(9,575,636) = \$2,698,126,955.72

During one year it will earn, interest of

(\$2,698, 126,955.72)(.03) = \$80,943,808.67

The total amount of money in the fund at the end of one year is then

\$2,698,126,955.72+\$80,943,808.67=\$2,779,070,764.39

Payments of \$100 to each of the l_{26} or 9,557,155, survivors will require

(\$100)(9,557,155) = \$955,715,500

This leaves a balance in the fund at the end of one year of

\$2,779,070,764.39 – \$955,715,500 = \$1,823,355,264.39

The continued progress of the fund can be traced in Chart 8-1

CHART-1

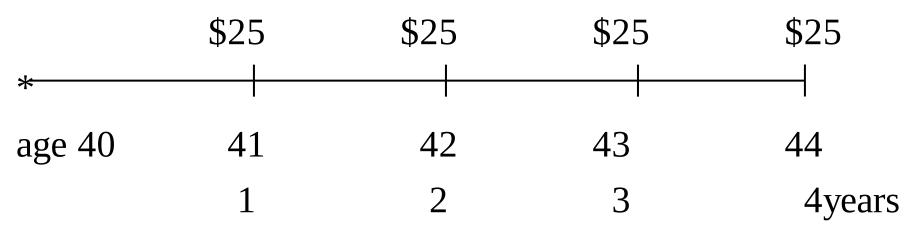
(1) Year	(2) *Fund* at *Beginning of* Year	(3) *Interest for One* Year *(Col. 2 ×* .03)	(4) *Total Fund at End of* Year *before Annuity Payments Are Mode (Col.* 2+ *Col.* 3)	(5) *Annuity Payments to Survivors*	(6) *Balance in Fund afterAnnuityPayments Are Made (Col. 4- Col.*5)
1	$2,698,].26.955.72	$80,943,808.67	$2,779,070.764.39	$955715500.00	$i,823,355,264.39
2	1,823,355,264,39	54,700,657.93	1.878,055,922.32	953,842,300.00	924,213,622.32
3	924,213,622.32	27,726408.67	951,940.030,99	951,944.20C.00	(-4,169.01)

The shortage of $4,169.01 represents less than $\frac{1}{20}$th of a cent for each of the survivors, and results from rounding off the individual contribution, $281.77, to two decimal places. If all calculations had been carried to more decimal places, the balance in the fund at the end of the third year would have been even closer to zero.

To Illustrate- Using the Annuity Table for 1949 and $2\frac{1}{2}\%$ interest, calculate the present value at age 40 of a life annuity of $25 per year for 4 years, first payment due at age 41.

Solution-

The line diagram for this life annuity appears as follows:



The expression for the present value will have *a numerator* representing the total to be paid out to the survivors at each age, with each such amount being discounted at interest to the evaluation date:

$$\$25(l_{41}\upsilon + l_{42}\upsilon^2 + l_{43}\upsilon^3 + l_{44}\upsilon^4)$$

The *denominator* is the number living on the evaluation date (l_{40}):

Basic equation

$$\text{Present Value} = \$25\left(\frac{l_{41}\upsilon + l_{42}\upsilon^2 + l_{43}\upsilon^3 + l_{44}\upsilon^4}{l_{40}}\right)$$

Substituting the values for the l's from Table II, for the υ's from Table I($2\frac{1}{2}$%)

$$= \$25\left(\frac{\begin{array}{c}(9,715,549)(.975610)\\ +(9,693,980)(.951814)\\ +(9,642,815)(.905951)\end{array}}{9,735,263}\right)$$

$$= \$25\left(\frac{9,478,587 + 9,226,866 + 8,979,486 + 8,735,918}{9,735,263}\right)$$

= \$93.53

TYPES OF LIFE ANNUITIES. Life annuities may be either temporary life annuities or whole life annuities. In a *temporary life annuity*, each payment is made only if a designated person is then alive, but the payments are limited to a- fixed number of years. In a *whole life annuity*, the payments continue for the entire lifetime of a designated person.

The three-year and four-year life annuities calculated above are examples of *temporary life annuities*. Each payment is made only if a designated person is then alive, but the number of such payments is limited to a definite number The first payment is made one period following the date on which the present value is calculated.

There are also temporary life annuities in which the first payment is made at the *beginning,* that is, on the same date on which the present value is calculated. These are known as *temporary life annuities due*. The use of the word “due” is analogous to its use in annuities certain.

The line diagrams of two five-payment life annuities, one immediate and one due, look like this:

		$1	$1	$1	$1	$1
Temporary life annuity	*					
	age x	$x+1$	$x+2$	$x+3$	$x+4$	$x+5$
		$1	$1	$1	$1	
Temporary life annuity due	*					
	age x	$x+1$	$x+2$	$x+3$	$x+4$	$x+5$

To Illustrate- Using the 1958 C.S.O. Table and 3% interest calculate the present value at age 25 of a 3-year life annuity due of $100 per year.

Solution-

The line diagram for this life annuity due appears as follows:

	$100	$100	$100	
*				
	age 25	26	27	28
		1	2	3 years

The expression for the present value will have a numerator representing the total to be paid out to the survivors at each age, with each such amount being discounted at interest to the evaluation date. The first payment is due upon the evaluation date. Hence, its present value is simply $\$100(l_{25})$; it is not multiplied by any discounting factor. The *denominator* is the number living at the evaluation date:

Basic equation

$$\text{Present Value} = \$100\left(\frac{l_{25} + l_{26}\upsilon + l_{27}\upsilon^2}{l_{25}}\right)$$

Substituting the values for the l's from Table III, for the υ's from Table I(3%)

$$= \$100\left(\frac{\begin{array}{l}(9{,}557{,}636)\\+(9{,}557{,}155)(.970874)\\+(9{,}538{,}423)(.942596)\end{array}}{9{,}575{,}636}\right)$$

$$= \$100\left(\frac{9{,}575{,}636 + 9{,}278{,}793 + 8{,}990{,}879}{9{,}575{,}636}\right)$$

$$= \$290.79$$

The present value of a three-year life annuity identical to this one, except that the first payment was made at the *end* of the first year, was calculated earlier in this section to be \$281.77. This value is less than the present value of the life annuity due (\$290.79), because each payment in the annuity immediate is paid one year later than its counterpart in the annuity due. Hence, there is a smaller probability that it will have to be paid, and there is a greater number of years in which interest is earned.

Life annuities wherein the payments continue for the entire lifetime of a designated person are known as *whole life annuities.* Without the word "due," this name implies that the first payment is made one period following the date on which the present value is calculated. Whole life annuities in which the first payment is made at the *be ginning,* that is, on the same date on which the present value is calculated, are known as *whole life annuities due.*

The present value of whole life annuities is calculated by exactly the same procedure as that shown above for temporary life annuities. In the ease of whole life annuities, the payments are included to the end of the mortality table. It is thus assumed that all will die before a certain age; therefore, the number of payments to include in the calculation is actually a limited number, just as for temporary life annuities.

To Illustrate- Using the Annuity Table for 1949 and $2\frac{1}{2}\%$ interest, calculate the present value at age **106** of a whole life annuity of \$50 per year.

Solution-

The first payment is due 1 year after age 106 (at age 107). Payments will continue for the person's entire lifetime. However, the Annuity Table for 1949 (shown in Table II) assumes that no persons live beyond the age of 109. Hence, the line diagram for this annuity appears as follows:

```
              $50    $50    $50
        * ------|------|------|
  age 106     107    108    109
                1      2      3 years
```

The expression for the present value will have a *numerator* representing the total to be paid out to the survivors at each age, with each such amount being discounted at interest to the evaluation date. The *denominator* is the number living on the evaluation date:

Basic equation

$$\text{Present Value} = \$50\left(\frac{l_{107}\upsilon + l_{108}\upsilon^2 + l_{109}\upsilon^3}{l_{106}}\right)$$

Substituting the values for the l's from Table II, for the υ's from Table I($2\frac{1}{2}\%$)

$$= \$50\left(\frac{(54)(.975610) + (16)(.951814) + (\ 4)(.928599)}{167}\right)$$

$$= \$50\left(\frac{52.6829 + 15.2290 + 3.7144}{167}\right)$$

$$= \$21.45$$

There is no rule for the number of decimal places to keep in rounding the answers obtained by the actual multiplications in the

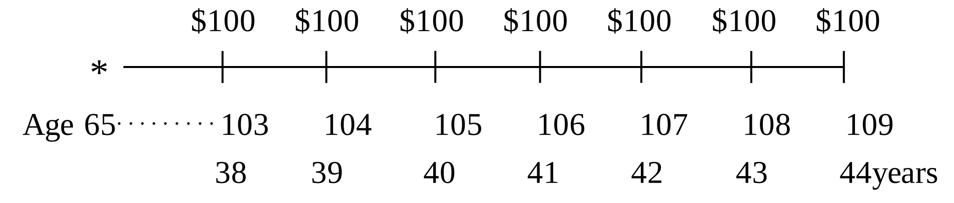
numerator. It is desirable to keep only a sufficient number of digits to have a meaningful answer. In the above illustration, each multiplication answer was rounded to four decimal places. In previous illustrations (involving much larger numbers for the *l's),* each multiplication answer was rounded to the nearest whole number.

The present value of the annuity calculated in the above illustration ($21.45) is less than the amount of one year's payment ($50). This is a phenomenon encountered when only the very high ages are being used. In this illustration it indicates that a considerable portion of those paying for the annuity at age 106 will die before receiving even one payment.

The calculation of the present value of whole life annuities at the younger ages would become very laborious if the above procedure were followed. Therefore, in actual practice this calculation is done by using *corn mutation functions.* This method will be explained later in Section 8.7.

A *deferred life annuity* is a life annuity in which the first payment is *postponed* one or more periods. Once payments commence, they may continue for the remaining lifetime of the designated person, or they may be limited to a specified number of payments.

As an example of a deferred life annuity, consider the problem of finding the present value at age 65 of a whole life annuity of $100 per year, the first payment being made 38 years after the evaluation date. This means that the first payment is due at age 65 + 38 = age 103 (if the person is then alive). If the table used were the Annuity Table for 1949, the final payment would be at age 109, because this table shows none living after that age:



The total amount to be paid out to the survivors at age 103 would be $\$100(l_{103})$

The present value is being calculated as of a time 38 years prior to the date of this payment. Hence, the present value is this total multiplied by the factor for finding present value:

$$\$100(l_{103}\upsilon^{38})$$

The present value of each of the payments is similarly calculated, and the total is divided by the number living at age 65 to derive the amount each must pay in. The expression for the present value of this deferred life annuity is

$$\$100\left(\frac{l_{103}\upsilon^{38}+l_{104}\upsilon^{39}+l_{105}\upsilon^{40}+l_{106}\upsilon^{41}+l_{107}\upsilon^{42}+l_{108}\upsilon^{43}+l_{109}\upsilon^{44}}{l_{106}}\right)$$

To Illustrate- Using the Annuity Table for 1949 and $2\frac{1}{2}\%$ interest, calculate the present value at age 40 of a temporary life annuity of \$1,500 per year, first payment at age 50 and the last payment at age 53.

Solution-

The line diagram for this life annuity appears as follows:

```
              $1,500 $1,500 $1,500 $1,500
         * ---------|------|------|------|
age 40 ········50      51      52      53
       ········10      11      12      13 years
```

The amount payable at age 50 is due 10 years after the evaluation date; the amount payable at age 51 is due 11 years after the evaluation date; etc. Following the above procedure of expressing the present value as the total of the present values of the individual payments:

Basic equation

$$\text{Present Value} = \$1{,}500\left(\frac{l_{50}\upsilon^{10}+l_{51}\upsilon^{11}+l_{52}\upsilon^{12}+l_{53}\upsilon^{13}}{l_{40}}\right)$$

Substituting the values for the l's from Table II the υ's from Table I($2\frac{1}{2}$ %)

$$= \$1,500 \left(\frac{(9,388,071)(.781198) + (9,326,513)(.762145) + (9,258,644)(.743556) + (9,184,223)(.725420)}{9,735,263} \right)$$

$$= \$1,500 \left(\frac{7,333,942 + 7,108,155 + 6,884,320 + 6,662,419}{9,735,263} \right)$$

= \$4,312.49

RELATIONSHIPS AMONG LIFE ANNUITIES. There are certain relationships among types of annuities which are important.

The first of these is the relationship between a *whole life annuity* and a *whole life annuity due*. The following line diagrams show a whole life annuity and a whole life annuity due, both evaluated at age *x*:

$1 $1 $1 ········(for life)
whole Life annuity * ——————|——————|——————|——————————————
age x x+1 x+2 x+3···············

$1 $1 $1 $1 ·······(for life)
whole life annuity due *——————|——————|——————|——————————————
age x x+l x+2 x+3

The diagrams show that the only difference between the two types is the one payment at age *x* in the second annuity. This illustrates that the *present valve of the whole life annuity due is equal to the present valve of the whole life annuity plus the amount of one payment.* In equation form, this is

$$\begin{pmatrix}\text{Present Value at Age } x\\ \text{of Whole Life Annuity Due}\end{pmatrix} = \begin{pmatrix}\text{Present Value at Age } x\\ \text{of Whole Life Annuity}\end{pmatrix} + 1$$

For a payment of $10 per year, the equation would be

$$\$10\begin{pmatrix}\text{Present Value at Age } x\\ \text{of Whole Life Annuity Due}\end{pmatrix} = \$10\left[\begin{pmatrix}\text{Present Value at Age } x\\ \text{of Whole Life Annuity}\end{pmatrix} + 1\right]$$

$$= \$10\begin{pmatrix}\text{Present Value at Age } x\\ \text{of Whole Life Annuity}\end{pmatrix} + \$10$$

To Illustrate- Using the Annuity Table for 1949 and $2\frac{1}{2}\%$ interest, calculate the present value at age 106 of a whole life annuity due of $50 per year.

Solution

The line diagram for this life annuity due appears as follows (with 109 being the highest age in this particular mortality table):

```
       $50    $50    $50    $50
        * ------|------|------|
   age 106    107    108    109
                1      2      3 years
```

In an earlier illustration, the present value at age 106 of a whole life annuity of $50 per year (first payment one year following age 106) was calculated to be $21.45. Hence, the above relationship can be used, with $21.45 substituted in the calculation.

Basic equation

$$\boxed{\$50\begin{pmatrix}\text{Present Value at Age 106}\\ \text{of Whole Life Annuity Due}\end{pmatrix} = \$50\left[\begin{pmatrix}\text{Present Value at Age 106}\\ \text{of Whole Life Annuity}\end{pmatrix} + 1\right]}$$

$$= \$50\begin{pmatrix}\text{Present Value at Age 106}\\ \text{of Whole Life Annuity}\end{pmatrix} + \$50$$

= \$2145 + \$50

= \$71.45

This desired present value could also have been calculated by using the other procedure, as follows (remembering that since the first payment is due upon the evaluation date, it is not multiplied by any present value factor):

Basic equation

$$\text{Present Value} = \$50\left(\frac{l_{106} + l_{107}\upsilon + l_{108}\upsilon^2 + l_{109}\upsilon^3}{l_{106}}\right)$$

Substituting the values for the l's from Table II for the υ's from Table I($2\frac{1}{2}$%)

$$= \$50\left(\frac{\begin{array}{l}(167)\\ +(\ 54)(.975610)\\ +(\ 16)(.951814)\\ +(\ \ 4)(.928599)\end{array}}{167}\right)$$

$$= \$50\left(\frac{167.0000 + 52.6829 + 15.2290 + 3.7144}{167}\right)$$

= \$71.45

This answer agrees with that obtained by the use of the relationship between a whole life annuity and a whole life annuity due.

The second important relationship is that between *temporary life* annuities and *temporary life annuities due*. The following line diagrams show, as a specific example, a \$500 19-year life annuity and a \$500 20-year life annuity due, both evaluated at age 30:

		$500	$500	$500	$500 · ·	$500	
19-years life annuity	*						
	age 30	31	32	33	34 · · · · · ·	49	50

	$500	$500	$500	$500	$500 · ·	$500	
20-years life annuity due	*						
	age 30	31	32	33	34 · · · · · ·	49	50

The only difference between them is the one payment of $500 on the evaluation date. This example illustrates that *the present value of a temporary life annuity due is equal to the present value of a temporary life annuity having one fewer total periods plus the amount of one payment.* In equation form, this is

$$\begin{pmatrix} \text{Present Value at Age } x \\ \text{of } n\text{ - years Life Annuity Due} \end{pmatrix} = \begin{pmatrix} \text{Present Value at Age } x \\ \text{of } (n\text{ - }1)\text{ - years Life Annuity} \end{pmatrix} + 1$$

To Illustrate- Using the 1958 C.S.O. Table and 3% interest calculate the present value at age 25 of a 4-year life annuity due of $100 per year.

Solution

The line diagram for this temporary life annuity due appears as follows:

$100	$100	$100	$100	
*				
age 25	26	27	28	29
	1	2	3	4 years

In an earlier illustration, the present value at age 25 of a 3-year life annuity of $100 per year (first payment due 1 year following age 25) was calculated to be $281.77. Using the above relationship and substituting $281.77, gives

Basic equation

$$\$100\begin{pmatrix}\text{Present Value at Age } x\\ \text{of } n\text{ - years Life Annuity}\end{pmatrix} = \$100\left[\begin{pmatrix}\text{Present Value at Age } x\\ \text{of } (n-1)\text{ - years Life Annuity}\end{pmatrix}+1\right]$$

Substituting 25 for x, 4 for n

$$\$100\begin{pmatrix}\text{Present Value at Age 25}\\ \text{of 4 - years Life Annuity}\end{pmatrix} = \$100\left[\begin{pmatrix}\text{Present Value at Age 25}\\ \text{of 3 - years Life Annuity}\end{pmatrix}+1\right]$$

$$= \$100\begin{pmatrix}\text{Present Value at Age 25}\\ \text{of 3 - years Life Annuity}\end{pmatrix}+\$100$$

Substituting present value calculated above

= $281.77 + $100

= $381.77

This desired present value could have been calculated by using the other procedure, as follows:

Basic equation

$$\text{Present Value} = \$100\left(\frac{l_{25}+l_{26}\upsilon+l_{27}\upsilon^2+l_{28}\upsilon^3}{l_{25}}\right)$$

Substituting the values for the l's from Table III for the υ's from Table I(3%)

$$= \$100\left(\frac{\begin{array}{l}(9{,}575{,}636)\\+(9{,}557{,}155)(.970574)\\+(9{,}538{,}423)(.942596)\\+(9{,}519{,}442)(.915142)\end{array}}{9{,}575{,}636}\right)$$

$$= \$100\left(\frac{9{,}575{,}636+9{,}278{,}793+8{,}990{,}879+8{,}711{,}641}{9{,}575{,}636}\right)$$

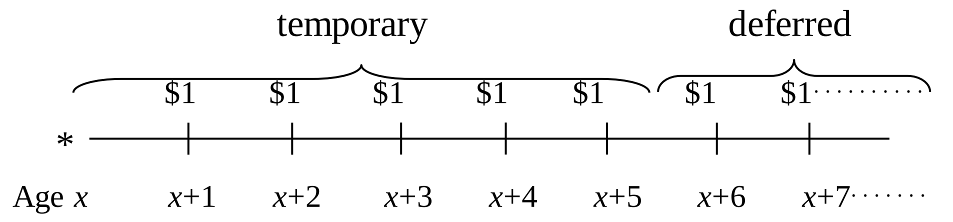
= $381.77

This answer agrees with that obtained by using the relationship between an n-year life annuity due and an (n-l)-year life annuity.

The third important relationship involves *whole life annuities, temporary life annuities, and deferred life annuities,* This relationship is as follows: *A temporary life annuity plus a deferred life annuity (deferred the same number of years for which the temporary annuity runs) equals a whole life annuity*. The reasoning here is the same as that for annuities certain ("second method"). In equation form, this is

$$\begin{pmatrix}\text{Present Value}\\ \text{at Age } x \text{ of}\\ n\text{ - years Temporary}\\ \text{Life Annuity}\end{pmatrix}+\begin{pmatrix}\text{Present Value}\\ \text{at Age } x\\ \text{of Life Annuity}\\ \text{Deferred } n\text{ - years}\end{pmatrix}=\begin{pmatrix}\text{Present Value}\\ \text{at Age } x\\ \text{of Whole Life}\\ \text{Annuity}\end{pmatrix}$$

For example, a 5-year temporary life annuity plus a life annuity deferred 5 years equals a whole life annuity. This is shown in the following line diagram:



It is useful to know this relationship because published tables of life annuity values usually show only whole life annuities and temporary life annuities. The values of deferred life annuities must be obtained by some other means.

To Illustrate- If a published table gives the present value of a whole life annuity of $1 per year to a man age 40 as $18.80, and the present value of a 15-year temporary life annuity of $1 per year to a man age 40 as $12.84, find the present value of a $100 life annuity deferred for 15 years to a man age 40.

Solution

Using the present value relationship above:

Basic equation

$$\$100\begin{pmatrix} n\text{-years} \\ \text{Temporary} \end{pmatrix} + \$100\begin{pmatrix} \text{Deferred} \\ n\text{-years} \end{pmatrix} = \$100(\text{Whole Life})$$

Substituting 15 for *n*

$$\$100\begin{pmatrix} 15\text{-years} \\ \text{Temporary} \end{pmatrix} + \$100\begin{pmatrix} \text{Deferred} \\ 15\text{-years} \end{pmatrix} = \$100(\text{Whole Life})$$

$$\$100(12.84) + \$100\begin{pmatrix} \text{Deferred} \\ 15\text{years} \end{pmatrix} = \$100(18.80)$$

Substituting the present values given

$$\$100(12.84) + \$100\begin{pmatrix} \text{Deferred} \\ 15\text{years} \end{pmatrix} = \$100(18.80)$$

$100(12.84) from each side

$$\$100\begin{pmatrix} \text{Deferred} \\ 15\text{years} \end{pmatrix} = \$100(18.80) - \$100(12.84)$$

$$= \$100\ (5.96)$$

$$= \$596$$

Both life annuities (first payment at end of' one period) and life annuities due (first payment at beginning) have considerable practical use in life insurance company operations. For example, annuities are widely sold whereby the buyer pays a lump sum to the insurance company, and the company then pays back a periodic income as long as the buyer lives. Here, the first payment is usually made to the buyer at the end of the first period. Hence, this is an example of a life annuity. An example of a life annuity due would be the payment of premiums on a life insurance policy. They constitute a life annuity due because money changes hands only if a designated person is alive at the time each premium is payable, this premium being payable at the beginning of each period.

EXERCISES

(Use Table II and $2\frac{1}{2}$% interest, unless specified differently)

1- Write an expression (using symbols) for the present value at age 10 of $250 due in 25 years, with benefit of survivorship.

2- Write an expression (using symbols) for the present value at age 65 of a 4-year temporary life annuity due of $50 per year.

3- Write an expression (using symbols) for the present value at age 20 of a deferred life annuity having 3 payments of $750 each, the first one of which is payable at age 42.

4- Write an expression (using symbols) for the present value at age 9Gof a whole life annuity of $1,000 per year, assuming the mortality table which will be used is the 1958 C.S.O, Table.

5- Calculate the present value at age 70 of a $40 payment due at age 80, with benefit of survivorship.

6- If it is assumed that females will always show the same mortality experience as males 3 years younger, calculate the value for a female at age 25 of $100 due 15 years later, with benefit of survivorship. (Hint: The 3-year "setback" means that instead of using l_{25} and l_{40}, use l_{22} and l_{37})

7- Calculate the amount that a man age 22 should pay for the right to receive $10 per year, first payment due at age 30 and last payment due at age 33.

8- Calculate the present value at age 100 of a deferred life annuity of $1,000 per year, first payment at age 107 (payments continue for life).

9- Using the 1958 C.S.O. Table (Table III)•and 3% interest, calculate the present value at age 97 of a whole life annuity due of $100 per year.

10- Calculate the present value at age 40 of a whole life annuity, deferred for 10 years, of $15 per year, using the following present value factors for I per year:
Present Value at Age 40 of Whole Life Annuity = 18.713
Present Value at Age 40 of 10-year Life Annuity = 8.509

ACCUMULATED VALUE OF LIFE ANNUITIES

The applications of the accumulated value of a life annuity are much less extensive than those of the present value. However, accumulated value is a useful tool in the calculation of reserves, to be presented in Chapter 11.

The *present value* of a life annuity means, in general, that amount which would be paid at the beginning to provide future payments, with benefit of survivorship. On the other hand, the *accumulated value* of a life annuity means, in general, that amount payable to a surviving person to which past payments have accumulated, with benefit of survivorship.

The development is similar to that shown for present values. To begin by accumulating a single payment, the question is asked:

"If a man deposited $1 at age 35, how much money would he receive at age 60 if he must be alive to receive it?"

If Table II is used, the number shown as living at each of the two ages is

$$l_{35} = 9{,}814{,}474$$

$$l_{60} = 8{,}465{,}043$$

Since $1 is to be paid in by each of the l_{35} men, the total amount that will be paid in altogether is

$$\$1(l_{35}) = \$9{,}814{,}474$$

Twenty-five years later, l_{60} men will receive the money. The original question now may be stated: 'How much will each receive?" The money paid in will earn interest over the 25-year period. If the rate of interest is $2\frac{1}{2}$%, then the original $9,814,474 will accumulate as follows:

Basic equation for accumulating

$$S = A(1+i)^n$$

Substituting $9,814,474 for A, .025 for i, 25 for n

$$= \$9{,}814{,}474(1.025)^{25}$$

Substituting the value of $(1.025)^{25}$ from Table I

$$= (\$9{,}814{,}474)(1.853944)$$

$$= \$18{,}195{,}485.19$$

This sum is then divided among the l_{60} men:

$$\frac{\$18{,}195{,}485.19}{l_{60}} = \frac{\$18{,}195{,}485.19}{8{,}465{,}043}$$

$$= \$2.15$$

This $2.15 is greater than the accumulated value calculated at interest only. The latter would be

$$S = \$1(1.025)^{25}$$

$$= \$1(1.853944)$$

$$= \$1.85$$

$1.85 is the amount each man would receive at the end of the 25 years if all those who contributed originally were to share at the end. The $2.15 payment with benefit of survivorship is larger because in that case only those who survive are to share in the accumulation.

In general terms, if x is the age when the deposit is made, and n is the number of years elapsed until the payment is returned to the survivors, then each survivor will receive

$$\$1\left[\frac{l_x(1+i)^n}{l_{x+n}}\right]$$

This expression states that $1 deposited by each of the l_x persons accumulates at interest for n years, and the total is then divided among the l_{x+n}, persons still alive. In equation form, this is written:

$$\begin{pmatrix} \text{Accumulated Value of \$1} \\ \text{at End of } n \text{ years to a} \\ \text{Life age } x \text{ at the Beginning} \\ \text{with Benfit of Survivorship} \end{pmatrix} = \$1\left[\frac{l_x(1+i)^n}{l_{x+n}}\right]$$

This accumulation factor corresponds to (1 + *i*) used in Chapter 3, except that now *contingent* payments are being considered.

The factors for calculating present values and accumulated values of a single payment both with interest only and with benefit of survivorship are shown in Chart 8-2. In each case, *n* is the number of years involved, *x* is the age at the *beginning* of the time, and *x*+*n* is the age at the *end* of the time:

CHART-2

	With Interest Only	*With Benefit of Survivorship*
Present Value Factor	$\frac{1}{(1+i)^n} or\, \upsilon^n$	$\left(\frac{l_{x+n}}{l_x}\right)\left[\frac{1}{(1+i)^n}\right] or\, \frac{l_{x+n}\upsilon^n}{l_x}$
Accumulation Factor	$(1+i)^n$	$\frac{l_x(1+i)^n}{l_{x+n}}$

The present value factor and the accumulation factor are the inverse of each other (i.e., numerator and denominator are switched). This is just as true when dealing with benefit of survivorship as when dealing with interest only.

The accumulated value of a series of payments may be handled as the total of accumulated value of each of the individual payments. For example, suppose it is desired to know the accumulated value, with benefit of survivorship, at age 65 for a three-year annuity due of $1. This means that $1 was deposited at the beginning of each of the last three years, and the accumulation will be paid to a designated person (at age 65) only if he is then alive to receive it. If he is then alive, how much will he receive? The deposits were made at ages 62,63, and 64:

$$\begin{array}{cccc} \$1 & \$1 & \$1 & \\ \vdash & + & + & * \\ \text{Age } 62 & 63 & 64 & 65 \end{array}$$

The accumulated value of each of the three payments can be calculated individually, as follows:

The Payment Due at Age 62

Basic equation

$$\text{Accumulated Value} = \$1\left[\frac{l_x(1+i)^n}{l_{x+n}}\right]$$

Substituting 62 for x (i.e., the age at the beginning), 3 for n (i.e., number of years), and .025 for i

$$= \$1\left[\frac{l_{62}(1.025)^3}{l_{65}}\right]$$

The Payment Due at Age 63

Basic equation

$$\text{Accumulated Value} = \$1\left[\frac{l_x(1+i)^n}{l_{x+n}}\right]$$

Substituting 63 for x, 2 for n, and .025 for i

$$= \$1\left[\frac{l_{63}(1.025)^2}{l_{65}}\right]$$

The Payment Due at Age 64

Basic equation

$$\text{Accumulated Value} = \$1\left[\frac{l_x(1+i)^n}{l_{x+n}}\right]$$

Substituting 64 for x, 1 for n, and .025 for i

$$= \$1\left[\frac{l_{64}(1.025)}{l_{65}}\right]$$

The accumulated value at age 65 of this annuity is the total of the three above expressions. The common multiplier ($1) can be factored out. The fractions to be added together have a common denominator (l_{65}). Hence, the accumulated value of the annuity can be expressed as

$$\text{Accumulated Value} = \$1\left[\frac{l_{62}(1.025)^3 + l_{63}(1.025)^2 + l_{64}(1.025)}{l_{65}}\right]$$

The *numerator* of this expression represents the total amount paid in by the survivors at each age, with each such amount being accumulated at interest to the evaluation date. The *denominator* represents the number of persons still alive on the evaluation date, among whom this total accumulated value will be allocated to be paid out.

To Illustrate- Using the Annuity Table for 1949 and $2\frac{1}{2}\%$ interest, calculate the accumulated value at age 40 of a life annuity of $i5 per year for 4 years, first payment due at age 36.

Solution

The line diagram for this life annuity appears as follows:

	$15	$15	$15	$15	*
Age	36	37	38	39	40

The expression for the accumulated value will have a *numerator* representing the total amount paid in by the survivors at each age, with each such amount being accumulated at interest to the evaluation date:

$\$15[l_{36}(1+i)^4 + l_{37}(1+i)^3 + l_{38}(1+i)^2 + l_{39}(1+i)]$

The *denominator* is the number living on the evaluation date (l_{40}).

Basic equation

$$\text{Accumulated Value} = \$15\left[\frac{l_{36}(1+i)^4 + l_{37}(1+i)^3 + l_{38}(1+i)^2 + l_{39}(1+i)}{l_{40}}\right]$$

Substituting values for the l's from Table II for the $(1+i)$'s from Table I($2\frac{1}{2}\%$)

$$= \$15\left(\frac{\begin{array}{l}(9{,}800{,}822)(1.103813) \\ +(9{,}786{,}180)(1.076891) \\ +(9{,}770{,}454)(1.050625) \\ +(9{,}753{,}522)(1.025000)\end{array}}{9{,}735{,}263}\right)$$

$$= \$15\left(\frac{10{,}818{,}275 + 10{,}538{,}649 + 10{,}265{,}083 + 9{,}997{,}360}{9{,}735{,}263}\right)$$

= \$64.13

This section on accumulated values of annuities has dealt only with the accumulations of temporary life annuities, because the accumulations of whole life or deferred life annuities have no application in practice.

LIFE ANNUITIES PAYABLE MORE THAN ONCE A YEAR

So far in this chapter, only life annuities payable once a year have been considered.

In actual practice, however, it is common for a series of payments (which are made only as long as the recipient is alive) to be paid more frequently than annually. Monthly social security benefits are one example. In life company operations, settlement option payments (to be presented in Section 8.5), which are most often made monthly, are another example.

Annuities of this type are difficult to analyze precisely because modality tables generally do not record the number living at fractional intervals of a year. However, there is a simple method for calculating the present value of such annuities which gives an answer very close to the true present value.

For *whole life annuities,* the method involves calculating the present value as if payments were annual, and then *adding* a fractional part of a year's payments to this value.

For *whole life annuities due,* the method involves calculating the present value as if payments were annual, and then *subtracting* a fractional part of a year's payments from this value.

These fractions are as follows:

$\frac{1}{4}$ if payments are semiannual

$\frac{3}{8}$ if payments are quarterly

$\frac{11}{24}$ if payments are monthly

The fraction has a numerator which is always one less than the number of payments per year, and a denominator which is always two times the number of payments per year. (The mathematics by which this method was derived is beyond the scope of this book.)

To Illustrate- Using the Annuity Table for 1949 and $2\frac{1}{2}\%$ interest, calculate the present value at age of a whole life annuity of $12.50 per quarter.

Solution

The total amount paid every year is ($1 2.50)(4) $50 In Section 8.2, the present value at age 106 of a whole life annuity of $50 per year (with annual payments) was calculated to be $21.45. Since payments are made quarterly in this illustration, the rule states that I of a year's payments must be added to this present value. Therefore, the present value of this annuity, when payments are quarterly, is

$$\text{Present Value} = \begin{pmatrix} \text{present Value as If} \\ \text{Payments Were Annual} \end{pmatrix} + \frac{3}{8}\begin{pmatrix} \text{A Year's} \\ \text{Payments} \end{pmatrix}$$

$$= \$21.45 + \frac{3}{8}(\$50)$$

$$= \$21.45 + \$18.75$$

$$= \$40.20$$

Since $\frac{3}{8}$ of a year's payments is added above, the present value is larger than it would be for payments being made annually, even though the total payments each year are of the same amount. This is logical, because in the particular year that the designated person dies, he may already have received one or more quarters' payments prior to his death, whereas he would have received nothing in that particular year if payments were being made annually (at the end of each year).

To Illustrate-Again- Using the Annuity Table for 1949 and $2\frac{1}{2}\%$ interest, calculate the present value at age 106 of a whole life annuity *due* of \$12.50 per quarter.

Solution

Here again the total amount paid every year is (12.50)(4) = \$50. In Section 8.2, the present value at age 106 of a whole life annuity *due* of \$50 per year (with annual payments) was calculated to be \$71.45. Since payments are made quarterly in this illustration, the rule states that $\frac{3}{8}$ of a year's payments must be subtracted from this present value. Therefore, the present value of this annuity due, where payments are quarterly, is

$$\text{Present Value} = \begin{pmatrix} \text{present Value as If} \\ \text{Payments Were Annual} \end{pmatrix} - \frac{3}{8}\begin{pmatrix} \text{A Year's} \\ \text{Payments} \end{pmatrix}$$

$$= \$71.45 - \frac{3}{8}(\$50)$$

$$= \$71.45 - \$18.75$$

$$= \$52.70$$

Since $\frac{3}{8}$ of a year's payments is subtracted above, the present value is smaller than it would be for payments made annually, even though the total payments each year are of the same amount, This is logical, because in the particular year that the designated person dies, he may have received a few quarterly payments prior to his death, whereas he would have received a whole year's payment in that particular year if payments were being made annually (at the beginning of each year).

In actual practice, *temporary* life annuities payable more than once a year are seldom seen, but *deferred* whole life annuities payable more than once a year are of great importance. This is particularly true of settlement option payments.

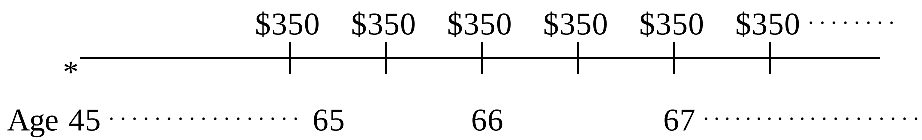
The method used to evaluate *deferred* whole life annuities payable more than once a year makes use of the rules given above for evaluating whole life annuities payable more than once a year. Briefly, the present value is first found *as of the end of the deferred period*, and this present value is then discounted back to the true evaluation date by using the regular factor for finding present values (with benefit of survivorship)

$$\frac{l_{x+n}\upsilon^n}{l_x}$$

To Illustrate- Using the Annuity Table for 1949 and $2\frac{1}{2}\%$ interest, calculate the present value at age 45 of a whole life annuity of \$350 every half-year, first payment due at age 65. (It is given that the present value at age 65 of a whole life annuity due of 1 per year is 12.49597.)

Solution

The line diagram appears as follows;



First, the present value will be calculated at age 65 instead of age 45. Viewed at age 65, it is *a whole life annuity due*. The total amount paid every year is ($350)(2) = $700. Since payments are being made semiannually, the rule states that $\frac{1}{4}$ of a year's payments must be subtracted from this present value. Therefore, the present value at age 65, when payments are semiannual, is

Basic equation

$$\begin{pmatrix}\text{Present Value}\\ \text{at Age 65}\end{pmatrix} = \begin{pmatrix}\text{present Value as If}\\ \text{Payments Were Annual}\end{pmatrix} - \frac{1}{4}\begin{pmatrix}\text{A Year's}\\ \text{Payments}\end{pmatrix}$$

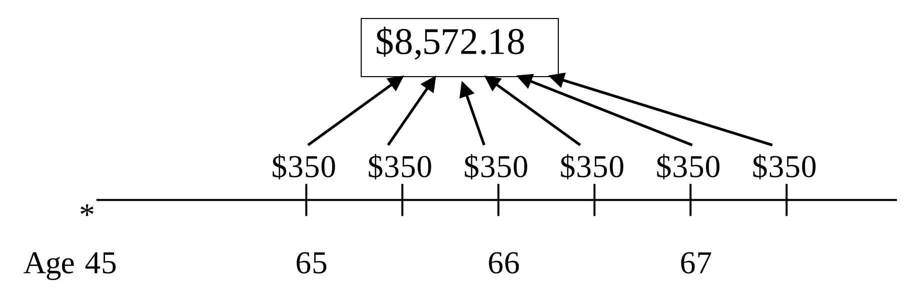
Using the given factor 12.49597 to calculate present value of an annual annuity

$$= \$700(12.49597) - \frac{1}{4}(\$700)$$

= $8,747.18 -$175.00

= $8572.18

This figure of $8,572.18 is the value as *of age 65:*



The problem is now similar to finding the present value (with benefit of survivorship) at age 45 of a *single payment* due in 20 years (at age 65):

Basic equation

$$\begin{pmatrix}\text{Present Value}\\ \text{at Age 65}\end{pmatrix} = \$8{,}572.18\left(\frac{l_{x+n}\upsilon^{n}}{l_x}\right)$$

(Present Value) = $8,572.18

Substituting 45 for x, 20 for n

$$= \$8{,}572.18\left(\frac{l_{65}\upsilon^{20}}{l_{45}}\right)$$

= $8,572.18 145)

Substituting the values for the l's from Table II for the υ^{20} from Table I($2\frac{1}{2}$%)

$$= \$8{,}572.18\left(\frac{(7{,}716{,}840)(.610271)}{9{,}612{,}083}\right)$$

$$= \$8{,}572.18\left(\frac{4{,}709{,}364}{9{,}612{,}083}\right)$$

= $4,199.87

10.5 TABLES OF SETTLEMENT OPTIONS

At the time a settlement is made under a life insurance policy, as when a death claim is paid, the person who receives the proceeds may be entitled to receive the proceeds in periodic payments rater than in one sum. The *present value* of these periodic payments must be equal to the one-sum proceeds.

Life insurance policies contain a provision spelling out the exact types of periodic payments which are available, including either the mortality and interest assumptions to be used, or tables which incorporate

these assumptions, or both. These tables are called "Tables of Settlement Options" and include options for periodic payments figured at interest only (payments *certain)* and also for periodic payments figured with benefit of survivorship *(contingent* payments).

Commonly, the options *involving interest only* are the following:

1- "Interest Option," which means that the company holds the proceeds and periodically pays out the interest thereon.
2- "Fixed Period Option," which means that the company pays out a series of equal payments for a certain period of time only, such as for 10 years (with no consideration of life contingencies). The recipient chooses the period of time, and the amount of the periodic payment is then so calculated that the present value of the series equals the proceeds.
3- "Fixed Payment Option," which means that the company pays out a series of equal payments of a certain amount, such as $100 per year (with no consideration of life contingencies). The recipient chooses the amount of the periodic payment, and the period of time is then so calculated that the present value of the series equals the proceeds.

As an example, the Tables of Settlement Options may provide that for each $1,000 of proceeds, the recipient may elect to receive instead $114.26 per year for 10 years. The $114.26 was calculated so that the present value of the 10 payments certain will equal $1,000 (using 21% interest). This may be verified by multiplying $114.26 by the factor for $a_{\overline{10}|2\frac{1}{2}}$ from Table I, which is 8.752064. Calculating such tables is a common practical application of the principles of Section 6.7, "Amortization Payments."

These options must be calculated at fairly low interest rates, such as $2\frac{1}{2}$%, because the company, by printing these tables in the policy, is actually guaranteeing that it will pay this rate of interest to somebody at some time many years in the future (if the recipient then wishes it). The

actual interest rate which the company will be able to afford to pay many years in the future is unknown at the time these guaranteed figures are printed in the policy.

Commonly, the options *involving contingent payments* are the following:

1- "Life Income Option," which means that the company pays out a series of equal payments for as long as the recipient lives.
2- "Life Income Option with Period Certain," which means that the company pays out a series of equal payments for a designated period of time (such as 10 years or 20 years), these payments being *certain* and not contingent; thereafter payments will continue for as long as the recipient lives.
3- "Life Income Option with Refund," which means that the company pays out a series of equal payments, and the payments are *certain* until such time as the total paid out equals the proceeds; thereafter payments will continue for as long as the recipient lives.

As an example of the "Life Income Option," consider the accompanying portion of a table which might appear in the policy. It shows the annual payment available for each $1,000 of proceeds. The payments would continue for as long as the recipient lives. The amounts are dependent upon the age of the recipient at the time he applies for the proceeds:

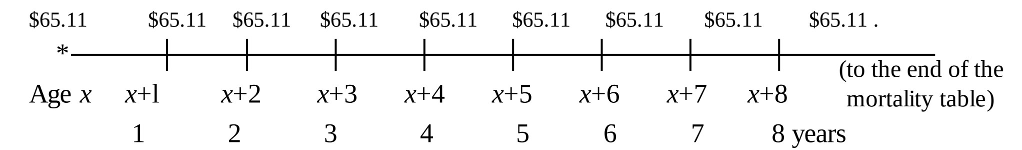
Age	*Annual Payment*
.	.
.	.
.	.
63	$80.85
64	83.81
65	86.99
.	.
.	.

The above annual payments are calculated so that the present value of the whole life annuity will equal $1,000 using Table LI. This may be verified for age 65 by multiplying $86.99 by the factor for the

present value at age 65 of a whole life annuity of 1 per year, which is 11.49597. This example assumes the first payment is made at the end of the first period. In practice, payments are more often calculated to provide the first payment at the beginning.

The "Life Income Option with Period Certain" represents a combination of a *temporary annuity certain* and a *deferred life annuity*. An example would be annual payments for five years certain and as long thereafter as the recipient lives:

annuity Certain deferred life annuity



The present value of the first five payments illustrated above, which are $65.11 each and constitute an annuity due, is

$$\$65.11\ddot{a}_{\overline{5|}\,i}$$

using interest only, because they are payments certain. The present value of the remainder of the payments is

$$\$65.11\left(\frac{l_{x+5}\upsilon^{5}+l_{x+6}\upsilon^{6}+l_{x+7}\upsilon^{7}+\text{.....to the end of the mortality table}}{l_x}\right)$$

The payments represent a deferred life annuity (first payment due at age x+5). The total of these two present values, then, must be equal to the proceeds which the person is entitled to receive at age x.

The "Life Income Option with Refund" provides a life income to the original recipient; but if this recipient should die before an amount equal 'to the total proceeds has been paid out, income installments will be continued to a successor-payee until this amount has been paid. The payments are *certain* until all the proceeds have been "refunded." Thereafter, the payments are contingent on the life of the original recipient. Under this option, the period during which the payments are certain is determined by the size of each payment. Conversely, the size of

each payment is determined by the period of time during which the payments are certain. This is because the present value of all the payments must equal the proceeds, and the number of payments affects this present value. Consequently, the calculation of a table showing installments payable under the Life Income Option with Refund requires the use of methods of higher mathematics. If a table is available showing the size of each payment, the period for which payments are certain is found by dividing the proceeds by the amount of each payment.

The "Life Income Option with Refund" described above is sometimes called the *Installment Refund* option. A slight variation of it is the *Cash Refund* option, which provides that if the recipient dies before the proceeds have been "refunded" the balance necessary to completely refund the proceeds is all paid at once by the company. The calculations involved in this second type are also beyond the scope of this text.

Since the present value of a *certain* payment is greater than the present value of a *contingent* payment, it follows that the amount of each payment will be smaller under options providing for more certain payments. For example, a typical Settlement Option Table might show the following monthly payments available at a certain age, per $1,000 of proceeds:

$6.68 Life Income (no payments certain)

$6.22 Life Income with 120 months certain

$5.15 Life Income with 240 months certain

As the number of certain payments *increases*, the amount of each payment *decreases*. In addition, the above table might show:

$5.78 Life Income with refund

The period during which these $5.78 payments are certain is found as follows:

$1,000 ÷ $5.78 = approximately 173

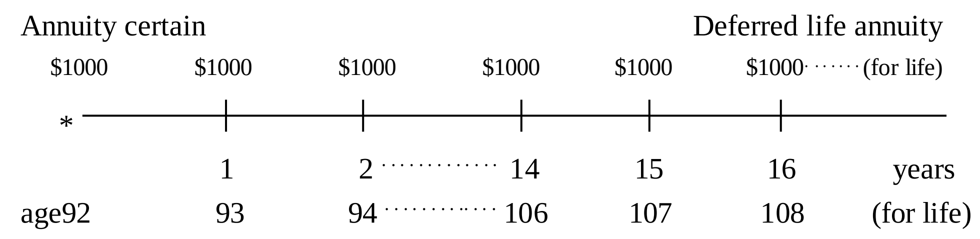
That is, in 173 months the $1,000 proceeds will have been "refunded." This also follows the above pattern, since the $5.78 falls between $6.22 (the 120-month payment) and $5.15 (the 240-month payment).

It is well to remember that in the complete array of figures printed in a Table of Settlement Options, every figure shares the common characteristic that the present value of those payments equals $1,000 (because the tables show figures per $1,000 of proceeds). In actual practice, companies are usually willing to set up payments in almost any manner the recipient desires, as long as the present value equals the proceeds available.

To Illustrate- Using the Annuity Table for 1949 and $2\frac{1}{2}\%$ interest, calculate the amount of annual payment for age 92, per $1,000 of proceeds, in a settlement option which provides payments for 15 years certain and for life thereafter.

Solution

If it **is** assumed that the first payment is made at the beginning, then the line diagram appears as follows;



The present value of the 15 payments certain is

Basic equation

Present Value = (Payment) $\ddot{a}_{\overline{15}|2\frac{1}{2}}$

Using the equation $\ddot{a}_{\overline{n}|i} = a_{\overline{n-1}|i} + 1$

$= (\text{Payment})(a_{\overline{15}|2\frac{1}{2}\%} + 1)$

Substituting the value for $a_{\overline{14}|2\frac{1}{2}\%}$ from Table I

$= (\text{Payment})(11.690912 + 1)$

$= (\text{Payment})(12.690912)$

The present value of the life annuity due, deferred 15 years (first payment at age 107), is

Basic equation (age 109 is highest age in Annuity Table for 1949)

$$\text{Present Value} = (\text{Payment})\left[\frac{l_{107}\upsilon^{15} + l_{108}\upsilon^{16} + l_{109}\upsilon^{17}}{l_{92}}\right]$$

Substituting values for the l's from Table II for the υ's from Table I($2\frac{1}{2}$ %)

$$= (\text{Payment})\left(\frac{\begin{matrix}(54)(.690466)\\ +(16)(.673625)\\ +(\ \ 4)(.657195)\end{matrix}}{565{,}326}\right)$$

$$= (\text{Payment})\left(\frac{37.2852 + 10.7780 + 2.6288}{565{,}326}\right)$$

= (Payment)(.000090)

The total of the two present values must equal $1,000:

(Payment)(12.690912) + (Payment)(.000090) = $1,000

Factoring out the common multiplier, ‘Payment”

(Payment)(12.6909l2 + .000090) = $1,000

(Payment)(l2.691002) = $1,000

Payment = $78.80

In addition to the settlement options described in this section, options are sometimes available which are even more complicated, such as life annuities depending on one or more of several persons being alive.

LIFE ANNUITIES PAYABLE CONTINUOUSLY

Life annuities have been discussed which are payable more often than once a year, such as monthly. It is interesting to see what happens when the periodic payments become even more frequent than monthly. For instance, payments might be made weekly, or daily, or hourly, or every minute, etc. The ultimate frequency would be that payments would be made *continuously*. This concept is only theoretical and could not exist in practice, but there are places where this concept is used by life insurance companies in making certain calculations.

In evaluating life annuities payable more than once a year, the amount of the periodic payment has been multiplied by the number of payments per year to arrive at the total amount paid in a year's time. Obviously, this approach will not be possible in dealing with life annuities payable continuously, because the number of payments per year is infinitely large and the amount of each such payment is infinitesimally small. Instead, the total amount paid in a year's time must always be given. For example, 'a whole life annuity of $25 payable continuously" means that the total of all the infinitesimally small payments being paid continuously is $25 each year.

To calculate the present value of such annuities, the same approximate method is applied that is used for other life annuities payable more than once a year. As set forth in Section 8.4, the fraction of a year's payments which is added or subtracted has a numerator which is always 1 less than the number of payments per year, and a denominator which is always 2 times the number of payments per year. As the number of payments per year gets bigger and bigger, this fraction gets closer and closer to being $\frac{1}{2}$ By using some methods of higher mathematics, it can be proved that when the payments are made continuously, the fraction does equal $\frac{1}{2}$. Hence, the fraction to be used in calculation is $\frac{1}{2}$ if payments are continuous.

To Illustrate- Using the Annuity Table for 1949 and $2\frac{1}{2}\%$ interest, calculate the present value at age 106 of a whole life annuity of $50 per year payable continuously.

Solution

When an annuity is payable continuously, it can be looked upon as either an annuity immediate or an annuity due; it makes no difference because, the terms become meaningless. The calculation will be presented both ways to show that the answer is the same either way.

In Section 8.2, the present value at age 106 of a whole life annuity *immediate* of $50 per year (with annual payments) was calculated to be $21.45. Since payments are made continuously in this illustration, *f* of a year's payments must be *added to this present value.* Therefore, the present value of this annuity, where payments are continuous, is

$$\text{Present Value} = \begin{pmatrix}\text{Present Value as If}\\ \text{Payments Were Annual}\end{pmatrix} + \frac{1}{2}\begin{pmatrix}\text{A Years}\\ \text{Payments}\end{pmatrix}$$

$$= \$21.45 + \frac{1}{2}(\$50)$$

$$= \$21.45 + \$25$$

$$= \$46.45$$

Also in Section 8.2, the present value at age 105 of a whole life annuity *due* of $50 per year (with annual payments) was calculated to be $71.45. Since payments are made continuously in this illustration, $\frac{1}{2}$ of a year's payments must be *subtracted from this present value.*

Therefore, the present value of this annuity due, where payments are continuous, is

$$\text{Present Value} = \begin{pmatrix}\text{Present Value as If} \\ \text{Payments Were Annual}\end{pmatrix} - \frac{1}{2}\begin{pmatrix}\text{A Years} \\ \text{Payments}\end{pmatrix}$$

$$= \$71.45 - \frac{1}{2}(\$50)$$

$$= \$71.45 - \$25$$

$$= \$46.45$$

This answer agrees with the first answer

Calculation of the present value of a *deferred* life annuity payable continuously follows exactly the same method as shown for other life annuities. The present value is first calculated *as of the end of the deferred period* (so it is treated like a whole life annuity). Then the present value of this value at the evaluation age is found by multiplying by the factor for finding present value:

$$\frac{l_{x+n}\upsilon^n}{l_x}$$

EXERCISES

(Use Table II and $2\frac{1}{2}$% interest, unless specified differently)

1- Write an expression (using symbols) for the accumulated value at age 65 of $1,000 paid in at age 20, with benefit of survivorship.

2- Write an expression (using symbols) for the accumulated value at age 25 of a 4-year life annuity of $25 per year, First payment at age 21.

3- A man deposits $100 with an insurance company at age 65. What amount should the company pay him 5 yehs later, if the payment is conditioned on his being alive to receive it?'

4- It is given that the present value at age 33 of a whole life annuity (immediate) of 1 per year is 24.764575. Calculate the present value of a whole life annuity (immediate) of $100 per year at age 33 if the payments are made: semiannually, quarterly, monthly, continuously.

5- Calculate the same present values requested in Exercise 4, but for an annuity due.

6- Using the factor given in Exercise 4, calculate the present value at age 20 of a whole life annuity due of $2 per month beginning at age 33.

7- Using the factor given in Exercise 4, calculate the amount of annual payment which would be shown in a Table of Settlement Options for the whole life annuity due option at age 3.3 (first payment at beginning).

8- If the beneficiary of a life insurance policy elects to receive the proceeds in yearly payments of $565 each (first payment at once) for 5 years certain, find the amount of the proceeds.

COMMUTATION FUNCTIONS

FOR EVALUATING A SINGLE PAYMENT. When dealing with payments *certain,* tabulated values for the present value and accumulated value factors (such as Table I) aid in performing calculations. When benfit of survivorship is involved,, these factors are much more complicated. For example, the factor for the present value of a payment of 1 (with benefit of survivorship)

$$\frac{l_{x+n}\upsilon^n}{l_x}$$

Is different for each *age* and *number of years.* Tabulated values of this factor, as well as factors for the present value of temporary and whole life annuities, are generally published for the common mortality tables and interest rates. Such tabulations are quite voluminous, however. In practice, another aid in performing calculations is widely used, namely, *commutation functions* (sometimes called *commutation symbols).*

To see how commutation functions are used and how they simplify the work, consider again the expression for finding present values of a contingent payment due in *n* years to a life now age *x*:

$$\frac{l_{x+n}\upsilon^n}{l_x}$$

As will be seen later, it will be very useful to have the numerator and denominator look similar to each other. This is accomplished by multiplying numerator and denominator both by υ^x:

$$\frac{(l_{x+n}\,\upsilon^n)(\upsilon^n)}{(l_x)(\upsilon^x)} = \frac{l_{x+n}\upsilon^n}{l_x\upsilon^x}$$

The value of the fraction is unchanged by multiplying both the numerator and the denominator by the same amount. In the numerator, υ^n multiplied by υ^x equals υ^{x+n} (adding exponents when multiplying). The numerator and denominator above now look similar to each other, since in each case the subscript of the l is the same as the exponent of the υ.

Obviously, then, it would be extremely useful to have $l_x\upsilon^x$ already calculated for all values of x (based on a desired mortality table and interest rate). This value of l_x, multiplied by υ^x is represented by the commutation symbol D_x.

The following, then, is the definition of the D_x symbol:

$$\boldsymbol{D_x = l_x\upsilon^x}$$

In Table II and Table IV, columns of D_x are shown for the Annuity Table for 1949 at $2\frac{1}{2}$% and for the 1958 C.S.O. Table at 3%, respectively.

As an example, using age 20 in Table IV, the value of D_{20} can be verified by multiplying l_{20} (from Table III) by υ^{20} at 3% (from Table I):

Basic equation

$$D_x = l_x\upsilon^x$$

Substituting 20 for x

$$D_{20} = l_{20}\upsilon^{20}$$

Substituting the values for l_{20} and υ^{20} from the tables

$$= (9{,}664{,}994)(.553676)$$

$$= 5{,}351{,}275$$

The values of D_x in Table IV were derived using more decimal places in υ^n. Nevertheless, the above answer is very close to that shown in the Table for D_{20}.

Above, the factor for finding present values of a contingent payment due in *n* years to a life now age *x* was finally expressed as

$$\frac{l_{x+n}\upsilon^{x+n}}{l_x\upsilon^x}$$

The numerator is equal to D_{x+n}, since the definition of a *"D"* is *l* multiplied by υ (the subscript of the *l* being the same as the exponent of the υ, and this then being the subscript of the *D).* The denominator is equal to D_x. Hence the factor for finding present values may be expressed

$$\begin{pmatrix}\text{Present Value of \$1} \\ \text{Due in } n \text{ Years to a} \\ \text{Life Now Age } x\text{, with} \\ \text{Benefit of Survivorship}\end{pmatrix} = \$1\left(\frac{D_{x+n}}{D_x}\right)$$

In the above expression, the subscript of *D* in the *numerator* is the age when the contingent payment is to be made. The subscript of *D* in the *denominator* is the age at which the present value is being evaluated.

The value of this factor

$$\frac{D_{x+n}}{D_x}$$

Is the same as the value of the factor used previously

$$\frac{l_{x+n}\upsilon^{n}}{l_x}$$

but the *D's* are easier to use in making calculations.

In the example given in Section 8.1, the present value at age 35 of $100 payable at age 60 (with benefit of survivorship) would be

$$\$100\left(\frac{D_{60}}{D_{35}}\right)$$

Using values of *D* from Table II, this becomes

$$\$100\left(\frac{1{,}923{,}965}{4{,}135{,}535}\right) = \$46.52$$

The answer is the same as before, but the calculation is simplified.

To Illustrate- Using Table IV, calculate the present value at age 20 of $400 due in 15 years if the person is then still alive.

Solution

This is the same problem as shown in the illustration in Section 8.1. The solution will now be given using commutation functions.

Basic equation

$$\text{Present Value} = \$400\left(\frac{D_{x+n}}{D_x}\right)$$

Substituting 20 for *x*, 15 for *n*

$$= \$400\left(\frac{D_{35}}{D_{20}}\right)$$

Substituting the values for D_{35} and D_{20} from Table IV

$$= \$400\left(\frac{3{,}331{,}295}{5{,}351{,}273}\right)$$

$$= \$249.01$$

This answer agrees with that calculated in Section 8.1.

The factor for calculating the *accumulated* value of a single payment by using commutation functions is the *inverse* of the *present value* factor (i.e., numerator and denominator are switched):

$$\left(\begin{array}{c}\text{Accumulated Value of \$1} \\ \text{at End of } n \text{ Years to a} \\ \text{Life Age } x\text{, at the Beginning} \\ \text{with Benefit of Survivorship}\end{array}\right) = \$1\left(\frac{D_x}{D_{x+n}}\right)$$

Note that in this expression, it is still true that the subscript of D in the *numerator* is the age when the contingent payment is to be made. The subscript of D in the *denominator* is the age at which the accumulated value is being evaluated.

FOR EVALUATING AN ANNUITY. The present value at age x of a whole life annuity due of $1 per year may be expressed as the total of the present values of the individual payments:

$$\text{Present Value} = \$1\left(\frac{D_x}{D_x}\right) + \$1\left(\frac{D_{x+1}}{D_x}\right) + \$1\left(\frac{D_{x+2}}{D_x}\right) + \ldots\left(\begin{array}{c}\text{to the end} \\ \text{of the} \\ \text{mortality table}\end{array}\right)$$

The common multiplier ($1) can be factored out. The fractions to be added together alt have a common denominator (D_x). Hence, the present value of the annuity can be expressed as follows:

$$\text{Present Value} = \$1\left[\frac{D_x + D_{x+1} + D_{x+2} + \ldots \text{to the end of the mortality table}}{D_x}\right]$$

In order to avoid the necessity of adding together all the D's to the end of the mortality table, this total is also tabulated. This total of the D's to the end of the modality table is represented by the commutation symbol N_x

The subscript of the N being the same as that of the first Din the series.

The following, then, is the definition of the N_x symbol:

$$N_x = (D_x + D_{x+1} + D_{x+2} + \ldots \text{to the end of the mortality table})$$

In Table II and Table IV, columns of N_x are shown for the Annuity

Table for 1949 at $2\frac{1}{2}\%$ and for the 1958 C.S.O. Table at 3%, respectively.

As an example, using age 104 in Table II, the value of N_{104} can be verified by adding the D's starting with D_{104} to the end of the mortality table:

Basic equation

$N_x = (D_x + D_{x+1} + D_{x+2}$ +...to the end of the mortality table)

Substituting 104 for x

$$N_{104} = D_{104} + D_{105} + D_{106} + D_{107} + D_{108} + D_{109}$$

Substituting the values for the D's from Table II

$$= 89 + 35 + 12 + 4 + 1 + 0$$

$$= 141$$

This value agrees with that shown in the Table for N_{104}

The commutation function Nx can also be used to simplify the, calculation of *temporary* life annuities. In the example given in Section 8.2, the present value at age 25 of a life annuity of \$100 per year for three years, first payment due at age 26, would be

$$\$100\left(\frac{D_{26} + D_{27} + D_{28}}{D_{25}}\right)$$

Here the total of the D's to the end of the modality table is not needed, but only the total for three years. This can be found by taking N_{26} (the total of the D's from age 26 to the end of the table) and *subtracting* N_{29} (the total of the D's from age 29 to the end of the table). What remains after the subtraction is $D_{26} + D_{27} + D_{28}$ That is,

$$\$100\left(\frac{D_{26}+D_{27}+D_{28}}{D_{25}}\right)=\$100\left(\frac{N_{26}-N_{29}}{D_{25}}\right)$$

Using values of N and D from Table IV, this becomes

$$\$100\left(\frac{108{,}616{,}223-95{,}729{,}800}{4{,}573{,}377}\right)=\$281.77$$

The answer is the same as that calculated in Section 8.2.

A general statement may be made that the factor to use in evaluating a life annuity will be of the form $\frac{N-N}{D}$, where the subscript of the first N is the age when the first payment is due, the subscript of the second N is the first age when there are no more payments due (i.e., one greater than the age when the last payment is due), and the subscript of the D is the age at which the annuity is being evaluated (or paid for). Also, the difference between the subscripts of the two N's equals the actual number of payments. If the payments are to be made for life, the second N does not appear.

To Illustrate- Using Table II, calculate the present value at age 40 of a deferred temporary life annuity of $1,500 per year, first payment at age 50 and last payment at age 53.

Solution

This is the same problem as shown in an illustration in Section 8.2. The solution will now be given by using commutation functions:

Basic equation; subscript of first N is. age at first payment; subscript of second N is one age greater than when last payment is due: subscript of D is evaluation age

$$\text{Present Value}=\$1{,}500\left(\frac{N_{50}-N_{54}}{D_{40}}\right)$$

Substituting the Values for N_{30}, N_{34} and D_{40} from Table II

$$= \$1,500\left(\frac{51,853,713 - 41,429,812}{3,625,710}\right)$$

$$= \$4,312.49$$

This answer agrees with that calculated in Section 8.2.

One commutation symbol standing by itself has no usefulness. When commutation functions are used, they must be involved in a fraction; that is, one or more commutation functions must be divided by one or more other commutation functions. The reason underlying this is the fact that present values or accumulated values (with benefit of survivorship) always involve probabilities of living or dying. Such probabilities are calculated by dividing some number of persons living (or dying) by some number of persons living.

EXERCISES

(Use Table II, unless specified differently)

1- Write an expression (using commutation functions) for the present value at age 24 of $1,000 payable at age 62 (if alive); calculate the value.

2- Write all expression (us n g commutation functions) for the accumulated value, with benefit of surv~vorship, at age 63 of $500 deposited with an insurance company when a man is age 21; calculate the value.

3- Calculate the values of $\frac{D_x}{D_{25}}$ for x = 35, 40, 45 and 50. Compare the results with the corresponding values of the present value factor, υ^n for n = 10, 15, 20 and 25 at $2\frac{1}{2}$% using Table I.

4- If females will exhibit the same mortality as males who are 5 years younger, calculate the amount which a woman now age 30 would receive back 20 years later if she deposits $100 now, to accumulate with benefit of survivorship. (Hint: The .5-year "setback" means that instead of using $\frac{D_{30}}{D_{50}}$, use $\frac{D_{25}}{D_{45}}$

5- Write expressions (us big commutation functions) for the present value at age 23 of a whole life annuity of $1 per year, with the first payment at age 23; age 24; age 55. Calculate the values.

6- Write an expression (using commutation functions) for the present value at age 36 of a deferred 10-year life annuity of 8100 per year, first payment due at age 46; calculate the value.

7- Calculate the value at age :35 of a whole life annuity due of $1,500 per year.

8- Calculate the amount a man age 45 should pay for a life annuity due of $1,000 per year, which has payments for 10 years only.

9- What is the present value to a man age 64 of a temporary life annuity of $100 per year for 3 years?

10- Construct a schedule showing that the value found in Exercise will provide the benefits specified.

11- An insurance company has sold a life insurance policy to a man age 25 with premiums of $18.09 payable every year for life. What is the present value at the time the policy is sold of all the future premiums the company will receive?

12- State in words what each of the following expressions represents:

$$\$100\left(\frac{N_{69}}{D_{20}}\right) \qquad \$100\left(\frac{N_{69}-N_{79}}{D_{20}}\right)$$

CHART-3

Present Value at Age *x* of Life Annuity of 1 per Year

Type of Life Annuity	*Symbol for Present Value*	*Present Value Using Commutation Functions*
Whole Life Annuity	a_x	$\frac{N_{x+1}}{D_x}$
Whole Life Annuity Due	$\ddot{a}_x$	$\frac{N_x}{D_x}$
Temporary Life Annuity for *n* Years	$a_{x:\overline{n}\rceil}$	$\frac{N_{x+1}-N_{x+n+1}}{D_x}$
Temporary Life Annuity Due for *n* Years	$\ddot{a}_{x:\overline{n}\rceil}$	$\frac{N_x-N_{x+n}}{D_x}$
Whole Life Annuity, Deferred for *m* Years	${}_{m\|}a_x$	$\frac{N_{x+m+1}}{D_x}$
Whole Life Annuity Due, Deferred for *m* Years	${}_{m\|}\ddot{a}_x$	$\frac{N_{x+m}}{D_x}$
Temporary Life Annuity for *n* years, Deferred for *m* Years	${}_{m\|}a_{x:\overline{n}\rceil}$	$\frac{N_{x+m+1}-N_{x+m+n+1}}{D_x}$
Temporary Life Annuity Due for *n* Years, Deferred for *m* Years	${}_{m\|}\ddot{a}_{x:\overline{n}\rceil}$	$\frac{N_{x+m+1}-N_{x+m+n}}{D_x}$

Chart-3 displays certain internationally used symbols, each of which represents the present value at age x of a life annuity of 1 per year. (These present values are also shown as they would be calculated using commutation functions.)

The use of “a” with the number of years under an “angle” is analogous with the symbol $a_{n\ i}$ given in Chapter 6 for the present value of an annuity certain. For life annuities, however, the age at which the present value is calculated also becomes a part of the symbol. For example, $a_{n\ i}$ (read “*a* sub *x* angle *n”)* represents the present value at age *x* of an *n*-year life annuity.

The use of two dots over the *"a"* is also analogous to the usage in annuities certain, indicating an annuity due.

Life annuity symbols wherein a *bar* is placed over the “*a*” represent annuities payable continuously. For example: $\bar{a}_x$

is the internationally used symbol for the present value at age *x* of a whole life annuity of 1 per year payable continuously.

CHAPTER 10

NET SINGLE PREMIUMS

In Chapter 9, payments were discussed which are made only if a designated person is alive. Life insurance involves a payment to be made only when a designated person dies. The amount of such a payment is called the amount of insurance or the death benefit.

It is common in life insurance for alt of the figures quoted in connection with such insurance to be based upon a death benefit of $1,000. Accordingly, for a policy with a $25,000 death benefit, all such figures would be multiplied by 25. This was seen in connection with Tables of Settlement Options in Section 8.5. In discussing premiums in this book, it will also be assumed that the unit policy is $1,000.

NET SINGLE PREMIUM FOR ONE YEAR OF LIFE INSURANCE – THE NATURAL PREMIUM

The present value of the benefits offered by a particular insurance policy is equal to the *net single premium.* This amount is calculated using a designated mortality table and a specified interest rate. The net single premium does not include any amount for expenses or profits.

It may be desired, for example, to calculate the net single premium that a man age 25 should pay for $1,000 of life insurance covering a one-year period. Under such insurance, if he dies before reaching age 26, $1,000 will be paid to his beneficiary. To begin solving the problem, it is necessary to consult a mortality table, If Table LII is used, the numbers shown living and dying at age 25 are

$l_{25} = 9{,}575{,}636$

$d_{25} = 18{,}481$

This means that if there is a group of 9,575,636 men alive at age 25, then 18,481 men of this group may be expected to die during the year (before reaching age 26). In order to find the net single premium, it is assumed that all of these men are individually involved, that is, that each one who dies that year will receive $1,000 (to be paid to the recipient designated). For purposes of simplifying some of the calculations, it is customary to assume that all such payments are made at the end of the year in which death occurs, In actual practice, however, such payments are made very soon after death occurs. The methods of calculation used when payments are assumed to be made “at the moment of death” will be in Section 9.7.

Since a $1,000 benefit is to be paid for each of the d_{25} men, the total amount that will he paid out as benefits is

$$\$1{,}000 d_{25} = (\$1{,}000)(18{,}481)$$

$$= \$18{,}481{,}000$$

One year earlier, l_{25} men will pay the money in. The original problem may now be stated: "How much will each pay?” The total amount paid in is

$$\begin{pmatrix}\text{Amount Each}\\ \text{Pays In}\end{pmatrix}(l_{25}) = \begin{pmatrix}\text{Amount Each}\\ \text{Pays In}\end{pmatrix}(9{,}575{,}636)$$

$$\begin{pmatrix}\text{Amount each}\\ \text{pays in}\end{pmatrix}(9{,}575{,}636) \qquad (\$1{,}000)(18{,}481)$$

$$= \$18{,}481{,}000$$

*

age 25 age 26

The money paid in will earn interest over the one-year period. For this example, the rate will be assumed to be 3%. The basic equation for finding present value can In, used to show that *all the money paid in equal the present value of all the money to be paid out one year later*. The "amount each pays in' can then be solved for:

$$A = S\upsilon^n$$

Substituting $\begin{pmatrix}\text{Amount Each}\\ \text{Pays In}\end{pmatrix}$ (9,575,636) for A, \$18,481,000 for S and the value of υ^1 at 3% from the table

$$\begin{pmatrix}\text{Amount Each}\\ \text{Pays In}\end{pmatrix}(9{,}575{,}636) = (\$18{,}481{,}000)(.970874)$$

$$\begin{pmatrix}\text{Amount Each}\\ \text{Pays In}\end{pmatrix}(9{,}575{,}636) = \$17{,}942{,}722$$

$$\begin{pmatrix}\text{Amount Each}\\ \text{Pays In}\end{pmatrix} = \$1.87$$

It can be demonstrated that \$1.87 is the desired net single premium at age 25 to provide \$1,000 of insurance for a one-year period, as follows:

$$\text{Total amount paid in} = \$1.87(l_{25})$$
$$= \$1.87(9{,}575{,}636)$$
$$= \$17{,}906{,}439.32$$

Total amount accumulated at 3% to the end of the year

$$= \$17{,}906{,}439.32(1 + i)$$
$$= \$17{,}906{,}439.32(1.03)$$
$$= \$18{,}443{,}632.50$$

Amount payable for each who dies during the year (the accumulated fund divided by the number who die)

$$= \$18{,}443{,}632.50 \div d_{25}$$
$$= \$18{,}443{,}632.50 \div 18{,}481$$
$$= \$998 \text{ approximately}$$

(The missing \$2 results from rounding off \$1.87 to the nearest cent, instead of using more decimal places.)

This net single premium for one year of life insurance at age 25 is also called the "natural premium" at age 25. It could be written as

$$\$1{,}000\left(\frac{d_{25}}{l_{25}}\right)\upsilon \qquad \text{Or as} \qquad \frac{\$1{,}000\, d_{25}\, \upsilon}{l_{25}}$$

Both these expressions for the natural premium permit interesting verbal interpretations. In the first, $\left(\frac{d_{25}}{l_{25}}\right)$ equals the probability that a person age 25 will die before reaching age 26 (q_{25}). The first expression says that the \$1,000 is multiplied by the probability of dying and also by the regular factor for finding present values at interest. The second expression says that the \$1,000 payable to each of d_{25} persons is discounted at interest for one year, and this amount is divided among the l_{25} persons to find out how much each must pay in.

Using more general terms, the equation for the natural premium is

$$\begin{pmatrix} \text{Net Single Premium for} \\ \text{\$1,000 Death Benefit} \\ \text{to a Life Age } x\text{, If} \\ \text{Death Occurs in 1 Year} \end{pmatrix} = \$1{,}000\left(\frac{d_x \upsilon}{l_x}\right)$$

To Illustrate- Using the 1958 C.S.O. Table (Table III) and 3% interest, calculate the net single premium for 1 year of life insurance of $1,000 at age 40; also at age 60; also at age 80.

At *Age* 40

Basic equation

$$\begin{pmatrix}\text{Net Single}\\ \text{Premium}\end{pmatrix} = \$1{,}000\left(\frac{d_x \upsilon}{l_x}\right)$$

Substituting 40 for x

$$= \$1{,}000\left(\frac{d_{40}\upsilon}{l_{40}}\right)$$

Substituting values from the tables

$$= \$1{,}000\left(\frac{(32{,}622)(.970874)}{9{,}241{,}359}\right)$$

$= \$3.43$

At *Age* 60

$$\begin{pmatrix}\text{Net Single}\\ \text{Premium}\end{pmatrix} = \$1{,}000\left(\frac{d_x \upsilon}{l_x}\right)$$

Substituting 60 for x

$$= \$1{,}000\left(\frac{d_{60}\upsilon}{l_{60}}\right)$$

Substituting values from the tables

$$= \$1{,}000\left(\frac{(156{,}592)(.970874}{7{,}698{,}698}\right)$$

$$= \$19.75$$

At *Age* 80

Basic equation

$$\begin{pmatrix}\text{Net Single}\\ \text{Premium}\end{pmatrix} = \$1{,}000\left(\frac{d_x \upsilon}{l_x}\right)$$

Substituting 80 for *x*

$$= \$1{,}000\left(\frac{d_{80} \upsilon}{l_{80}}\right)$$

Substituting values from the tables

$$= \$1{,}000\left(\frac{(288{,}848)(.970874}{2{,}626{,}372}\right)$$

$$= \$10678$$

It can be seen that the net single premium for one year of insurance, the natural premium, increases sharply at the older ages. This is similar to the age-by-age increase in the values of q~, (the probability of dying within one year) shown in Chapter 7.

NET SINGLE PREMIUMS FOR TERM INSURANCE

Life insurance which provides a benefit if death occurs during a specified period of years is known as *term insurance*. The one-year insurance considered in the above section is one-year term insurance.

To determine the net single premiums for life insurance for longer periods, the procedure is basically the same. For example, it may be desired to find the net single premium at age 25 for $1,000 of insurance during a period of three years (between ages 25 and 28), i.e., for three-year term insurance. The total amount to be paid out for those who die during the first year is $\$1{,}000\ d_{25}$

Assuming such payments are made at the *end of the year*, the present value at age 25 of those payments is $\$1{,}000\ d_{25}\upsilon$

The total amount to be paid out for those who die during the second year is $\$1{,}000\ d_{26}$

Assuming such payments are made at the end of the year, the present value at age 25 of those payments is $\$1{,}000\ d_{25}\upsilon^2$

The total amount to be paid out for those who die during the third year is $\$1{,}000\ d_{27}$

Assuming such payments are made at the end of the year, the present value at age 25 of those payments is $\$1{,}000\ d_{27}\upsilon^3$

(In each case the exponent of the υ is the number of years between age 25 and the date when the death benefit is paid.)

The present value at age 25 of all the death benefits paid during the three-year period is the total of the three individual present values. The common multiplier ($1,000) can be factored out:

Present Value = \$1,000 ($d_{25}\upsilon + d26\upsilon^2 + d_{27}\upsilon^3$)

This amount is paid in at age 25 by the 125 persons. Hence, the above expression should be divided by l_{25} to find out how much each must pay in (the net single premium):

$$\begin{pmatrix}\text{Net Single}\\ \text{Premium}\end{pmatrix} = \$1{,}000\left(\frac{d_{25}\upsilon + d_{26}\upsilon^2 + d_{27}\upsilon^3}{l_{25}}\right)$$

The *numerator* of this expression represents the total to be paid out for those who die in each of the three years, with each such amount being discounted at interest from the end of the year of death to the evaluation date. The *denominator* represents the number of persons alive on the evaluation date, among whom this total present value to be paid in must be allocated.

If, for example, the 1958 C.S.O. Table and 3% interest are used, the value of this net single premium can be calculated as follows:

From above

$$\begin{pmatrix}\text{Net Single}\\ \text{Premium}\end{pmatrix} = \$1{,}000\left(\frac{d_{25}\upsilon + d_{26}\upsilon^2 + d_{27}\upsilon^3}{l_{25}}\right)$$

Substituting values from the tables

$$= \$1{,}000\left(\frac{\begin{array}{c}(18{,}481)(.970874)\\ +(18{,}732)(.942596)\\ +(18{,}981)(.915142)\end{array}}{9{,}575{,}636}\right)$$

$$= \$1{,}000\left(\frac{17{,}943 + 17{,}657 + 17{,}370}{9{,}575{,}636}\right)$$

= \$5.53

It can be demonstrated that if each of the *I*,~ persons pays $5.53,

The resulting fund will provide $1000 (at the end of the year of death) for all who die before age 28. At the beginning of the first year, the amount paid in is

$$\$5.53\ l_{25} = (\$5.53)(9{,}575{,}636)$$
$$= \$52{,}953{,}267.08$$

At the end of one year, interest earned on the fund is equal to

$$(\$52{,}953{,}267.08)(.03) = \$1{,}588{,}598.01$$

Hence, the total fund at that time is

$$\$52{,}953{,}267.08 + \$1{,}588{,}598.01 = \$54{,}541{,}865.09$$

From this total fund, $1,000 is deducted for each person who has died during the first year:

$$\$1{,}000\ d_{25} = (\$1{,}000)(18{,}481)$$
$$= \$18{,}481{,}000$$

This leaves a balance in the fund of

$$\$54{,}541{,}865.09 - \$18{,}48l{,}000 = \$36{,}060{,}865.09$$

The continued operation of the fund for succeeding years may be traced in the accompanying schedule (Chart 9-1). The final shortage in the fund represents less than is for each person dying the final year, and results from rounding off the'net single premium to two decimal places.

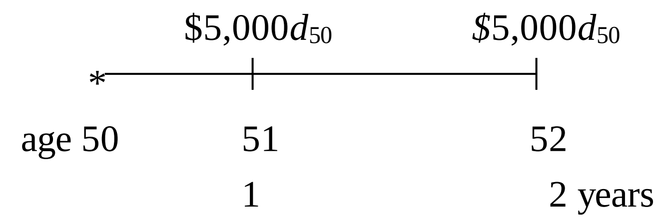
CHART-1

(1) Year	(2) Fund at Beginning of year (Col. 6 of Previous Yeas)	(3) Interest for One Year (Col. 2 × .03)	(4) Total Fund at End of Year before Payment of Death Claims (Col.2+Col.3)	(5) Claims Paid at End of Year (Number of Deaths × $1000)	(6) Balance of Fund at End of Year After Payment of Claims (Col.4-Col.5)
1	$52,953,267.08	$1,588,598.01	$54,541,865.09	$18,481,000	$36,060,865.09
2	36,060,865.09	1,081,825.95	37,142,691.04	18,732,000	18,410,691.04
3	18,410,691.04	552,320.73	18,963,011.77	18,981,000	- 17,988.23

To Illustrate- Using the 1958 C.S.O. Table (Table III) and 3% interest, calculate the net single premium at age 50 for $5,000 of 2-year term insurance.

Solution

The line diagram for this life insurance appears as follows:



The expression for the net single premium is a fraction with a *numerator* equal to the total of the amounts to be paid out for those who die in each of the 2 years, discounted at interest from the end of each year of death to the evaluation date:

$\$5000d_{50}\upsilon + \$5000d_{51}\upsilon^2$

or

$\$5000\ (d_{50}\upsilon + d_{51}\upsilon^2$

The *denominator* of the fraction is the number living on the evaluation date at age 50:

Basic equation

$$\begin{pmatrix}\text{Net Single}\\ \text{Premium}\end{pmatrix} = \$5{,}000\left(\frac{d_{50}\upsilon + d_{51}\upsilon^2}{l_{50}}\right)$$

Substituting values from the tables

$$= \$5{,}000\left(\frac{(72{,}902)(.970874) + (79{,}160)(.942596)}{8{,}762{,}306}\right)$$

$$= \$5{,}000\left(\frac{70{,}779 + 74{,}616}{8{,}762{,}306}\right)$$

$$= \$82.97$$

NET SINGLE PREMIUMS FOR WHOLE LIFE INSURANCE

Under whole life insurance, the death benefit will be paid whenever death occurs; that is, the period of years covered by the insurance extends to the end of the mortality table. Thus, whole life insurance may be looked upon as term insurance covering a period of years equal to those remaining in the mortality table.

The calculation of the net single premium for whole life insurance follows exactly the same procedure as that shown above for term insurance. In the case of whole life insurance, the years included extend to the end of the mortality table.

To Illustrate- Using the 1958 C.S.O Table and 3% interest, calculate the net single premium at age 96 for $1,000 of whole life insurance.

Solution

The period of years covered by this insurance extends for the person s entire lifetime. However, since the 1958 C.S.O. Table assumes that no persons live beyond the age of 100, it is assumed that the insurance ends at age 100. The line diagram for this whole life insurance appears as follows:

	$\$1,000d_{96}$	$\$1,000d_{97}$	$\$1,000d_{98}$	$\$1,000d_{99}$
*				
age96	97	98	99	100
	1	2	3	4years

The expression for the net single premium is a fraction with a *numerator* representing the total to be paid out for those who die in each of the years, with each such amount being discounted at interest from the end of the year of death to the evaluation date. The *denominator* of the fraction is the number living on the evaluation date:

Basic equation

$$\begin{pmatrix}\text{Net Single}\\ \text{Premium}\end{pmatrix} = \$1,000\left(\frac{d_{96}\upsilon + d_{97}\upsilon^2 + d_{98}\upsilon^3 + d_{99}\upsilon^4}{l_{96}}\right)$$

Substituting values from the tables

$$= \$1,000\left(\frac{\begin{array}{l}(25,250)(.970874)\\+(18,456)(.942596)\\+(12,916)(.915142)\\+(\ 6,415)(.888487)\end{array}}{63,037}\right)$$

$$= \$1,000\left(\frac{24,515+17,397+11,820+5,700}{63,037}\right)$$

$$= \$942.81$$

The calculation of net single premiums for whole life insurance at the younger ages would become very laborious if the above procedure were used. Therefore, in actual practice this calculation is usually done by using commutation functions. The commutation functions which apply in net single premium calculations will be explained in Section 9.8.

NET SINGLE PREMIUMS FOR A PURE ENDOWMENT

A *pure endowment* is an amount which is paid on a certain date only if a designated person is then alive to receive it. It is, therefore, the opposite of life insurance. It is, in fact, the same as a payment which is made with benefit of survivorship, as described in Chapters. Pure endowments are often combined with life insurance, as will be shown in Section 9.5. In that context, the term "net single premium for a pure endowment" is used instead of "present value of a single payment with benefit of survivorship".

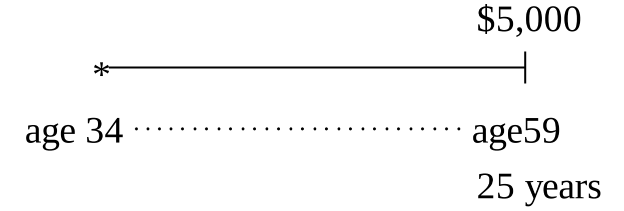
Since the principles and equations for such a payment were presented in Section 8.1, the following equation will be given here without further explanation:

$$\begin{pmatrix} \text{Net Single Premium} \\ \text{\$1,000 Pure Endowment} \\ \text{to a Life Age } x\text{, Due} \\ \text{at the End of } n \text{ Years} \end{pmatrix} = \$1{,}000\left(\frac{l_{x+n}\,\upsilon^{n}}{l_x}\right)$$

To Illustrate- Using the 1958 C.S.O. Table and 3% interest, calculate the net single premium for a female age 34 for a $5,000 pure endowment due in 25 years, using a ‘‘3-year setback’’ for females.

Solution

The line diagram for this pure endowment appears as follows:



The female's age at the date the pure endowment is due is 34+25=59. The use of a "3-year setback" means that 3 years must be subtracted from the age before using the Table. The problem must be treated as if the age were 34-3=31, and the pure endowment were payable at age 59-3 = 56:

Basic equation

$$\begin{pmatrix}\text{Net Single}\\\text{Premium}\end{pmatrix} = \$5{,}000\left(\frac{l_{x+n}\upsilon^n}{l_x}\right)$$

Substituting 31 for x, 25 for n

$$= \$5{,}000\left(\frac{l_{56}\upsilon^{25}}{l_{31}}\right)$$

Substituting values from the tables

$$= \$5{,}000\left(\frac{(8{,}223{,}010)(.477606)}{9{,}460{,}165}\right)$$

$= \$2{,}075.73$

NET SINGLE PREMIUMS FOR ENDOWMENT INSURANCE

Endowment insurance means that the benefit will be paid if death occurs during a specified number of years, or the benefit will be paid at the end of that period if the person is then alive. Therefore, endowment insurance is a combination of two benefits already presented: *term insurance* and *pure*

endowment. The payment on death constitutes term insurance, while the payment on survival constitutes a pure endowment.

To Illustrate- Using the 1958 C.S.O. Table and 3% interest, calculate the net single premium at age 62 for a $7,500 endowment-at-age-6[5] insurance policy.

Solution

The policy provides that $7,500 will be paid if death occurs during the period between ages 62 and 65. It also provides that $7,500 will be paid at age 65 if the person is then alive. The line diagram for this endowment insurance policy appears as follows:

			$\$7,500\ l_{65}$
	$\$7,500 d_{62}$	$\$7,500 d_{63}$	$\$7,500 d_{64}$
*			
age.62	63	64	65
	1	2	3years

The expression for the net single premium for the *term insurance part* has a *numerator* representing the total to be paid out for those who die each year, with each such amount being discounted at interest from the end of the year of death to the evaluation date. The *denominator* is the number living on the evaluation date. The common multiplier ($7,500) can be factored out

$$\begin{pmatrix} \text{Net Single Premium} \\ \text{for Term} \\ \text{Insurance Part} \end{pmatrix} = \$7,500\left(\frac{d_{62}\upsilon + d_{63}\upsilon^2 + d_{64}\upsilon^3}{l_{62}}\right)$$

The expression for the net single premium for the *pure endowment part* follows from the equation given in Section 9.4:

$$\begin{pmatrix}\text{Net Single Premium for}\\ \text{Pure Endowment Part}\end{pmatrix} = \$7{,}500\left(\frac{l_{x+n}\upsilon^{n}}{l_x}\right)$$

The expression for the net single premium for the entire *endowment insurance policy* is the total of the above two expressions. The two expressions can be readily added, since they already have a common denominator (l_{62}). The common multiplier ($7,500) can be factored out:

Adding the above expressions

$$\begin{pmatrix}\text{Net Single}\\ \text{Premium}\end{pmatrix} = \$7{,}500\left(\frac{d_{62}\upsilon + d_{63}\upsilon^{2} + d_{64}\upsilon^{3} + d_{65}\upsilon^{3}}{l_{62}}\right)$$

Substituting values from the tables

$$= \$7{,}500\left(\frac{\begin{array}{l}(179{,}271)(.970874)\\ +(191{,}174)(.942596)\\ +(203{,}394)(.915142)\\ +(\ 6{,}800{,}531)(.915142)\end{array}}{7{,}374{,}370}\right)$$

$$= \$7{,}500\left(\frac{174{,}050 + 180{,}200 + 186{,}134 + 6{,}223{,}452}{63{,}037}\right)$$

$$= \$6{,}879.06$$

It should be noted that the exponents on the last two υ's are the same in the expression above. *(d*$_{64}$ and l_{65} are both multiplied by υ^{3}.) This is done because the two benefits are payable on the same date: the death benefit for those who die during the final year, and the pure endowment benefit for those still alive at the end of the final year.

The two separate parts of the net single premium for endowment insurance (term insurance and pure endowment) can be calculated separately if it is desired to know the relative contribution of each to the total net premium. For example, Chart 9-2 shows such figures (according to the 1958 C,S,O. Table at 3%) for 20-year endowment insurance issued at ages 20, 40, and 60.

CHART-2

Net Single Premiums per $1,000

	Age 20	*Age* 40	*Age* 60
20-Year Tern ……………………………….	$ 3177	$115.08	$474.22
20-Year Pure Endowment	529.41	461.25	188.88
TOTAL = 20-Year Endowment Pnhcy..........	$561.18	$576.33	$663.10

THE ACCUMULATED COST OF INSURANCE

The net single premium which would have to be paid at the *end* of the term of coverage (by the survivors) to provide the death benefits for those who had died during the term is called the *accumulated cost of insurance.* Although this sort of arrangement may seem impractical, useful application of the accumulated cost of insurance will be demon stated in Chapter 11.

It may be desired, for example, to calculate the accumulated cost of insurance at age 28 for $1,000 of insurance during a period of three years (i.e., between ages 25 and 28). The total amount to be paid out for those who die during the first year is $\$1,000\ d_{25}$

Assuming the payments are made at the end of the year, these payments, accumulated at interest to the evaluation date (two years later at age 28), would amount to $\$1,000 d_{25}(1+i)^2$

The total amount to be paid for those who die during the second year is $\$\ 1{,}000d_{26}$

Accumulated at interest from the end of the year to the evaluation date (one year later at age 28), this would amount to $\$1{,}000d_{26}(1+i)$

The total amount to be paid to those who die during the third year is $\$1{,}000d_{27}$

Assuming such payments are made at the end of the year, the value at age 28 of those payments is $\$\ 1{,}000d_{27}$

Because payment would be made upon the evaluation date (at age 28). In each case the exponent of the $(1+i)$ is the number of years between the date when the death benefit is paid and age 28.

The accumulated value at age 28 of all the death benefits paid during the three-year period is the total of the three individual accumulated values. The common multiplier ($1,000) can be factored out.

Accumulated Value = $\$1{,}000[d_{25}(1+i)^2+d_{26}(1+i)+d_{27}]$

This amount is paid in at age 28 by the survivors. Hence, the above expression should be divided by l_{28} to find out how much each must pay in (the accumulated cost of insurance):

$$\begin{pmatrix}\text{Accummulated}\\\text{Cost of Insurance}\end{pmatrix} = \$1{,}000\left(\frac{d_{25}(1+i)^2 + d_{26}(1+i) + d_{27}}{l_{28}}\right)$$

The *numerator* of this expression represents the total to be paid out for those who die in each of the years, with each such amount being accumulated at interest from the end of the year of death to the evaluation date. The *denominator* represents the number of persons alive on the evaluation date, among whom this total accumulated cost must be allocated to be paid in.

If, for example, the 1958 C.S.O. Table and 3% interest are used, the value of this accumulated cost can be calculated as follows:

Equation above

$$\begin{pmatrix}\text{Accummulated}\\ \text{Cost of Insurance}\end{pmatrix} = \$1{,}000\left(\frac{d_{25}(1+i)^2 + d_{26}(1+i) + d_{27}}{l_{28}}\right)$$

Substituting values from the tables

$$= \$1{,}000\left\{\frac{\begin{array}{l}(18{,}481)(1.060900)\\ +(18{,}732)(1.030000)\\ +(18{,}981)\end{array}}{9{,}519{,}442}\right\}$$

$$= \$1{,}000\left(\frac{19{,}606 + 19{,}294 + 18{,}981}{9{,}519{,}442}\right)$$

$$= \$6.08$$

In Section 9.2, the net single premium (payable at age 25) for this same term insurance was calculated to be \$5.53. The accumulated cost calculated above (payable at age 28) can be verified by multiplying the net single premium by the factor for accumulating with benefit of survivorship (described in Section 8.3):

Basic equation

$$\boxed{\begin{pmatrix}\text{Accummulated}\\ \text{Cost of Insurance}\end{pmatrix} = \begin{pmatrix}\text{Net Single}\\ \text{Premium}\end{pmatrix}\left(\frac{l_x(1+i)^n}{l_{x+n}}\right)}$$

Substituting 25 for x, 3 for n

$$= \begin{pmatrix}\text{Net Single}\\ \text{Premium}\end{pmatrix}\left(\frac{l_{25}(1+i)^3}{l_{28}}\right)$$

Substituting \$5.53 for net single premium, and values from the tables

$$= \$5{,}53\left(\frac{(9{,}575{,}636)(1.092727)}{9{,}519{,}442}\right)$$

$$= \$6.08$$

These two calculations agree.

To Illustrate- Using the 1958 C.S.O. Table and 3% interest, calculate the accumulated cost of insurance at age 65 for \$15,000 of 2-year term insurance.

Solution

The line diagram for this life insurance appears as follows:

$\$15{,}000d_{63}$ $\$15{,}000d_{64}$

age63 64 65

The expression for the accumulated cost will have a *numerator* representing the total to be paid out for those who die in each of the years, with each such amount being accumulated at interest from the end of the year of death to the evaluation date:

$$\$15{,}000d_{63}(1+i) + \$15{,}000d_{64}$$

The common multiplier (\$15,000) can be factored out. The *denominator* is the number living on the evaluation date (l_{65}):

Basic equation

$$\left(\begin{array}{c}\text{Accummulated}\\\text{Cost of Insurance}\end{array}\right) = \$15{,}000\left(\frac{d_{63}(1+i)+d_{64}}{l_{65}}\right)$$

Substituting values from the tables

$$= \$15{,}000\left(\frac{(191{,}174)(1.030000) + (203{,}394)}{6{,}800{,}531}\right)$$

$$= \$15{,}000\left(\frac{196{,}909 + 203{,}394}{6{,}800{,}531}\right)$$

$$= \$882.95$$

INSURANCE. PAYABLE AT THE MOMENT OF DEATH

The net single premiums and accumulated costs of insurance discussed so far in this chapter have involved the assumption that all payments for deaths are made at the *end of the year* in which death occurs. In actual practice, such payments are made very soon after death occurs.

If it is assumed that deaths are uniformly distributed throughout any year, then it is possible to assume that all deaths during any year occur at the middle of the year. The deaths during the first half of the year counterbalance those taking place in the last half of the year.

If all payments for deaths are made at the middle of the year, such payments are made one half-year earlier than at the end of the year of death. Consequently, the life insurance company is losing interest for one half of a year on the amount of the death benefit.

A common company procedure, which is approximately correct, is to calculate the net single premium on the basis of a death benefit which is

increased by one-half year's interest. That is, when assuming that payments for deaths are made at the moment of death, the death benefit is multiplied by $\left(1+\frac{1}{2}i\right)$

If, for example, the interest rate is .03, half of this interest rate is .015, and therefore the death benefit is multiplied by 1.015.

It is important to note, however, that in the case of endowment insurance the *pure endowment* part of the net single premium is not so multiplied. This is because the actual payment of this benefit always takes place at the end of the year.

To Illustrate- Using the 1958 C.S.O. Table and 3% interest, calculate the net single premium at age 25 for a $1,000 3-year term insurance policy, assuming the insurance is payable at the moment of death.

Solution

Assuming the insurance is payable at the moment of death, the death benefit ($1,000) is multiplied by $\left(1+\frac{1}{2}i\right)$ for purposes of calculation:

$$\text{Death Benefit} = \$1{,}000\left(1+\frac{1}{2}i\right)$$

$$= \$\ 1{,}000\left(1+\frac{1}{2}\text{ of } .03\right)$$

$$= \$1{,}000(1.015)$$

The net single premium for this same 3-year term insurance policy, with the death benefit of $1,000 assumed to be paid at the end of the year, was calculated in Section 9.2 to be $5.53. Hence, if the death benefit is 1.015 times as great, the net single premium would be

$$(\$5.53)(1.015) = \$5.61$$

To Illustrate Again- Using the 1958 C.S.O. Table and 3% interest, calculate the net single premium at age 60 for a $1,000 20-year endowment policy, assuming the insurance is payable at the moment of death.

Solution

The *term* insurance part is payable at the moment of death. Therefore, its net single premium (calculated previously on the assumption that insurance is payable at the end of the year) is multiplied by $\left(1+\frac{1}{2}i\right)$. The *pure endowment* part is payable at the end of the year (at age 60+20=80). Therefore, its net single premium is not so multiplied.

The net single premiums for the two parts of this same $1,000 endowment policy, with the death benefit assumed to be paid at the end of the year, were given in Section 9.5 to be

Term Part ………………………… $474.22

Pure Endowment Part ……………… 188,88

Hence, if the death benefit for purposes of net premium calculation is $\left(1+\frac{1}{2}i\right)$ times as great, the net single premium would be

Term Part ………….. $474.22(1.015) = $481.33

Pure Endowment Part ………………... <u>188.88</u>

Total …………………………… $670.21

EXERCISES

(Use Table III and 3% interest for all of the following)

1- Write an expression (using symbols) for the net single premium at age 25 for $4,000 of 1-year term insurance. Calculate this premium.

2- Write an expression (using symbols) for the net single premium at age 45 for $1,000 of 3-year term insurance. Calculate this premium.

3- Write an expression (using symbols) for the net single premium at age 97 for $10,000 of whole life insurance. Calculate this premium.

4- Write an expression (using symbols) for the net single premium at age 50 for a $1,000 pure endowment payable at age *65*. Calculate this premium.

5- Write an expression (using symbols) for the net single premium at age 20 for a $5,000 3-year endowment insurance policy. Calculate this premium.

6- Calculate separately the term insurance part and the pure endowment part of the answer to Exercise 5.

7- Construct a schedule showing that the net single premium calculated in Exercise 5 is sufficient to provide all of the benefits of the policy.

8- Write an expression (using symbols) for the accumulated cost of insurance at age 22 for $1,000 of 3-year term insurance. Calculate this cost.

9- The net single premium at age 25 for $1,000 of whole life insurance (1958 C.S.O. Table with 3% interest) is $279.14. Calculate the net single premium for this same insurance policy, assuming that the insurance is payable at the moment of death.

10- Calculate the net single premium for the endowment insurance policy described in Exercise 5, assuming that the insurance is payable at the moment of death. (Hint: Use the information developed in working Exercise 6.)

COMMUTATION FUNCTIONS

FOR ONE YEAR TERM INSURANCE. To see how commutation functions are used in calculating net single premiums, consider again the expression for the net single premium for a death benefit of 1 to a life age x if death occurs in one year (the natural premium):

$$\frac{d_x \upsilon^1}{l_x}$$

The same procedure is followed here as was shown in Chapter 8, namely, both the numerator and the denominator are multiplied by υ^x:

$$\frac{(d_x \upsilon^1)(\upsilon^x)}{(l_x)(\upsilon^x)} = \frac{d_x \upsilon^{x+1}}{l_x \upsilon^x}$$

The value of the fraction is unchanged by multiplying both the numerator and the denominator by the same amount. In the numerator, & multiplied by υ^x equals υ^{x+1} (adding exponents when multiplying).

Looking at the numerator, it is seen that it would be useful to have $d_x\upsilon^{x+1}$ already calculated for all values of x (based on a desired mortality table and interest rate). This value of d_x multiplied by υ^{x+1} is represented by the commutation symbol: C_x

The following, then, is the definition of the C_x symbol:

$$C_x = d_x \upsilon^{x+1}$$

In Table IV, columns of C_x are shown for the 1958 C.S.O. Table at 3%. As an example, using age 20 in Table IV, the value of C_{20} can be verified by multiplying d_{20} (from Table III) by υ^{21} at 3% (from Table I):

Basic equation

$$C_x = d_x \upsilon^{x+1}$$

Substituting 20 for x

$C_{20} = d_{20} \upsilon^{21}$

Substituting values from the tables

= (17,300)(.537549)

= 9,300

This agrees with the value given in Table IV

Above, the factor for finding the net single premium for one year of insurance to a life age x was finally expressed as

$$\frac{d_x \upsilon^{x+1}}{l_x \upsilon^x}$$

The numerator is equal to C_x (because C_x is defined as d_x multiplied by υ^{x+1}). The denominator is equal to D_x (because D_x is defined as l_x multiplied by υ^x) Hence, the factor for finding the net single premium for one-year term insurance may be expressed

$$\begin{pmatrix} \text{Net Single Premium for} \\ \text{\$1,000 Death Benefit to} \\ \text{a Life Age } x\text{, If Death} \\ \text{Occurs in 1 Year} \end{pmatrix} = \$1{,}000\left(\frac{C_x}{D_x}\right)$$

The value of this factor $\frac{C_x}{D_x}$

is the same as the value of the factor for the natural premium used previously:

$$\frac{d_x \upsilon^1}{l_x}$$

but the commutation functions are easier to use in making calculations.

In the example given in Section 9.1, the net single premium at age 25 for $1,000 of one-year term insurance would be

$$\$1{,}000\left(\frac{C_{25}}{D_{25}}\right)$$

Using values for the commutation functions from Table IV, this becomes

$$\$1{,}000\left(\frac{8{,}570}{4{,}573{,}377}\right) = \$1.87$$

The answer is the same as before, but the calculation is simplified.

To Illustrate- Using Table IV, calculate the net single premium at age 40 for one year of life insurance of $1,000.

Solution

This is the same problem as shown in the illustration in Section 9.1. The solution will now be given by using commutation functions:

Basic equation

$$\left(\begin{matrix}\text{Net Single}\\ \text{Premium}\end{matrix}\right) = \$1{,}000\left(\frac{C_x}{D_x}\right)$$

$$= \$1{,}000\left(\frac{C_{40}}{D_{40}}\right)$$

Substituting values from Table IV

$$= \$1{,}000\left(\frac{9{,}709}{2{,}833{,}002}\right) = \$3.43$$

This answer agrees with that calculated in Section 9.1

FOR OTHER BENEFITS The net single premium at age *x* for $1,000 of *whole life insurance* may be expressed as the following total of the net single premiums for the individual years insurance (with the subscripts of the *D's* all being the age at the evaluation date):

$$\begin{pmatrix}\text{Net Single}\\ \text{Premium}\end{pmatrix} = \$1{,}000\left(\frac{C_x}{D_x}\right) + \$1{,}000\left(\frac{C_{x+1}}{D_x}\right) + \$1{,}000\left(\frac{C_{x+2}}{D_x}\right) + \ldots\ldots\begin{pmatrix}\text{to the end of the}\\ \text{mortality table}\end{pmatrix}$$

The common multiplier ($1,000) can be factored out. The fractions to be added together all have a common denominator (D_x). Accordingly, the net single premium for the whole life insurance can be expressed as

$$\begin{pmatrix}\text{Net Single}\\ \text{Premium}\end{pmatrix} = \$1{,}000\left[\frac{C_x + C_{x+1} + C_{x+2} + \ldots\text{to the end of the mortality table}}{D_x}\right]$$

In order to avoid the necessity of adding together all the *C*'s to the end of the mortality table, this total is also tabulated. The total of the C's to the end of the mortality table is represented by the commutation symbol M_x

the subscript of the M being the same as that of the first C in the series. The following, then, is the definition of the M_x symbol:

$$M_x = (C_x + C_{x+1} + C_{x+2} + \ldots \text{ to the end of the mortality table})$$

In Table IV, columns of M_x are shown Dir the 1958 C.S.O. Table at 3%. The commutation function M_x can also be used to simplify the calculation of *term insurance* and *endowment insurance* net single premiums. In the example given in Section 9.2, the net single premium at age 25 for \$1,000 of three-year term insurance is

$$\$1{,}000\left(\frac{C_{25}+C_{26}+C_{27}}{D_{25}}\right)$$

Here the total of the C's to the end of the mortality table is not needed, but only the total for three years. This total can be found by taking M_{25} (the total of the C's from age 25 to the end of the table) and subtracting M_{28} (the total of the C's from age 28 to the end of the table). What remains after the subtraction is $C_{25}+C_{26}+C_{27}$. That is,

$$\$1{,}000\left(\frac{C_{25}+C_{26}+C_{27}}{D_{25}}\right) = \$1{,}000\left(\frac{M_{25}-M_{28}}{D_{25}}\right)$$

Using values of M and ID from Table IV, this becomes

$$\$1{,}000\left(\frac{1{,}276{,}590-1{,}251{,}291}{4{,}573{,}377}\right) = \$5.53$$

The answer is the same as that calculated in Section 9.2.

A general statement may be made that the factor to use in calculating a net single premium or an accumulated cost of insurance will be of the form $\frac{M - M + D}{D}$, where the subscript of the first M is the age when the insurance coverage begins; the subscript of the second M is the age at which the

insurance coverage stops (i.e., one greater than the last age covered); the subscript of the *D* in the numerator is the age at which a pure endowment would be paid; and the subscript of the *D* in the denominator is the age at which this net single premium or accumulated cost of insurance is paid. The difference between the subscripts of the Ms equals the actual number of years of insurance coverage. If there is no pure endowment involved, the *D* in the numerator does not appear. If the insurance is for the whole of life, the second *M* does not appear.

To Illustrate- Using Table IV, calculate the net single premium at age 50 for $5,000 of [2]-year term insurance.

Solution

This is the same problem as shown in the illustration in Section 9.2. The solution will now be given by using commutation functions. In the general expression given above, the *D* in the numerator will not appear, because there is no pure endowment involved:

Basic equation (subscripts of the *M*'s define the period of coverage; subscript of *D* is the evaluation age)

$$\begin{pmatrix}\text{Net Single}\\ \text{Premium}\end{pmatrix} = \$5{,}000\left(\frac{M_{50} - M_{52}}{D_{50}}\right)$$

Substituting values from Table IV

$$= \$5{,}000\left(\frac{1{,}028{,}986 - 995{,}821}{1{,}998{,}744}\right) = \$82.96$$

This answer is only 1 cent different from that calculated in Section 9.2.

To Illustrate Again- Using Table IV, calculate the net single premium at age 96 for $1,000 of whole life insurance.

Solution

This is the same problem as shown in the illustration in Section 9.3. The solution will now be given by using commutation functions. In the general expression for net single premiums, the second *M* will not appear, because the insurance is for the whole of life. Also, the *D* in the numerator will not appear, because there is no pure endowment involved:

Basic equation

$$\begin{pmatrix} \text{Net Single} \\ \text{Premium} \end{pmatrix} = \$1{,}000 \left(\frac{M_{96}}{D_{96}} \right)$$

Substituting values from Table IV

$$= \$1{,}000 \left(\frac{3{,}481}{3{,}692} \right)$$

$$= \$942.85$$

This answer is only 4 cents different from that calculated in Section 9.3.

To Illustrate Again- Using Table IV, calculate the net single premium at age 62 for a $7,500 endowment-at-age-65 insurance policy.

Solution

This is the same problem as shown in the illustration in Section 9.5. The solution will now be given by using commutation functions:

Basic equation (subscripts of the *M*'s define the period of coverage; subscript of *D* in numerator is age of pure endowment; subscript of *D* in denominator is evaluation age)

$$\begin{pmatrix}\text{Net Single}\\ \text{Premium}\end{pmatrix} = \$7,500\left(\frac{M_{62} - M_{65} + D_{65}}{D_{62}}\right)$$

Substituting values from Table IV

$$= \$7,500\left(\frac{773,206 - 686,750 + 995,688}{1,179,823}\right)$$

$$= \$6,879.07$$

This answer is only one cent different from that calculated in Section 9.5.

The form $\frac{M - M + D}{D}$ is also used to compute the accumulated cost of insurance. However, the evaluation date for the accumulated cost of insurance is at the *end* of the term of coverage, whereas the net single premium is evaluated at the *beginning*. This is because the accumulated cost of insurance represents the amount that would have to be paid by the survivors at the end of the term of coverage, while the net single premium is the amount to be paid by those living at the beginning of the term of coverage. Therefore, when the accumulated cost is computed, the subscript of the ID in the denominator is the highest age.

To Illustrate- Using Table IV, calculate the accumulated cost of insurance at age 28 for $1,000 of 3-year term insurance.

Solution

This is the same S-year term policy for which the net single premium was calculated on page 208 to be $5.53. There the denominator used in the calculation was D_{25} because the net single premium is evaluated at the

beginning of the insurance. Now to calculate the accumulated cost of insurance, the denominator is D_{28} because the evaluation date is at the end. (The D in the numerator will not appear, because there is no pure endowment involved.)

Basic equation (subscripts of the M's define the period of coverage; subscript of D is the evaluation age)

$$\left(\begin{array}{l}\text{Accumulated Cost}\\\text{of Insurance}\end{array}\right) = \$1{,}000\left(\frac{M_{25} - M_{28}}{D_{28}}\right)$$

Substituting values from Table IV

$$= \$1{,}000\left(\frac{1{,}276{,}590 - 1{,}251{,}291}{4{,}160{,}727}\right)$$

$$= \$6.08$$

This answer agrees with that calculated in Section 9.6 for the same policy without the use of commutation functions.

It should be noted that the accumulated cost of insurance ($6.08) is higher than the net single premium ($5.53). This is as expected since the number of persons paying in at the beginning of the term of coverage is greater than the number of survivors who would pay at the end.

EXERCISES

(Use Table IV for all of the following. Assume insurance is payable at the end of the year unless otherwise indicated.)

1- Write an expression (using commutation functions) for the net single premium at age 10 for $10,000 of 1-year term insurance. Calculate the value.

2- Write an expression (using commutation functions) for the net single premium at age 10 for $10,000 of term-to-age-$4^0$ insurance. Calculate the value.

3- Write an expression (using commutation functions) for the net single premium at age 65 for $1,000 of whole life insurance. Calculate the value.

4- Write an expression (using commutation functions) for the net single premium at age 5 for a $2,000 pure endowment due 25 years thereafter. Calculate the value.

5- Write an expression (using commutation functions) for the net single premium at age 40 for a $5,000 30-year endowment insurance policy. Calculate the value,

6- Write an expression (using commutation functions) for the accumulated cost at age 30 for a $10,000 12-year term insurance policy, Calculate the value.

7- Write an expression (using commutation functions) for the net single premium at age 15 for a $1,000 terni-fo-age-65 insurance policy, assuming the insurance is payable at the moment of death, Calculate the value.

8- Describe in words what is represented by each of the following expressions:

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$a)\ \$1{,}000\left(\frac{C_{43}}{D_{43}}\right)$

$b)\ \$1{,}000\left(\frac{C_{43}+C_{44}+C_{45}+C_{46}}{D_{43}}\right)$

$c)\ \$1{,}000\left(\frac{M_{43}+M_{47}}{D_{43}}\right)$

$d)\ \$5{,}000\left(\frac{M_{62}}{D_{62}}\right)$

$e)\ \$15{,}000\left(\frac{M_{25}-M_{50}+D_{50}}{D_{25}}\right)$

$f)\ \$1{,}500\left(\frac{D_{65}}{D_{25}}\right)$

$g)\ \$..{,}500\left(1+\frac{1}{2}i\right)\left(\frac{M_{0}-M_{40}}{D_{0}}\right)$

SYMBOLS FOR NET SINGLE PREMIUMS

Chart 9-3 displays certain internationally used symbols, each of which represents the net single premium at age x for $1 of life insurance. (These net single premiums are also shown as they would he calculated using commutation functions.)

The capital letter *"A"* is used with a subscript for the evaluation age, and the number of years under an "angle." In general, the symbol $A_{x:n}$. refers to n-year endowment insurance. If the reference is to term insurance or to a pure endowment, then a "$1\rceil$" is placed over the age or the number of years, respectively.

CHART-3

Net Single Premium at Ago *x* for $1 of Life Insurance

Type of Life Insurance	*Symbol for Net Single Premium*	*Net Single Premium Using Commutation Functions*
Whole Life Insurance	A_x	$\frac{M_x}{D_x}$
n-Year Term Insurance	$A^1_{x:\overline{n}\rceil}$	$\frac{M_x - M_{x+n}}{D_x}$
Pure Endowment Due in n Years	$A_{x:\overset{1}{\overline{n}\rceil}}$	$\frac{D_{x+n}}{D_x}$
n-Year Endowment Insurance	$A_{x:\overline{n}\rceil}$	$\frac{M_x - M_{x+n} + D_{x+n}}{D_x}$

Net single premium symbols wherein a *bar* is placed over the *"A"* represent insurance payable at the moment of death. For example,

$$\overline{A}_x$$

is the internationally used symbol for the net single premium at age x for $1 of whole life insurance payable at the moment of death.

CHAPTER 11

ANNUAL PREMIUMS

INTRODUCTION

The purchase of life insurance by the payment of a single premium when the policy is issued is relatively uncommon, because few people are financially able to do so. The purchase of life insurance by payment of the one-year term insurance premium each year is also uncommon, because this premium increases sharply at the older ages.

A more practical procedure has been devised for paying premiums whereby the premium is paid annually but its amount is the same each year. The calculation of such annual level premiums is based on this principle: at *the date the policy is issued, the present value of the premiums must be equal to the present value of the benefits.*

Annual premiums so calculated are called *net annual premiums.* The word "net" means that the premium calculation involves only rates of interest and modality, with no consideration of expenses or profits. (Annual premiums which do include an amount for expenses and profits are called *gross annual premiums.* These will be considered in Section 9.7.)

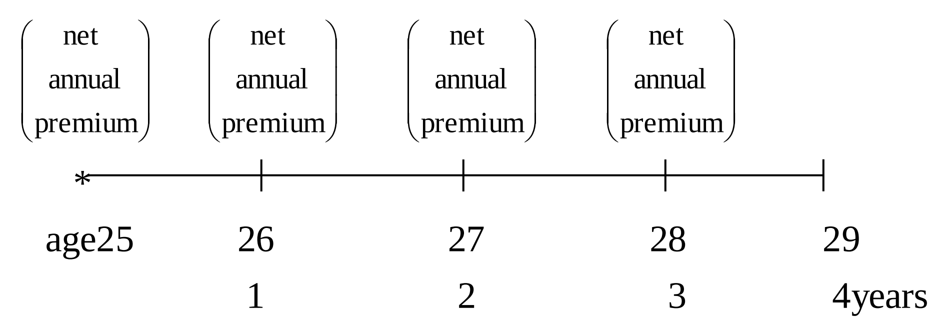
Life insurance policies may be issued on any date during a calendar year. The first annual premium is due on that date, the second annual premium is due one year later, etc. The period of time between such anniversaries is known as a *policy year,* to distinguish it from a calendar year (i.e., January 1 to December 31). In this book, references to "years" in connection with insurance policies will mean *policy years.*

Annual premiums are always paid at the beginning of the year, such payments taking place each year only if the person insured is then alive to pay. Therefore, annual premiums for a policy constitute a *life annuity due.* Such premiums may be paid for the same number of years as the insurance benefit covers, or a fewer number of years. Thus, they may constitute either a *whole life annuity due* or a *temporary life annuity due.* In describing a certain policy, if the premium paying period is not specified, it is generally understood that premiums are payable for as long as there is life insurance coverage.

NET ANNUAL PREMIUMS FOR TERM INSURANCE

A term life insurance policy which covers a stated number of years normally requires an annual premium payable at the beginning of each of those years. The calculation of the net annual premiums is based upon the principle stated above: at *the date the policy is issued, the present value of the net premiums must be equal to the present value of the benefits.*

For example, it may be desired to calculate the net annual premium (per $1,000 of insurance) for a four-year term insurance policy issued to a person age 25. Since no premium-paying period is specified, it is understood that these premiums are payable for four years. In line diagram form, this series of net annual premiums appears as follows:



The net annual premiums constitute a *temporary life annuity due.* Their present value, at the date the policy is issued, may be calculated by consulting a mortality table and assuming that all persons enumerated therein are individually involved. At age 25, their present value is

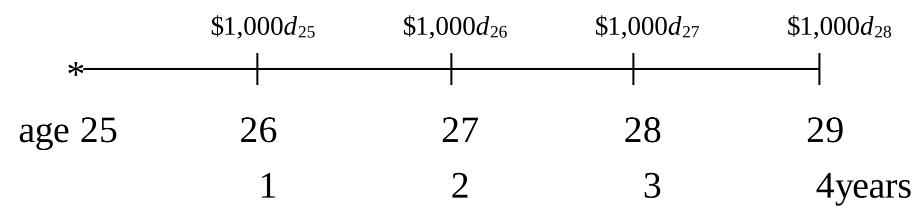
$$\begin{pmatrix}\text{Present Value of}\\ \text{Net Annual Premiums}\end{pmatrix} = \begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}(l_{25} + l_{26}\upsilon + l_{27}\upsilon^2 + l_{28}\upsilon^3)$$

This equation shows that the present value of the net annual premiums equals the total of each of the net annual premiums paid by survivors at each age, with each such amount being discounted at interest

to the evaluation date. The first item inside the parentheses, namely l_{25}, is not multiplied by any υ factor because it represents those net annual premiums which are payable upon the evaluation date.

The above expression would be divided by the number living on the evaluation date (l_{25}) to find the present value per person (as was done in Chapter 8). However, in this case, this step is not necessary. The above expression represents a total present value for all the l_{25} persons. Calculations which follow will demonstrate how it4s used to find a net annual premium per person.

Similarly, the present value of the *benefits* may be calculated. In line diagram form, the death benefits to be paid appear as follows:



At age 25, the present value is

$$\begin{pmatrix}\text{Present Value}\\ \text{of Benfits}\end{pmatrix} = \$1{,}000(d_{25}\upsilon + d_{26}\upsilon^2 + d_{27}\upsilon^3 + d_{28}\upsilon^4)$$

That is, the $1,000 death benefit is paid for those who die at each age, with each such amount being discounted at interest from the end of the year of death to the evaluation date.

The above expression would be divided by the number living on the evaluation date (l_{25}) to find this present value per person, i.e., the net single premium (as was done in Chapter 9). In this case, this step is not necessary. The above expression represents a total net single premium for all the l_{25} persons. It will be used in the calculation below to find a net annual premium per person.

The expression for the present value of net annual premiums is equal to the expression for the present value of the benefits:

$$\begin{pmatrix}\text{Present Value of}\\ \text{Net Annual Premiums}\end{pmatrix} = \begin{pmatrix}\text{Present Value}\\ \text{of Benfits}\end{pmatrix}\begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}$$

$$(l_{25} + l_{26}\upsilon + l_{27}\upsilon^2 + l_{28}\upsilon^3)$$

$$= \$1{,}000(d_{25}\upsilon + l_{26}\upsilon^2 + d_{27}\upsilon^3 + d_{28}\upsilon^4)$$

The equation can be solved for the Net Annual Premium, which will be the net annual premium per person. If, for example, the 1958 C.S.O. Table and 3% interest were being used to calculate the above net annual premiums, the present value of the net annual premiums (the left side) would be evaluated as follows;

From above

$$\begin{pmatrix}\text{Present Value of}\\ \text{Net Annual Premiums}\end{pmatrix} = \begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}(l_{25} + l_{26}\upsilon + l_{27}\upsilon^2 + l_{28}\upsilon^3)$$

Substituting the values for the l's from Table III, for the υ's from Table 1(3%)

$$= \begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}\begin{bmatrix}(9{,}575{,}636)\\ +(9{,}557{,}423)(.970874)\\ +(9{,}538{,}423)(9.42596)\\ +(9{,}519{,}442)(.915142)\end{bmatrix}$$

$$= \begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}\begin{bmatrix}9{,}575{,}636 + 9{,}278{,}793\\ +8{,}990{,}879 + 8{,}711{,}641\end{bmatrix}$$

$$= \begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}[36{,}556{,}949]$$

The same mortality table and interest rate used to calculate the present value of the net annual premiums are used to calculate the present value of the benefits. The right side of the equation would be evaluated as follows;

From above

$$\begin{pmatrix}\text{Present Value}\\ \text{of Benfits}\end{pmatrix} = \$1{,}000(d_{25}\upsilon + d_{26}\upsilon^2 + d_{27}\upsilon^3 + d_{28}\upsilon^4)$$

Substituting the values for the d's from Table III, for the υ's from Table 1(3%)

$$= \$1{,}000\begin{bmatrix} (18{,}481)(.970874) \\ +(18{,}732)(.942596) \\ +(18{,}981)(.915142) \\ +(19{,}324)(.888487) \end{bmatrix}$$

$$= \$1{,}000(17{,}943+17{,}657+17{,}370+17{,}169)$$

$$= \$70{,}139{,}000$$

The net annual premium per person can then be found:

Basic equation

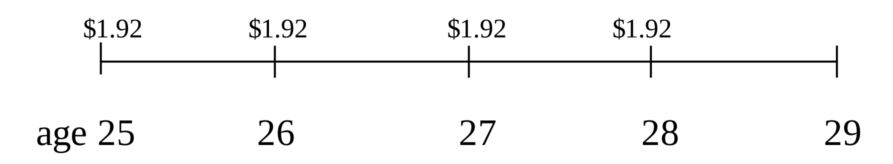
$$\begin{pmatrix}\text{Present Value of}\\ \text{Net Annual Premiums}\end{pmatrix} = \begin{pmatrix}\text{Present Value}\\ \text{of Benfits}\end{pmatrix}$$

Substituting values calculated above

$$\begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}[36{,}556{,}949] = \$70{,}139{,}000$$

$$\begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix} = \$1.92$$

In line diagram form, this series of net annual premiums appears as follows:



It can be demonstrated that the payment of these net annual premiums will provide $1,000 (at the end of the year of death) for all who die between ages 25 and 29. Using Table III, the amount of premium paid in at the beginning of the first year is

$$\$1.92(l_{25}) = \$1.92(9{,}575{,}636)$$

$$= \$18{,}385{,}221.12$$

At the end of one year, the accumulated value of this amount is

$$(\$18{,}385{,}221.12)(1.03) = \$18{,}936{,}777.75$$

From this fund is deducted $1,000 for each who have died during the first year:

$$\$1{,}000(d_{25}) = \$1{,}000(18{,}481)$$

$$= \$18{,}481{,}000$$

This leaves a balance in the fund of

$$\$18{,}936{,}777.75 - \$18{,}481{,}000 = \$455{,}777.75$$

The continued operation of the fund for succeeding years may be traced in the accompanying schedule (Chart 10-1). The final excess in the fund represents less than $3 for each person dying the final year, and results from rounding off the net annual premium to two decimal places.

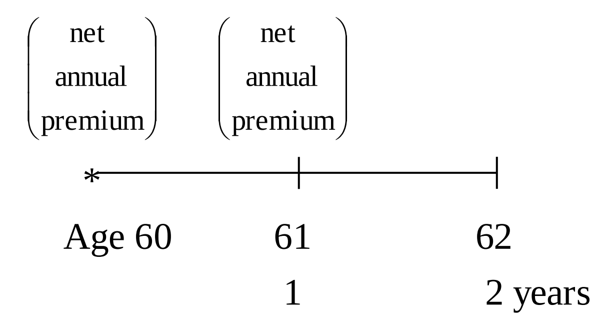
CHART-1

(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*
Year	*Premiums Paid at Beginning of Year*	*Total Fund at Beginning of Year (Col.6, Previous Year, plusCol.2)*	*Fund Accumulated for One Year (Col 3× 1.03)*	*Claims Paid at End of Year (Number of Deaths× $1,000)*	*Balance in Fund at End of Year after Payment of Claims (Col 4- Col.5)*
1	$18,385,221.12	$18,385,221.12	$18,936,777.75	$18,481,000	$455,777.75
2	18,349,737.60	18,805,515.35	19,369,680.81	18,732,000	637,680.81
3	18,313,772.16	18,951.452.97	19,519,996.56	18,981,000	538,996.56
4	18,277,328.64	18,816,325.20	19,380,814.96	19,324,000	56,814.96

To Illustrate- Using Table III and **3%** interest, calculate the net annual premium (per $1,000) for a 2-year term insurance policy issued at age 60.

Solution

In line diagram form, the *net annual premiums* for this policy appear as follows:



Their total present value is equivalent to the net annual premiums paid by the survivors at each age, with each such amount being discounted at interest to the evaluation date.

Basic equation

$$\begin{pmatrix}\text{Present Value of}\\ \text{Net Annual Premiums}\end{pmatrix} = \begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}(l_{60} + l_{61}\upsilon)$$

Substituting the values for the Tables

$$= \begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}\begin{bmatrix}(7{,}698{,}698)\\ +(7{,}452{,}106)(.970874)\end{bmatrix}$$

$$= \begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}[7{,}698{,}698 + 7{,}322{,}435]$$

$$= \begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}[15{,}021{,}133]$$

In line diagram form, the *benefits* for this policy appear as follows:

$$
\begin{array}{lcc}
 & \$1{,}000d_{60} & \$1{,}000d_{61} \\
* \text{———————} & | \text{———————} & | \\
\text{Age } 60 & 61 & 62 \\
 & 1 & 2 \text{ years}
\end{array}
$$

Their total present value is equivalent to the amounts paid for those who die at each age, with each amount being discounted at interest from the end of the year of death to the evaluation date.

Basic equation

$$\begin{pmatrix}\text{Present Value}\\ \text{of Benfits}\end{pmatrix} = \$1{,}000(d_{60}\upsilon + d_{61}\upsilon^2)$$

Substituting the values for the Tables

$$= \$1{,}000\begin{bmatrix}(156{,}592)(.970874)\\ +(167{,}736)(.942596)\end{bmatrix}$$

$$= \$1{,}000(152{,}031 + 158{,}107)$$

$$= \$310{,}138{,}000$$

The net annual premium is then found by substituting these values in the basic equation:

$$\begin{pmatrix}\text{Present Value of}\\ \text{Net Annual Premiums}\end{pmatrix} = \begin{pmatrix}\text{Present Value}\\ \text{of Benfits}\end{pmatrix}$$

$$\begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}[15{,}021{,}133] = \$310{,}138{,}000$$

$$\begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix} = \$20.65$$

The net annual premiums for a whole life insurance policy may be paid either

1. For the entire lifetime of the person insured. They thus constitute a *whole life annuity due*. This kind of insurance policy is known as an *ordinary life* policy.

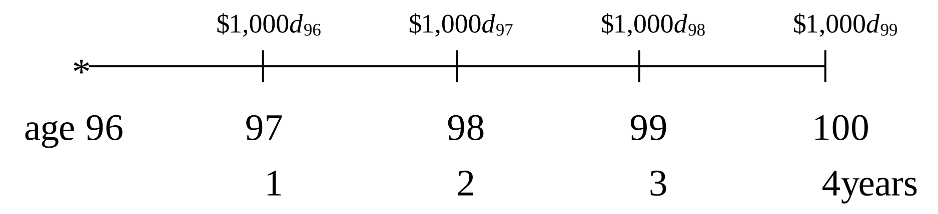
or

2. For a number of years which is *less* than the entire lifetime, say for *n* years. They thus constitute a *temporary life annuity due*. This kind of insurance is known as an *n-payment life* policy.

To Illustrate- Using the 1958 C.S.O. Table and 3% interest, calculate the net annual premium (per $1,000) for an ordinary life insurance policy issued at age 96; also for a 2-payment life policy issued at that age.

Solution

The *present value of the benefits* is the same for both policies because both provide insurance coverage for the whole of life. The line diagram for these benefits appears as follows (remembering that all persons die before age 100, according to the 1958 C.S.O. Table):



Their total present value is equivalent to the amounts of benefits paid for those who die at each age, with each such amount being discounted at interest from the end of the year of death to the evaluation date.

Basic equation

$$\begin{pmatrix}\text{Present Value}\\ \text{of Benfits}\end{pmatrix} = \$1{,}000(d_{96}\upsilon + d_{97}\upsilon^2 + d_{98}\upsilon^3 + d_{99}\upsilon^4)$$

Substituting the values form the Tables

$$= \$1{,}000\begin{bmatrix}(25{,}250)(.970874)\\ +(18{,}456)(.942596)\\ +(12{,}916)(.915142)\\ +(\ \ 6{,}415)(.888487)\end{bmatrix}$$

$$= \$1{,}000(24{,}515+17{,}397+11{,}820+5{,}700)$$

$$= \$59{,}432{,}000$$

The line diagrams for the *net annual premiums* for these two policies appear as follows:

Ordinary Life

(net annual premium)	(net annual premium)	(net annual premium)	(net annual premium)	
*				
Age96	97	98	99	100
	1	2	3	4years

Two-Payment Life

(net annual premium)	(net annual premium)			
*				
Age96	97	98	99	100
	1	2	3	4years

In each case, their total present value is equivalent to the net annual premiums paid by the survivors at each age, with each such amount being discounted at interest to the evaluation date.

Ordinary Life

Basic equation

$$\begin{pmatrix}\text{Present Value of} \\ \text{Net Annual Premiums}\end{pmatrix} = \begin{pmatrix}\text{Net Annual} \\ \text{Premium}\end{pmatrix} (l_{96} + l_{97}\upsilon + l_{98}\upsilon^2 + l_{99}\upsilon^3)$$

Substituting values from the tables

$$= \begin{pmatrix}\text{Net Annual} \\ \text{Premium}\end{pmatrix} \begin{bmatrix} (63{,}037) \\ +(37{,}787)(.970874) \\ +(19{,}331)(9.42596) \\ +(\ 6{,}415)(.915142) \end{bmatrix}$$

$$= \begin{pmatrix}\text{Net Annual} \\ \text{Premium}\end{pmatrix} [63{,}037 + 36{,}686 + 18{,}221 + 5{,}871]$$

$$= \begin{pmatrix}\text{Net Annual} \\ \text{Premium}\end{pmatrix} [123{,}815]$$

Two-Payment Life

Basic equation

$$\begin{pmatrix}\text{Present Value of} \\ \text{Net Annual Premiums}\end{pmatrix} = \begin{pmatrix}\text{Net Annual} \\ \text{Premium}\end{pmatrix} (l_{96} + l_{97}\upsilon)$$

Substituting values from the tables

$$= \begin{pmatrix}\text{Net Annual} \\ \text{Premium}\end{pmatrix} \begin{bmatrix} (63{,}037) \\ +(37{,}787)(.970874) \end{bmatrix}$$

$$= \begin{pmatrix}\text{Net Annual} \\ \text{Premium}\end{pmatrix} [63{,}037 + 36{,}686]$$

$$= \begin{pmatrix}\text{Net Annual} \\ \text{Premium}\end{pmatrix} [99{,}723]$$

The net annual premium for each policy is then solved for, as follows:

Ordinary Life

Basic equation

$$\begin{pmatrix}\text{Present Value of} \\ \text{Net Annual Premiums}\end{pmatrix} = \begin{pmatrix}\text{Present Value} \\ \text{of Benfits}\end{pmatrix}$$

Substituting values calculated above

$$\begin{pmatrix}\text{Net Annual} \\ \text{Premium}\end{pmatrix} [123{,}815] = \$59{,}432{,}000$$

$$\begin{pmatrix}\text{Net Annual} \\ \text{Premium}\end{pmatrix} = \$480.01$$

Two-Payment Life

Basic equation

$$\begin{pmatrix}\text{Present Value of} \\ \text{Net Annual Premiums}\end{pmatrix} = \begin{pmatrix}\text{Present Value} \\ \text{of Benfits}\end{pmatrix}$$

Substituting values calculated above

$$\begin{pmatrix}\text{Net Annual} \\ \text{Premium}\end{pmatrix} [99{,}723] = \$59{,}432{,}000$$

$$\begin{pmatrix}\text{Net Annual} \\ \text{Premium}\end{pmatrix} = \$595.97$$

As the premium-paying period is shortened, the amount of the net annual premium for the same benefits increases. This can be seen by comparing the net annual premium for the ordinary life policy above ($480.01) with the net annual premium for the two-payment life policy ($595.97).

The calculation of net annual premiums for whole life insurance at the younger ages would become very laborious if the procedure described were used. In actual practice, calculations for all kinds of net annual premiums are usually simplified by using commutation functions. How commutation functions are used to compute net annual premiums will be explained in Section 9.6.

NET ANNUAL PREMIUMS FOR ENDOWMENT INSURANCE

The net annual premiums for an endowment insurance policy may be paid either

1. For the same number of years as the insurance covers, say for *n* years. They thus constitute a *temporary life annuity due for n years*. This kind of insurance policy is known as an *n-year endowment* policy.

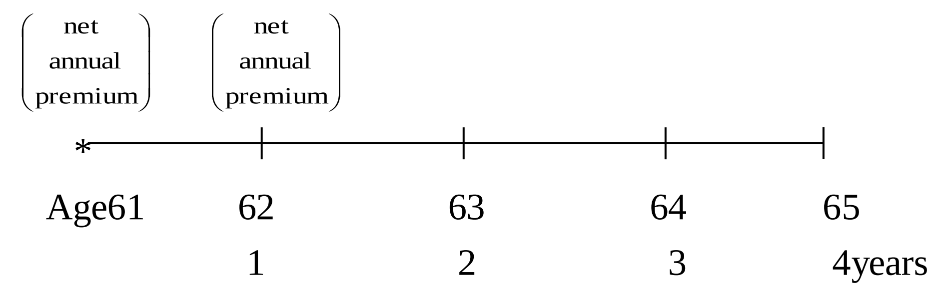
or

2. For a number of years which is *less* than the period that the insurance covers, say for *m* years (with the insurance coverage for *n* years). They thus constitute a *temporary life annuity due for m years*. This kind of insurance policy is known as an *in-payment n-year endowment* policy.

To Illustrate- Using the 1958 C.S.O. Table and 3% interest, calculate the net annual premium for a $1,000 2-payment endowment-at age-65 policy issued to a man age 61. Construct a schedule proving that this premium will provide the benefits of the policy.

Solution

The number of years of insurance equals 65 - 61 = 4. Hence, this is a 2-payment 4-year endowment policy. In line diagram form, the *net annual premiums* appear as follows:



Their total present value is equivalent to the net annual premiums paid by the survivors at each age, with each such amount being discounted at interest to the evaluation date.

Basic equation

$$\begin{pmatrix}\text{Present Value of}\\ \text{Net Annual Premiums}\end{pmatrix}=\begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}(l_{61}+l_{62}\upsilon)$$

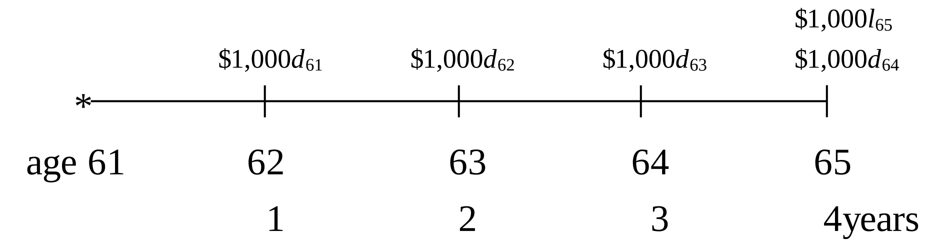
Substituting values from the tables

$$=\begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}\begin{bmatrix}(7{,}452{,}106)\\ +(7{,}374{,}370)(.970874)\end{bmatrix}$$

$$=\begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}[7{,}542{,}106+7{,}159{,}584]$$

$$=\begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}[14{,}701{,}690]$$

In line diagram form, the *benefits* for this policy appear as follows:



The amounts paid for those who die at each age are payable at the end of the year of death. The pure endowment is paid to the survivors at the end of the 4-year period. All amounts paid are discounted at interest to the evaluation date. Notice that in the expression below, the exponents on the last two υ's are the same. Both d_{64} and d_{65} are multiplied by υ^4. This is because the two benefits are payable on the same date: the death benefit for those who die during the final year, and the pure endowment benefit to those still alive at the end of the final year.

Basic equation

$$\begin{pmatrix}\text{Present Value}\\ \text{of Benfits}\end{pmatrix}=\$1{,}000(d_{61}\upsilon+d_{62}\upsilon^2+d_{63}\upsilon^3+d_{64}\upsilon^4+l_{65}\upsilon^4)$$

Substituting the values for the Tables

$$= \$1{,}000 \begin{bmatrix} (167{,}736)(.970874) \\ +(179{,}271)(.942586) \\ +(191{,}174)(.915142) \\ +(203{,}394)(.888487) \\ +(6{,}800{,}531)(.888487 \end{bmatrix}$$

=$1,000(162,854 + 168,980 + 174,951
+ 180,713 + 6,042,183)

= $6,729,678,000

The net annual premium is then found:

Basic equation

$$\begin{pmatrix} \text{Present Value of} \\ \text{Net Annual Premiums} \end{pmatrix} = \begin{pmatrix} \text{Present Value} \\ \text{of Benfits} \end{pmatrix}$$

Substituting values calculated above

$$\begin{pmatrix} \text{Net Annual} \\ \text{Premium} \end{pmatrix} [14{,}701{,}690] = \$6{,}729{,}678{,}000$$

$$\begin{pmatrix} \text{Net Annual} \\ \text{Premium} \end{pmatrix} = \$457.75$$

The accompanying schedule (Chart 10-2) demonstrates that this premium will provide the benefits of the policy.

CHART-2

(1) Year	(2) Premiums Paid at Beginning of Year	(3) Total Fund at Beginning of Year (Col.6 Previous Year, plus Col.2)	(4) Fund Accumulated for One Year (Col.3× 1.03)	(5) Claims Paid at End of Year (Number of Death × $1,000)	(6) Balance in Fund at End of Year after Payment of Claims (Col.4 - Col.5)
1	$3,452,399,021.50	$3,452,399,021.50	$3,555,970,992.15	$ 167,736,000	$3,388,234,992.15
2	3,375,617,867.50	6 ,763,852,859.65	6,966,768,445.44	179,271,000	6,787,497,445.44
3	None	6,787,497,445.44	6,991,122,368.80	191,174,000	6,799,948,368.80
4	None	6,799,948,368.80	7,003,946,819.86	203,394,000	6,800,552,819.86

At the end of the four years, the number of survivors is

$l_{65} = 6{,}800{,}531$

Therefore, the total pure endowments to be paid to the survivors at that time is

$\$\ 1{,}000(6{,}800{,}531) = \$6{,}800{,}531{,}000$

This leaves a balance in the hind, after the pure endowments are paid, equal to

$\$6{,}800{,}552{,}819.86 - \$6{,}800{,}531{,}000 = \$21{,}819.86$

This represents only about $\frac{1}{3}$ of a cent for each of the survivors, and is the result of rounding off the net annual premium to 2 decimal places.

RELATIONSHIPS BETWEEN NET PREMIUMS AND LIFE ANNUALS

The basis for the derivation of all the net annual premiums discussed in this chapter has been the principle: at *the date the policy is issued, the present value of the net annual premiums must be equal to the present value of the benefits.* The calculations above were made on a total basis, that is, the total present values for all persons living on the evaluation date.

Net premiums can also be calculated on a per person basis. The present value of the net annual premiums may be expressed as the net annual premium multiplied by the present value at age *x* of a temporary life annuity due of 1. This latter present value is known as the "annuity factor." Tables of such life annuity factors at various ages have been prepared and are usually available.

This relationship can be expressed by the following equation:

$$\begin{pmatrix}\text{Net Annual}\\ \text{Premiums}\end{pmatrix}\begin{pmatrix}\text{Annuity}\\ \text{Factor}\end{pmatrix}=\begin{pmatrix}\text{Present Value of}\\ \text{Net Annual Premiums}\end{pmatrix}$$

Since the present value of the benefits is the same as the net single premium, the basic statement of equality may be written:

$$\begin{pmatrix}\text{Net Annual}\\ \text{Premiums}\end{pmatrix}\begin{pmatrix}\text{Annuity}\\ \text{Factor}\end{pmatrix}=\begin{pmatrix}\text{Net Single}\\ \text{Premiums}\end{pmatrix}$$

From this equation, the value of any one of the items in the equation may be calculated if the values of the other two are known.

To Illustrate- Calculate the present value at age 40 of a 10-year life annuity due of 1 per year (the annuity factor), given the following values:

$57.84 = Net Annual Premium per $1,000 at Age 40 for a 10-Payment Life Policy

$502.64 = Net Single premium per $1,000 at Age 40 for Whole Insurance

Solution

Basic equation

$$\begin{pmatrix}\text{Net Annual}\\ \text{Premiums}\end{pmatrix}\begin{pmatrix}\text{Annuity}\\ \text{Factor}\end{pmatrix}=\begin{pmatrix}\text{Net Single}\\ \text{Premiums}\end{pmatrix}$$

Substituting values given

$$\$57.84\begin{pmatrix}\text{Annuity}\\ \text{Factor}\end{pmatrix}=\$502.64$$

$$\begin{pmatrix}\text{Annuity}\\ \text{Factor}\end{pmatrix}=\$8.69$$

To Illustrate Again- Calculate the net single premium per $1,000 at age 40 for a 10-year pure endowment, given the following values:

\$90.68 = Net Annual Premium per \$1,000 at Age 40 for a 10-Year Endowment Policy

\$70.90 = Net Single Premium per \$1,000 at Age 40 for a 10-Year Term Insurance Policy

8.69 = Present Value at Age 40 of a 10-Year Life Annuity Due of 1 per Year (Annuity Factor)

Solution

It must be remembered that the net single premium for a 10-year endowment policy consists of two parts:

1. Net single premium for term insurance

and

2. Net single premium for a pure endowment

The value of the first part is given; the value of the second part must be solved for.

Basic equation, showing net single premium in two pads

$$\begin{pmatrix}\text{Net Annual}\\ \text{Premiums}\end{pmatrix}\begin{pmatrix}\text{Annuity}\\ \text{Factor}\end{pmatrix} = \begin{pmatrix}\text{Net Single Premiums}\\ \text{for Pure Insurance}\end{pmatrix} + \begin{pmatrix}\text{Net Single Premiums}\\ \text{for Pure Endowment}\end{pmatrix}$$

Substituting values given

$$(\$90.68)(8.69) = \$70.90 + \begin{pmatrix}\text{Net Single Premiums}\\ \text{for Pure Endowment}\end{pmatrix}$$

$$(\$90.68)(8.69) - \$70.90 = \begin{pmatrix}\text{Net Single Premiums}\\ \text{for Pure Endowment}\end{pmatrix}$$

$$\$717.11 = \begin{pmatrix}\text{Net Single Premiums}\\ \text{for Pure Endowment}\end{pmatrix}$$

EXERCISES

1- Write an equation (using symbols) showing the present value of the net annual premiums being equal to the present value of the benefits for each of the following $1,000 policies (on a *total* basis, not per person): Solve each equation for the net annual premium per person (using Table III and 3% interest):

a) A 1-year term insurance policy issued at age 75

b) A 3-year term insurance policy issued at age 69

c) An ordinary life policy issued at age 97

d) A 2-payment life policy issued at age 97

e) A 3-year endowment insurance policy issued at age 10

f) A 4-year endowment insurance policy issued at age 25, wherein premiums stop at age 27 (i.e., last premium is payable at age 26)

2- Construct a schedule for the policy in Exercise 1, part *(f)*, showing the adequacy of the net annual premium to provide the benefits.

3- Given the following values, calculate the net annual premium per $1,000 at age 69 for an ordinary life insurance policy:

$808.41 = Net Single Premium per $1,000 at Age 69 for Whole Life Insurance.

10.772 = Present Value at Age 69 of a Whole Life Annuity Due of 1 per Year.

4- Calculate the present value of a 20-year life annuity due of 1 per year, given the following values:

$808.41 = Net Single Premium per $1,000 at Age 69 for Whole Life Insurance.

$102.15 = Net Annual Premium per $1,000 at Age 69 for a 20-Payment Life Insurance Policy.

5- If the following values are given, calculate the net single premium per $1,000 at age 30 for endowment at age-65 insurance:

$21.18 = Net Annual Premium per $1,000 at Age 30 for an Endowment-at-Age-65 Policy.

19,47 = Present Value at Age 30 of a 35-Year Life Annuity Due of 1 per Year.

COMMUTATION FUNCTIONS

The commutation functions explained in Chapters 8 and 9 can be used to calculate net annual premiums and will generally simplify the work.

The basis for the calculation is the equation given in Section 10.5:

$$\begin{pmatrix}\text{Net Annual}\\ \text{Premiums}\end{pmatrix}\begin{pmatrix}\text{Annuity}\\ \text{Factor}\end{pmatrix}=\begin{pmatrix}\text{Net Single}\\ \text{Premiums}\end{pmatrix}$$

The $1,000 four-year term insurance policy issued at age 25, discussed in Section 10.2, will be used as an example. The left-hand side of the above equation may be written

$$\begin{pmatrix}\text{Net Annual}\\ \text{Premiums}\end{pmatrix}\left(\frac{N_{25}-N_{29}}{D_{25}}\right)$$

This follows from the general statement made in Chapter 8 that the factor to use in evaluating a *life annuity* will be of the form $\frac{N-N}{D}$ where the subscript of the first N is the age when the first payment is due, the subscript of the second N is the first age when there are no more payments due, and the subscript of the D is the age at which the annuity is being evaluated.

The right-hand side of the above equation (Net Single Premium) may be written

$$\$1{,}000\left(\frac{M_{25}-M_{29}}{D_{25}}\right)$$

This follows from the general statement made in Chapter 9 that the factor to use in calculating a *net single premium* will be of the form $\frac{M-M+D}{D}$, where the subscript of the first M is the age when the insurance coverage begins, the subscript of the second M is the age at which the insurance coverage stops, the subscript of the D in the numerator is the age at which a pure endowment would be paid, and the subscript of the *ID* in the denominator is the age at which this net single

premium is evaluated. (In this particular case, the D in the numerator does not appear, because no pure endowment is involved.)

The entire equation then appears as follows:

$$\begin{pmatrix}\text{Net Annual}\\ \text{Premiums}\end{pmatrix}\left(\frac{N_{25}-N_{29}}{D_{25}}\right)=\$1{,}000\left(\frac{M_{25}-M_{29}}{D_{25}}\right)$$

The commutation symbol D_{25} appears in the denominator on both sides of the equation. If both sides are multiplied by D_{25}, the denominators are eliminated. The equation can then be solved for (Net Annual Premium) by dividing both sides by $(N_{25} - N_{29})$. The result is

$$\begin{pmatrix}\text{Net Annual}\\ \text{Premiums}\end{pmatrix}=\$1{,}000\left(\frac{M_{25}-M_{29}}{N_{25}-N_{29}}\right)$$

Using values of M and N from Table IV, this becomes

$$\begin{pmatrix}\text{Net Annual}\\ \text{Premiums}\end{pmatrix}=\$1{,}000\left(\frac{1{,}276{,}590-1{,}243{,}091}{113{,}189{,}600-95{,}729{,}800}\right)$$

$$= \$1.92$$

The answer is the same as that calculated in Section 10.2.

A general statement may be made that the factor to use in calculating a *net annual premium* will be of the form $\frac{M - M + D}{N - N}$ where the subscripts in the numerator define the benefits and follow the rule given in Chapter 9 for calculating net single premiums, and the subscripts of the N's in the denominator define the premium-paying period and follow the rule given in Chapter 8 for calculating life annuity factors.

To Illustrate- Using Table IV, calculate the net annual premium (per $1,000) for a 2-year term insurance policy issued at age 60.

Solution

This is the same problem as the illustration in Section 10.2. The solution will now be given by using commutation functions. In the general expression given above, the D in the numerator will not appear, because there is no pure endowment involved.

Basic equation; subscripts of the *M*'s define the period of coverage; subscripts of the *N*'s define the premium-paying period

$$\begin{pmatrix}\text{Net Annual}\\ \text{Premiums}\end{pmatrix} = \$1{,}000\left(\frac{M_{60} - M_{62}}{N_{60} - N_{62}}\right)$$

Substituting values from Table IV

$$= \$1{,}000\left(\frac{825{,}847 - 773{,}206}{16{,}510{,}076 - 13{,}960{,}493}\right)$$

$$= \$1{,}000\left(\frac{52{,}641}{2{,}549{,}583}\right)$$

$$= \$20.65$$

This answer agrees with that calculated in Section 10.2.

To Illustrate Again- Using Table IV, calculate the net annual premium (per $1,000) for an ordinary life policy issued at age 96.

Solution

This is the same problem as the illustration in Section 10.3. The solution will now be given by using commutation functions. In the general expression for net annual premiums, the second *M* in the numerator will not appear, because the insurance is for the whole of life. Also, the *D* in the numerator will not appear, because there is no pure endowment involved. In the denominator, the second N will not appear, because the premium payments are to be made for life.

Basic equation

$$\boxed{\begin{pmatrix}\text{Net Annual}\\ \text{Premiums}\end{pmatrix} = \$1{,}000\left(\frac{M_{96}}{N_{96}}\right)}$$

Substituting values from Table IV

$$= \$1{,}000\left(\frac{3{,}481}{7{,}251}\right)$$

$$= \$480.07$$

This answer is only 6 cents different from that calculated in Section 10.3. The difference is due to the fact that the commutation functions as shown in the tables are rounded off to the nearest whole number.

To Illustrate Again- Using Table IV, calculate the net annual premium for a \$1,000 2-payment endowment-at-age-65 policy issued to a man age 61.

Solution

This is the same problem as the illustration in Section 10.4. The solution will now be given by using commutation functions.

Basic equation; subscripts of the *M*,s define the period of coverage; subscript of *D* is age of pure endowment; subscripts of the *N*'s define the premium-paying period

$$\begin{pmatrix}\text{Net Annual}\\ \text{Premiums}\end{pmatrix} = \$1{,}000\left(\frac{M_{61} - M_{65} + D_{65}}{N_{61} - N_{63}}\right)$$

Substituting values from Table IV

$$= \$1{,}000\left(\frac{800{,}042 - 686{,}750 + 995{,}688}{15{,}203{,}325 - 12{,}780{,}670}\right)$$

$$= \$1{,}000\left(\frac{1{,}108{,}980}{2{,}422{,}682}\right)$$

$$= \$457.75$$

This answer agrees with that calculated in Section 10.4.

EXERCISES

(Use Table IV for all of the following.)

1- Write an expression (using commutation functions) -for the *net annual premium* (per $1,000) for each of the following policies. (If *a* student wishes, he can calculate the value of each.).

a) A 20-year term insurance policy issued at age 25.

b) A 1-year term insurance policy issued at age 65.

c) A tenn-to-age-65 insurance policy issued at age 40.

d) An ordinary life policy issued at age 0.

e) A 30-payment life policy issued at age 21.

f) A whole life insurance policy issued at age 30, wherein premiums stop at age 70 (i.e., last premium is payable at age 69).

g) A 25-year endowment insurance policy issued at age 28.

h) A 20-payment 30-year endowment insurance policy issued at age 15.

i) A 30-payment endowment-at-age-70 policy issued at age 22.

2- Calculate the net annual premium for a $20,000 ordinary life insurance policy issued to a girt age 15. Use a "3-year setback" for females.

3- Calculate the net annual premium for a $1,000 20-payment life policy issued at age 5, assuming the insurance is payable at the moment of death.

4- State in words what each of the following represents:

a) $\$1{,}000\left(\dfrac{M_5 - M_{35} + D_{35}}{N_5 - N_{25}}\right)$

b) $\$1{,}000\left(\dfrac{M_{60}}{N_{60} - N_{80}}\right)$

c) $\$1{,}000\left(\dfrac{M_{14} - M_{24}}{N_{14} - N_{19}}\right)$

d) $(\$5{,}000)(1.015)\left(\dfrac{M_{25} - M_{65}}{N_{25} - N_{65}}\right) + \$5{,}000\left(\dfrac{D_{65}}{N_{25} - N_{65}}\right)$

GROSS ANNUAL PREMIUMS

The *net* annual premiums considered thus far are sufficient, in terms of mortality and assumed interest rates, to provide the benefits guaranteed in the policy. However, they make no provision for the life insurance company's expenses of conducting business. Therefore, an amount called the *loading* must be added to these net premiums to provide for expenses, profits, and the possibility of unforeseen adversities. The total of the net premium and the loading is known as the *gross premium.* It is the gross premium which the policyowner pays to the insurance company.

Where the policy is "participating" (receives policy dividends), it is not necessary to have great refinement in the calculation of the loading. Savings from operations can be returned to the policyowners of participating policies in the form of dividends, and the dividend calculations can be changed when necessary to meet changing conditions. However, the amount of the loading must be reasonably conservative.

Where a policy is "nonparticipating," however, very detailed analysis of probable expenses is employed in calculating the loading. Because no policy dividends are returned to policyowners, the insurance company has no means of adjusting its income to allow for changes in expenses subsequent to issue of the policy.

The gross annual premium may be calculated by a variety of methods. Companies use many different formu1as for computing loadings, some very simple, some complex. As a result, the equations for calculating the gross premiums vary.

The expenses of a life insurance company fall into three principal categories:

1- Those expenses which are relatively *constant for each policy* regardless of the amount of the policy. These include the cost of issuing the policy, collecting the premiums, paying the claims, etc.

2- Those expenses which *vary with the amount of the premium.* These include state premium taxes and agents' commissions.

3- Those expenses which *vary with the amount of insurance,* i.e., those expenses which are usually higher for larger amount policies. These include costs of establishing whether applicants are in good health (such as medical examiners' fees), drawing up directions for payment of proceeds, etc.

Expenses which are relatively constant regardless of policy size (category 1) are frequently provided for by adding a certain charge for each policy, regardless of the amount of the policy. This is known as a "policy charge" or "policy fee." However, prior to about 1957, this method of loading was generally considered illegal because state laws prohibiting "discrimination between different policyholders of the same class" were interpreted to prohibit such a method of calculation. Therefore, a method of calculation was used in those days which was based on the determination of the amount of an average-size policy. The gross premiums per $1,000 were so calculated that such an average-size policy would yield the amount needed to pay for these particular expenses. The result was that large policies yielded more than enough to cover their expenses, while small policies yielded an insufficient amount. In total, however, approximately the correct amount was collected.

A compromise between the "old" and "new" methods of providing for these expenses which are relatively constant regardless of policy size is sometimes used. Under this method, gross premiums per $1,000 are quoted which vary according to the size group into which the policy falls. For example, these groups might be

Policies of less than $5,000 face amount
Policies of $5,000 to $9,999 face amount
Policies of $10,000 to $24,999 face amount
Policies of $25,000 to $99,999 face amount
Policies of $100,000 and over face amount

Within each such size group, an average-size policy is used to

calculate the gross premium per $1,000 for that group, such that the loading for this average-sized policy will cover the particular expenses referred to in category 1. The result is that the smaller-sized policies require a larger gross premium per $1,000 than the larger policies. This method is known as "band grading," or simply "banding."

In providing for those expenses which vary with the amount of the premium (category 2), a simple method is to add a percentage of the gross annual premium per $1,000 to the net annual premium. In equation form, this would be

Gross = Net + Percent of Gross

To Illustrate- Calculate the gross annual premium per $1,000 for a policy for which the net annual premium per $1,000 is $12.49, and a loading is needed of 25% of the gross annual premium. What gross annual premium would the policyowner pay for a $5,000 policy, assuming a charge is also made of $7.50 per policy?

Solution

Before computing the premium which the policyowner pays, it is necessary to find the gross premium *per $1,000* of insurance. The policy owner's gross premium is this gross premium per $1,000 multiplied by the number of thousands of insurance, plus any policy charge which is added by the company.

The calculation is first made to find the gross annual premium per $1,000:

Basic equation

Gross = Net + Percent of Gross

Substituting $12.49 for net, .25 for percent

Gross = $12.49 + (.25)(Gross)

Subtracting (.25)(Gross) from each side

$$\text{Gross} = (.25)(\text{Gross}) = \$12.49$$

$$\text{Gross } (1- .25) = \$12.49$$

$$\text{Gross } (.75) = \$12.49$$

$$\text{Gross} = \$16.65$$

For the $5,000 policy, the gross annual premium would be 5 times $16.65, plus the $7.50 charge:

$$\begin{pmatrix} \text{Gross Premium} \\ \text{For \$5,000} \end{pmatrix} = (5)(\$16.65) + \$7.50$$

$$= \$90.75$$

A common method of providing for those expenses which vary with the amount of insurance (category 3) is to use a constant amount per $1,000 of insurance. The expenses referred to in category 2, which vary with the amount of the premium, are then provided for by a percentage of gross annual premium. In equation form, this total would be

$$\text{Gross} = \text{Net} + \text{Constant} + \text{Percent of Gross}$$

To Illustrate- Calculate the gross annual premium per $1,000 for a policy for which the net annual premium per $1,000 is $31.28, if it is to be loaded $3 per $1,000 plus 20% of the gross annual premium. What would the gross annual premium be for a $15,000 policy, assuming a charge is also made of $10 per policy?

Solution

The calculation is first made to find the gross annual premium per $1,000:

Basic equation

$$\text{Gross} = \text{Net} + \text{Constant} + \text{Percent of Gross}$$

Substituting \$31.28 for net, \$3 for constant, and .20 for percent

$$\text{Gross} = \$31.28 + \$3.00 + (.20)(\text{Gross})$$

$$\text{Gross} - (.20)(\text{Cross}) = \$31.28 + \$3.00$$

$$\text{Gross}\ (1- .20) = \$31.28 + \$3.00$$

$$\text{Gross}\ (.80) = \$34.28$$

$$\text{Gross} = \$42.85$$

For the \$15,000 policy, the gross annual premium paid by the policy-owner would be 15 times \$42.85, plus the \$10 charge:

$$\begin{pmatrix}\text{Gross Premium}\\ \text{For } \$15{,}000\end{pmatrix} = (15)(\$42.85) + \$1.00$$

$$= \$652.75$$

In this case, the total \$652.75 gross annual premium which the policy-owner pays is made up of the net annual premium plus loading, as follows:

$$\begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix} = (15)(\$31.28) \qquad = \$469.20$$

$$\begin{pmatrix}\text{Expenses Constant}\\ \text{per Policy}\end{pmatrix} = \$10.00 \qquad = 10.00$$

$$\begin{pmatrix}\text{Expenses Varying}\\ \text{with Premium}\end{pmatrix} = (15)(\ 20)(\$42\ 85) \qquad = 128.55$$

$$\begin{pmatrix}\text{Expenses Varying}\\ \text{with Amount}\\ \text{of Insurance}\end{pmatrix} = (15)(\$3.00) \qquad = 45.00$$

\$652.75 Total

Since each insurance company determines its own method for calculating the loading, not all insurance companies use loading formulas which add loadings of each of the types described above. Companies sometimes use loading methods involving only one or two of the types of additions described, rather than all three, or they may use very complex methods of loading which are not given in this book.

FRACTIONAL PREMIUMS

Instead of paying for life insurance by annual premiums, many people prefer to pay premiums in installments during the year, either semiannually, quarterly, or monthly. These installments are known as *fractional premiums*. When premiums are paid in this manner, the company cannot invest the premium income so soon and thereby loses some interest. Also, the company incurs additional expenses for postage, clerical work, etc. The loss of interest and the additional expenses properly should be borne by the policyowners who pay such fractional premiums. Therefore, the semiannual premium charged will be more than one half of the annual premium, and the quarterly and monthly premiums will likewise be more than one fourth or one twelfth of the annual premium, respectively. The amount of the additional charges vary by company.

One practice that may be followed to obtain the gross *semiannual* premium is to increase the total gross annual premium (including any policy charge) by some percentage of itself, and divide the result by 2.

To Illustrate- If the gross annual premium for a certain policy is $34.89, calculate the semiannual premium. Assume that it is $\frac{1}{2}$ of the gross annual premium increased by $2\frac{1}{2}$%.

Solution

The gross annual premium increased by $2\frac{1}{2}$% is

$$(\$34.89)(1.025) = \$35.76$$

The semiannual premium is $\frac{1}{2}$ of this:

$$\text{Semiannual Premium} = \frac{1}{2}(\$35.76)$$

$$= \$17.88$$

If a great many semiannual premiums are to be calculated, it would be desirable to shorten the calculation by first dividing 1.025 by 2. This equals .5125. Each gross annual premium can be multiplied by this factor. By this procedure, only one multiplication is required to calculate each semiannual premium. The solution for the above illustration would then be written:

$$\text{Semiannual Premium} = (\$34.89)(.5125)$$
$$= \$17.88$$

This is the same answer as above.

To determine the *quarterly* premium, the gross annual premium may be increased by some larger percentage, such as 5%, and the result divided by 4.

To Illustrate- If the gross annual premium for a certain policy is $34.89, calculate the quarterly premium. Assume that it is $\frac{1}{4}$ of the gross annual premium increased by 5%.

Solution

The gross annual premium increased by 5% is

$$(\$34.89)(1.05) = \$36.63$$

The quarterly premium is I of this:

$$\text{Quarterly Premium} = \frac{1}{4}(\$36.63)$$
$$= \$9.16$$

Or, if 1.05 is divided by 4, the factor .2625 is obtained. The gross annual premium can be multiplied by this factor:

$$\text{Quarterly Premium} = (\$34.89)(.2625)$$
$$= \$9.16$$

The answer is the same by both methods.

To determine the *monthly* premium, the gross annual premium may be increased by a still greater percentage, such as 8%, and the result divided by 12.

To Illustrate- If the gross annual premium for a certain policy is $34.89, calculate the monthly premium. Assume that it is $\frac{1}{12}$ of the gross annual premium increased by 8%.

Solution

The gross annual premium increased by 8% is

$$(\$34.89)(1.08) = \$37.68$$

The monthly premium is $\frac{1}{12}$ of this:

$$\text{Monthly Premium} = \frac{1}{12}(\$37.68)$$

$$= \$3.14$$

Or, if 1.08 is divided by 12, the factor .09 is obtained. The gross annual premium can be multiplied by this factor:

$$\text{Monthly Premium} = (\$34.89)(.09)$$

$$= \$3.14$$

The answer is the same by both Methods.

EXERCISES

1- The net annual premium for a certain policy is $12.05 per $1,000. Calculate the gross annual premium per $i,000 by loading the net premium 20% of the gross premium.

2- Using the answer to Exercise 1, calculate the gross annual premium for a $10,000 policy, assuming an $8 polic3' charge is added to the premium.

3- Calculate the gross annual premium per $1,000 for a certain policy, given the following:

Net annual premium per $1,000 = $21.05
Loading (Except policy charge) = $2.50 per $1000, plus 10% of gross premium

4- Using the answer to Exercise 3, calculate the gross annual premium for a $15,000 policy, assuming a charge of $6 per policy is added.

5- Calculate the gross annual premium for a $10,000 policy for which the net annual premium per $1,000, $49.20, is to be loaded $5 per $1,000 plus 15% of the gross premium, and finally $10 per policy is added.

6- A certain company does not make an additional charge per policy, but instead charges a different gross annual premium per $1,000 for different-sized policies, as follows:

20-Payment Life; Age 30

Amount of Policy	*Gross Premium per $1,000*
Less than $10,000	$33.28
$10,000 to $49,999	32.02
$50,000 and over	31.86

What would be the gross annual premium for a $25,000 20-payment life policy issued at age 30?

7- If the gross annual premium for a certain policy is $314.95, calculate the monthly premium. Assume that it is $\frac{1}{12}$, of the gross annual premium increased by 5%.

8- If the gross annual premium for a certain policy is $14.10 per $1,000, plus a charge of $5 per policy, calculate the semiannual premium for a $10,000 policy. Assume that it is $\frac{1}{2}$ of the gross annual premium increased by 4%.

SYMBOL FOR NET ANNUAL PREMIUMS

Chart-3 displays certain internationally used symbols, each of which represents the net annual premium for $1 of life insurance issued at age *x*. (These net annual premiums are also shown as they would be calculated using commutation functions.)

The capital letter *"P"* is used with a subscript for the issue age, and the number of years under an "angle." In this respect, the subscripts are identical to those shown in Section 9,9 which appear with *"A"* for the various types of net single premiums. However, there is an additional subscript shown at the lower *left* of the *"F"* whenever the premium-paying period is shorter than the benefit period. For example, the symbol $_m p_{x:n}$ represents the net annual premium for a $1 *m*-payment *n*-year endowment policy issued at age *x*.

CHART-3

Net Annual Premium for $1 of Life Insurance Issued at Age *x*

Type of Life Insurance	*Symbol for Net Annual Premium*	*Net Annual Premium Using Commutation Functions*
Ordinary Life	P_x	$\frac{M_x}{N_x}$
m-Payment Life	$_mP_x$	$\frac{M_x}{N_x - N_{x+m}}$
n-Year Term Insurance	$P^1_{x:\overline{n}\rceil}$	$\frac{M_x - M_{x+n}}{N_x - N_{x+n}}$
m-Payment n-Year Term Insurance	$_mP^1_{x:\overline{n}\rceil}$	$\frac{M_x - M_{x+n}}{N_x - N_{x+m}}$
n-Year Endowment Insurance	$P_{x:\overline{n}\rceil}$	$\frac{M_x - M_{x+n} + D_{x+n}}{N_x - N_{x+n}}$
m-Payment *n*-Year Endowment Insurance	$_mP_{x:\overline{n}\rceil}$	$\frac{M_x - M_{x+n} + D_{x+n}}{N_x - N_{x+m}}$

CHAPTER 12

NET LEVEL PREMIUM RESERVES

TERMENAL RESERVES

It was pointed out in Section 9,1 that the net premium for one-year term insurance increases each year with the age of the insured. In Chapter 10 a method of calculating premiums under which the net premium remains uniform throughout the premium-paying period was described. Under the method, known as the net *level premium method,* the person is paying net premiums in the early policy years which are higher than required to pay death claims in those years. The excess accumulates from year to year to produce a fund known as the *net level premium reserve.* Because this fund will be needed to help pay benefits in later years, the total of the reserves for all the individual policies in force at any time represents the principal liability of a life insurance company. Since the insurance company has invested the premium income, it has accumulated assets which have a value at least large enough to provide for this liability.

As the name implies, such a reserve fund is calculated by using *net* level premiums, not gross premiums. The same mortality table and interest rate on which net premiums are based are used to compute the reserve. (Whenever the word “reserve” is used in this chapter, it will mean “net level premium reserve.” Reserves calculated by other than net level premiums will be described in Chapter 12.)

In Chapter 10, two schedules were shown which illustrated the year-by-year accumulation of such a fund (Sections 10.2 and 10.4).

The fund shown at the end of each year could be divided by the number of persons living at the end of that year to determine a reserve *per person* (per $1,000 of insurance). In this way, a reserve is actually determined each year for an individual policy.

Such reserves are calculated as of the end of each policy year, after payment of the year’s death claims. They are, therefore, known as *terminal reserves.*

As an example, consider a five-payment 10-year endowment policy issued to a person age 21. The net level annual premium per $1,000 is $158.752, rounding off to three decimal places for this example, instead of the customary two. Using Table III and 3% interest, and assuming tat each person in the Table has $1,000 of such insurance, the schedule which follows may be constructed. The premium income for each of the first five years is calculated by multiplying $158752 by the number living. The death claims each year are calculated by multiplying $1,000 by the number dying. It can then be seen how the total fund accumulates year by year. Column (8) of the schedule is the reserve per person (per $1,000 of insurance). It is calculated each year by dividing the total fund at the *end* of the year by the number of persons living at the *end* of the year (by the number living at the *beginning* of the *following* year). See Chart 13-1.

An inspection of the figures in column (8) shows that the reserve per person grows year by year until it reaches exactly $1,000 on the date when the $1,000 pure endowment is payable. It is also interesting to note that each person pays a premium the first year of $158.752, and that the reserve at the end of that year is $161.98. Since 3% interest is being credited, the $ 158.752 premium would have accumulated to

$158.752(1.03) = $163.51

using interest only. The difference represents the contribution which each person makes to pay that year's death claims:

$163.51 - $161.98 = $1.53

To Illustrate- Using the 1958 C.S.O. table and 3% interest, calculate the terminal reserve per $1,000 for a 4-year term insurance policy issued at age 25.

Solution

This is the same policy which was considered in Section 10.2. In that section, the year-by-year fund was calculated, and the schedule is repeated in Chart 11-2, which is shown on page 242. This schedule assumes that each person has $1,000 of insurance. To derive the terminal reserve per $1,000, the balance in the fund at the end of each year is divided by the number living at the end of that year (by the number living at the beginning of the following year):

Chart-1

Age 21-5pay 10-Year Endowment (1958C.S.O. 3%)

(1) Age x	(2) l_x	(3) *Premium Income* (*Premium* × l_x)	(4) *Total Fund Beginning Of Year* (*Col*.3+*Col*.7 *Previous Yr*.)	(5) *Fund Accumulated For 1 year* (*Col*.4 × 1.03)	(6) *Death Claims* (\$1,000 d_x)	(7) *Total Fund at End of year* (*Col*.5 –*Col*.6)	(8) *Reseve per Person* (*Col*.7 ÷ *Col*.2 *Following Yr*.)
21	9,647,694	$1,531,590,718	$1,531,590,718	$1,577,538,440	$17,655,000	$1,559,883,440	$161.98
22	9,630,039	1,528,787,951	3,088,671,391	3,181,331,533	17,912,000	3,163,419,533	329.11
23	9,612,127	1,525,944,386	4,689,363,919	4,830,044,837	18,167,000	4,811,877,837	501.55
24	9,593,690	1,523,060,338	6,334,938,175	6,524,986,320	18,324,000	6,506,662,320	697.50
25	9,575,636	1,520,151,366	8,026,813,686	8,267,618,097	18,481,000	8,249,137,097	863.14
26	9,557,155	0	8,249,137,097	8,496,611,210	18,732,000	8,477,879,210	888.81
27	9,538,423	0	8,477,879,210	8,732,215,586	18,981,000	8,713,234,586	915.31
28	9,519,442	0	8,713,234,586	8,974,631,624	19,324,000	8,955,307,624	942.65
29	9,500,118	0	8,955,307,624	9,223,966,853	19,760,000	9,204,206,853	970.87
30	9,480,358	0	9,204,853	9,480,333,059	20,193,00	9,640,140,059	1000.00
31	9,460,165	---	---	---	---	---	---

CHART-2

Age 26-4-Year Term Insurance (1968 C.S.O. 3%)

(1) Years	*(2)* Premiums Paid at Beginning of Year	*(3)* Total Fund at Beginning of Year (Col. 6 Previous Year + Col. 2)	*(4)* Fund Accumulated for One Year (Col. 3 × 1.03)	*(5)* Claims Paid at End of Year (Number of Deaths × $1,000)	*(6)* Balance in Fund at End of Year After Payment of Claims (Col.4 - Col. 5)
1	$18,385,221.12	$18,385,221.12	$18,936,777.75	$18,481,000	$445,777.75
2	18,349,737.60	18,805,515.35	19,369,680.81	18,732,000	637,68081
3	18,313,772.16	18,951,452.97	19,519,996.56	18,981,000	538,996.56
4	18,277,328.64	18,816,325.20	19,380,814.96	19,324,000	56,814.96

(Note: The final balance in the fund would be even closer to zero if the net annual premium were rounded off to more decimal places.)

End of each year is divided by the number living at the end of the years (by the number living at the beginning of the following years):

At the End of First Year

Balance in Fund = $455,777.75

Age at That Time = 26

Number Living at That Time = l_{26}

= 9,557,155

$$\text{Terminal Reserve per \$1,000} = \frac{\text{Balance in Fund}}{\text{Number Living}}$$

$$= \frac{\$455,777.75}{9,557,155}$$

= $ .05

At the End of Second Year

Balance in Fund = $637,680.81

Age at That Time = 27

Number Living at That Time = l_{27}

= 9,538,423

$$\text{Terminal Reserve per \$1,000} = \frac{\text{Balance in Fund}}{\text{Number Living}}$$

$$= \frac{\$637,680.81}{9,538,423}$$

$$= \$.07$$

At the End of Third Year

Balance in Fund = \$538,996.56

Age at That Time = 28

Number Living at That Time = l_{28}

= 9,519,442

$$\text{Terminal Reserve per \$1,000} = \frac{\text{Balance in Fund}}{\text{Number Living}}$$

$$= \frac{\$538,996.56}{9,519,442}$$

$$= \$.06$$

At the End of Fourth Year

Balance in Fund = \$56,814.96

Age at That Time = 29

Number Living at That Time = l_{29}

= 9,500,118

$$\text{Terminal Reserve per \$1,000} = \frac{\text{Balance in Fund}}{\text{Number Living}}$$

$$= \frac{\$56,814.96}{9,500,118}$$

= \$.01, but would be zero if net annual premium were rounded off to more decimal places

The terminal reserves on this term policy are much smaller than those calculated above on the endowment policy. The final reserve on this term policy is zero. This is logical because, at the end of the term, all of the net premium income should have been used to pay death claims. The company owes nothing further in benefits.

It is interesting to consider the accumulation of such a fund on a policy where the insurance extends for the whole of life. All mortality tables are constructed to show the number dying in the final *year* equal to the number living at the beginning of that year, leaving none living at the end. The accumulated fund will normally provide, therefore, exactly enough money in that final year to pay all these death claims, leaving no balance in the fund. From this viewpoint, a whole life insurance policy can be considered as a "term to age 100" policy, when based on the 1958 C.S.O. Table, which ends at age 100. If, as sometimes happens, an insured person actually lives to age 100, It is customary for the insurance company to pay the amount of insurance at that time.

The method explained above for calculating terminal reserves requires that a year-by-year accumulation be performed. To find the terminal reserve for any year, one must know the accumulations for all the preceding years. However, by another method, reserves may be calculated for any year desired without such a requirement. This second method uses the principle that the terminal reserve, at any specified time, is equal to the accumulated value of all the net premiums which have been received, less the accumulated cost of the insurance which has been provided.

This method is known as the *retrospective method*, because it involves the use of past happenings ("looking backwards"). In equation form, it may be written:

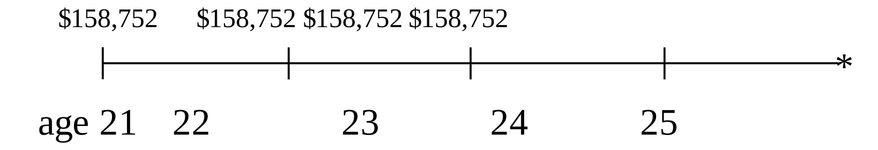
$$\begin{pmatrix}\text{Terminal}\\ \text{Reserve}\end{pmatrix} = \begin{pmatrix}\text{Accumulated Value}\\ \text{of Net Premiums}\\ \text{Received}\end{pmatrix} - \begin{pmatrix}\text{Accumulated}\\ \text{Cost of}\\ \text{Insurance}\end{pmatrix}$$

The two items on the right side of the equation make direct use of principles previously presented. The "accumulated value of net premiums received" represents the accumulated value of a temporary life annuity due (since premium payments are made each year only if the person insured is then alive), as discussed in Section 8.3. The "accumulated cost of insurance" was presented in Section 9.6.

To Illustrate- Using the 1958 C.S.O. Table and 3% interest, calculate the 4th terminal reserve for a $1,000 5-payment 10-year endowment policy issued at age 21, with a net annual premium of $158752.

Solution

This is the same policy for which terminal reserves were calculated in Section 11.1 by a year-by-year accumulation. The 4th terminal reserve (at attained age 25) will now be calculated by the retrospective method. The accumulated value of the net premiums received will be considered first. The line diagram of the net premiums *received during these first 4 years* appears as follows:



The expression for their accumulated value will have a *numerator* representing the total amount paid in by the survivors at each age, with each such amount being accumulated at interest to the evaluation date. The *denominator* is the number living on the evaluation date:

Basic equation

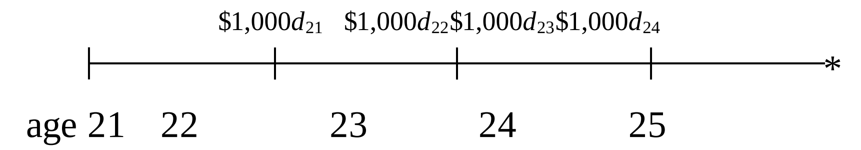
$$\begin{pmatrix}\text{Accumulated Value}\\ \text{of Net Premiums}\\ \text{Received}\end{pmatrix} = \$158.752\left[\frac{l_{21}(1+i)^4 + l_{22}(1+i)^3 + l_{23}(1+i)^2 + l_{24}(1+i)}{l_{25}}\right]$$

$$= \$158.752\left(\frac{\begin{array}{l}9,647,694(1.125509)\\+9,630,039(1.092727)\\+9,612,127(1.060900)\\+9,593,960(1.030000)\end{array}}{9,575,636}\right)$$

$$= \$158.752\left[\frac{10,858,566+10,523,004+10,197,506+9,881,779}{9,575,636}\right]$$

$$= \$687.37$$

The accumulated cost of insurance will be considered next. The line diagram of the life insurance benefits *paid during these first 4 years* appears as follows:



age 21 22 23 24 25

The expression for the accumulated cost will have a *numerator* representing the total to be paid out for those who die in each of the years, with each such amount being accumulated at interest from the end of the year of death to the evaluation date. The *denominator* is the number living on the evaluation date:

Basic equation

$$\begin{pmatrix}\text{Accumulated Value}\\\text{of Net Premiums}\\\text{Received}\end{pmatrix} = \$1,000\left[\frac{d_{21}(1+i)^3 + d_{22}(1+i)^2 + d_{23}(1+i) + d_{24}}{l_{25}}\right]$$

Substituting values from the tables

$$= \$1{,}000\left(\frac{\begin{array}{l}17{,}655(1.092727)\\+17{,}912(1.060900)\\+18{,}167(1.030000)\\+18{,}324\end{array}}{9{,}575{,}636}\right)$$

$$= \$1{,}000\left[\frac{19{,}292+19{,}003+18{,}712+18{,}324}{9{,}575{,}636}\right]$$

= \$7.87

Hence, the 4th terminal reserve, calculated by the retrospective method, is

Basic equation

$$\begin{pmatrix}\text{Terminal}\\\text{Reserve}\end{pmatrix} = \begin{pmatrix}\text{Accumulated Value}\\\text{of Net Premiums}\\\text{Received}\end{pmatrix} - \begin{pmatrix}\text{Accumulated}\\\text{Cost of}\\\text{Insurance}\end{pmatrix}$$

Substituting values calculated above

= \$687.37- \$7.87

= \$679.50

This answer agrees with the figure shown in column (8) for the 4th terminal reserve.

THE PROSPECTIVE METHOD

Terminal reserves can also be calculated by looking into the future of a life insurance policy. This method of calculating reserves is known as the *prospective method* (“looking ahead”).

At any particular time during the life of a policy, the company may look ahead at the benefits it will have to pay on that policy in the

future. The money necessary to pay those future benefits will come from two sources: the *reserve* currently being held, and the *future net premiums*. The following equation may be written to express this fact:

$$\begin{pmatrix}\text{Present Value}\\ \text{of Future Benfits}\end{pmatrix}=\begin{pmatrix}\text{Terminal}\\ \text{Reserve}\end{pmatrix}+\begin{pmatrix}\text{Present Value of}\\ \text{Future Net Premium}\end{pmatrix}$$

Solving for the terminal reserve results in the following equation which expresses the prospective method:

$$\begin{pmatrix}\text{Terminal}\\ \text{Reserve}\end{pmatrix}=\begin{pmatrix}\text{Present Value}\\ \text{of Future Benfits}\end{pmatrix}-\begin{pmatrix}\text{Present Value of}\\ \text{Future Net Premium}\end{pmatrix}$$

This is equivalent to the equation:

$$\begin{pmatrix}\text{Terminal}\\ \text{Reserve}\end{pmatrix}=\begin{pmatrix}\text{Net Single Premium}\\ \text{at Attained Age}\end{pmatrix}-\begin{pmatrix}\text{Present Value of}\\ \text{Future Net Annual}\\ \text{Premium at}\\ \text{Attained Age}\end{pmatrix}$$

The reserves calculated by the prospective method are equal to those calculated by the retrospective method.

It should be stressed that the two items on the right side of the equation for the terminal reserve are calculated at the *attained age,* and take into account only those benefits and premiums, respectively, which will be in effect *after* the date of the particular reserve which is being calculated. Both make direct use of principles previously presented. The "present value of future benefits" is the *net single premium,* presented in Chapter.9. In this case, it is the net single premium at the *attained age* for those benefits then remaining. The "present value of future net premiums" represents the present value of a life annuity due. The present value is calculated at the *attained age,* and includes only those premiums still to be paid. These principles were presented in Chapter 8.

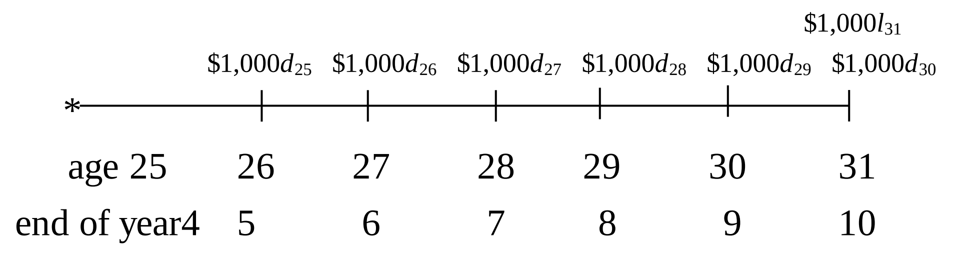
For example, if the third terminal reserve were being calculated for a 20-payment 30-year endowment insurance policy issued at age 25,

the attained age would be 25 + 3 = 28. The future benefits at age 28 would be the same as for a 27-year endowment insurance policy at that age. The future net premiums at age 28 would be 17 in number (three premiums having already been collected).

To Illustrate- Using the 1958 C.S.O. Table and 3% interest calculate the 4th terminal reserve for a $1,000 5-payment 10-year endowment policy issued at age 21, with a net annual premium of $158752.

Solution

This is the same policy for which the terminal reserves were calculated in Section 11.1 by a year-by-year accumulation, and in Section 11.2 by the retrospective method. This reserve will now be derived by the prospective method. If the 4th terminal reserve is to be calculated, then the *attained age* is 21 +4 = 25. The present value of future benefits will be considered first. This present value will be the net single premium at the attained age of 25 for a 6-year endowment policy. In line diagram form, these future benefits (those coming after the end of the 4th year) appear as follows:



The expression for the present value of these future benefits (the net single premium at the attained age) will have a *numerator* representing the total paid out for those who die each year, with each such amount being discounted at interest from the end of the year of death to the evaluation date, plus the total pure endowments paid to the survivors at the end of the period, discounted at interest to the evaluation date. The *denominator* is the number living on the evaluation date;

Basic equation

$$\begin{pmatrix}\text{Present Value}\\ \text{of Future Benfit}\end{pmatrix} \quad or \quad \begin{pmatrix}\text{Net Single Premium}\\ \text{at Attained Age 25}\end{pmatrix}$$

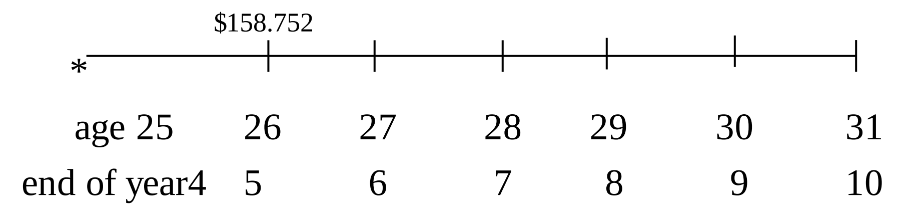
$$= \$1{,}000\left[\frac{d_{25}\upsilon + d_{26}\upsilon^2 + d_{27}\upsilon^3 + d_{28}\upsilon^4 + d_{29}\upsilon^5 + d_{30}\upsilon^6 + d_{31}\upsilon^6}{l_{25}}\right]$$

Substituting values from the tables

$$= \$1000\left(\frac{\begin{array}{l}18{,}481(.970874) + 18{,}732(.942596)\\ +18{,}981(.915142) + 19{,}324(.888487)\\ +19{,}760(.862609) + 20{,}193(.837484)\\ +9{,}460{,}165(.837484)\end{array}}{9{,}575{,}636}\right)$$

$$= \$838.256$$

The present value of future net annual premiums at the attained age will be considered next. The net annual premium was given as $158752. The present value of future net annual premiums will be the present value at age 25 of a life annuity due of $158752 per year for *one year*. (Since the policy requires only five premium payments, there is only one remaining at the end of four years.) In line diagrard form, the future net annual premiums (those coming after the end of the 4th year) appear as follows:



The expression for their present value will have *a numerator* representing the total amount paid in by the survivors at each age, with each such amount being discounted at interest to the evaluation date. The *denominator* is the number living on the evaluation date:

$$\begin{pmatrix}\text{Present Value of}\\ \text{Future Net Premiums}\end{pmatrix} = \$158.752\left(\frac{l_{25}}{l_{25}}\right)$$

In this instance, there is only one age to consider in the numerator. The item in the numerator is not multiplied by any t factor because it represents those net annual premiums which are payable upon the evaluation date. These particular premiums payable on the evaluation date are not discounted at interest for any period of time. Since the entire fraction in the parentheses equals 1, it can be dropped from this calculation.

$$\begin{pmatrix}\text{Present Value of}\\ \text{Future Net Premiums}\end{pmatrix} = \$158.752$$

Hence, the 4th terminal reserve, as calculated by the prospective method, is

Basic equation

$$\begin{pmatrix}\text{Terminal}\\ \text{Reserve}\end{pmatrix} = \begin{pmatrix}\text{Present Value of}\\ \text{Future Benefits}\end{pmatrix} - \begin{pmatrix}\text{Present Value of}\\ \text{Future Net Premiums}\end{pmatrix}$$

$$= \begin{pmatrix}\text{Net Single Premium}\\ \text{at Attained Age}\end{pmatrix} - \begin{pmatrix}\text{Present Value of}\\ \text{Future Net Annual}\\ \text{Premiums}\end{pmatrix}$$

Substituting values calculated above

= \$838256 - \$158,752

= \$679.50

This answer agrees with both the figure shown in column (8) of the schedule in Section 11.1 and the figure calculated in Section 11.2 by use of the retrospective method.

It is interesting to consider the equation for the terminal reserve ~using the prospective method) at the end of *zero years,* that is, at the time the policy is issued. The usual equation, as given before, is

$$\begin{pmatrix}\text{Terminal}\\ \text{Reserve}\end{pmatrix} = \begin{pmatrix}\text{Present Value of}\\ \text{Future Benefits}\end{pmatrix} - \begin{pmatrix}\text{Present Value of}\\ \text{Future Net Premiums}\end{pmatrix}$$

However, at *the time the policy is issued* the two items on the right side are equal to each other. This is because the basis for the calculation of net annual premiums is the principle that, *at the time the*

$$\begin{pmatrix}\text{Present Value of}\\ \text{Future Benefits}\end{pmatrix} = \begin{pmatrix}\text{Present Value of}\\ \text{Future Net Premiums}\end{pmatrix}$$

This equality is only true at the time the policy is issued, and hence at that time the value of the terminal reserve is zero.

Having two different methods for calculating terminal reserves (retrospective and prospective) enables one to choose the method which is simpler for the case at hand. Where the future benefits are complicated, it is often simpler to use the *retrospective* method (provided the net premium is known). On the other hand, if the reserve is desired at a date when there are no more premiums due (such as the 25th reserve on a 20-payment life policy), the *prospective* method is usually simpler because the "present value of future net premiums" is then zero. Other examples of situations for which the prospective method is preferable include policies where benefits in the early policy years are complicated, or where net annual premiums in the early policy years are not uniform year by year.

If published tables of net single premiums and life annuity factors are available, as is generally the case in practice, the terminal reserve can be calculated by the prospective method with less work.

To Illustrate- Calculate the 5th terminal reserve per $1,000 on an endowment-at-age-65 policy issued at age 20, given the following table:

Age	*Net Single Premium for $1,000 Endowment at Age 65*	*Present Value of a Temporary Life Annuity Due of 1 per Year to Ago §5*
20	$30475129	23.870206
21	312.66351	23.598552
22	320.80051	23.319182
23	329.17678	23.031596
24	337.80052	22.735515
25	346.68674	22.430421
26	355.84412	22.116018
27	365.27538	21.792211
28	374.98991	21.458679
29	384.99117	21,115303

(The values given in this table show several decimal places, in order to provide considerable accuracy in the answer.)

Solution-The first step is to calculate the net annual premium. The present value of the net premiums is a temporary life annuity due to age 65.

Basic equation

$$\begin{pmatrix}\text{Present Value of}\\ \text{Future Benefits}\\ \text{at Time of Issue}\end{pmatrix} = \begin{pmatrix}\text{Present Value of}\\ \text{Future Net Premiums}\\ \text{at Time of Issue}\end{pmatrix}$$

Expressing this equation in equivalent form

$$\begin{pmatrix}\text{Net Single}\\ \text{Premium}\end{pmatrix} = \begin{pmatrix}\text{Net annual}\\ \text{Premiums}\end{pmatrix}(\text{AnnuityFactor})$$

Substituting values given

$$\$304.75129 = \begin{pmatrix} \text{Net annual} \\ \text{Premiums} \end{pmatrix} (23.870206)$$

$$\frac{\$304.75129}{23.870206} = \begin{pmatrix} \text{Net annual} \\ \text{Premiums} \end{pmatrix}$$

$$\$12.76702 = \begin{pmatrix} \text{Net annual} \\ \text{Premiums} \end{pmatrix}$$

The second step is to determine the attained age. If the 5th terminal reserve is to be calculated, the attained age is 20+5= 25.

In the equation for finding the terminal reserve, the "present value of future benefits" will then be the net single premium at attained age 25 for endowment-at-age-65 insurance. The "present value of future net premiums" will he the present value at attained age 25 of a temporary life annuity due of the remaining net annual premiums to age 65.

Basic equation

$$\begin{pmatrix} \text{Terminal} \\ \text{Reserve} \end{pmatrix} = \begin{pmatrix} \text{Present Value of} \\ \text{Future Benefits} \end{pmatrix} - \begin{pmatrix} \text{Present Value of} \\ \text{Future Net Premiums} \end{pmatrix}$$

Expressing this equation in equivalent form

$$= \begin{pmatrix} \text{Net Single Premium} \\ \text{at Attained Age 25} \end{pmatrix} - \begin{pmatrix} \text{Net Annual} \\ \text{Premium} \end{pmatrix} \begin{pmatrix} \text{Annuity Factor} \\ \text{at Attained Age 25} \end{pmatrix}$$

Substituting values given

= \$346.68674 - \$12.76702(22.430421)

= \$34668674 - \$28636963 \$60.32

= \$60.32

EXERCISES

1- Using the following information, calculate the 5th terminal reserve, using both the retrospective and prospective methods. Compare the answers obtained by the two methods:

Present value (at end of 5 years) of future net premiums	= $412.77
Accumulated cost of insurance (at end of 5 years)	= 181.33
Present value (at end of 5 years) of future benefits	= 816.04
Accumulated value (at end of 5 years) of net premiums received	= 584.60

2- The net annual premium on a certain policy issued at age 19 is $19.92. Calculate the 7th terminal reserve, using the following Information:

Attained Age	*Present Value of Future Benefits*	*Present Value of Future Premiums (per $1)*
19	$406.11	$20.39
20	417.36	20.00
21	428.86	19.61
22	440.70	19.20
23	452.90	18.78
24	465.48	18.35
25	478.45	17.91
26	491.82	17.45
27	505.61	16.97
28	519.82	16.49
29	534.4’t	15.98

3- Using the following table, calculate the 38th terminal reserve on a $1,000 ordinary life policy issued at age 10:

Age	*Net Single Premium for $1,000 Whole Life Insurance*	*Present Value of Whole Life Annuity Due of 1 per year*
10	$351.30	33.083
11	357.06	32.790
12	362.99	32 .488
.	.	.
.	.	.
.	.	.
47	634.69	18.631
48	643.86	18.163
49	653.04	17.695

4- Write expressions for the 2nd year terminal reserve for each of the following $1,000 policies, using both the retrospective and prospective methods. Calculate the value of each, using Table III and 3% interest. (Hint: Line diagrams should be helpful.)

a) A 5-year term insurance policy issued at age 60 (Net annual premium = $23563)

b) An ordinary life policy issued at age 96 (Net annual premium = $479992)

c) A 2-payment life policy issued at age 96 (Net annual premium = $595950)

d) A 4-year endowment insurance policy issued at age 30 (Net annual premium = $232896)

e) A 4-payment 5-year endowment insurance policy issued at age 25 (Net annual premium = $226086)

COMUTATION FUNCTIONS

The commutation functions explained in Chapters 8, 9, and 10 can be used to calculate terminal reserves. This device can be used for either the retrospective or prospective method, and will generally simplify the work.

As a first step in the calculation, regardless of which method is being used, commutation functions can be used to calculate the *net annual premium.* The procedure was described in Section 10.6.

In the equation for the *retrospective method,* namely,

$$\begin{pmatrix}\text{Terminal}\\ \text{Reserve}\end{pmatrix} = \begin{pmatrix}\text{Accumulated Value}\\ \text{of Net Premiums}\\ \text{Received}\end{pmatrix} - \begin{pmatrix}\text{Accumulated}\\ \text{Cost of}\\ \text{Insurance}\end{pmatrix}$$

the items on the right side make use of principles previously presented. The "accumulated value of net premiums received" represents the accumulated value of a temporary life annuity due. Such a calculation, using commutation functions, was described in Section 8.7. The use of commutation functions to calculate "accumulated cost of insurance" was described it, Section 9.8.

To Illustrate- Using Table IV, calculate the 4th terminal reserve for a $1,000 5-payment 10-year endowment policy issued at age 21. The net annual premium is $158.752.

Solution

This is the same problem presented in the illustrations in Sections 13.2 arid 13.3. The solution will now be given using the retrospective method and commutation functions.

The "accumulated value of net premiums received" during the first **4** years may be written:

$$\begin{pmatrix}\text{Accumulated Value}\\ \text{of Net Premiums}\\ \text{Received}\end{pmatrix} = \$158.752\left(\frac{N_{21} - N_{25}}{D_{25}}\right)$$

This follows from the general statement made in Chapter 8 that the factor to use in evaluating a life annuity will be of the form $\frac{N - N}{D}$ where the subscript of the first IV is the age when the first payment is due, the subscript of the second N is the first age when there are no more payments due, and the subscript of the 13 is the age at which the annuity is being evaluated.

Equation given above

$$\begin{pmatrix}\text{Accumulated Value}\\ \text{of Net Premiums}\\ \text{Received}\end{pmatrix} = \$158.752\left(\frac{N_{21} - N_{25}}{D_{25}}\right)$$

Substituting values from Tabe IV

$$= \$158.752\left(\frac{132{,}991{,}534 - 113{,}189{,}600}{4{,}573{,}377}\right)$$

$$= \$158.752\left(\frac{19{,}801{,}934}{4{,}573{,}377}\right)$$

$$= \$687.37$$

The "accumulated cost of insurance" for the first 4 years may be written

$$\begin{pmatrix}\text{Accumulated Cost}\\ \text{of Insurance}\end{pmatrix} = \$1{,}000\left(\frac{M_{21} - M_{25}}{D_{25}}\right)$$

This follows from the general statement made in Chapter 11 that the factor to use in calculating an accumulated cost of insurance will be of

the form $\frac{M - M + D}{D}$ where the subscript of the first M is the age when the insurance coverage begins, the subscript of the second M is the age at which the insurance coverage stops, the subscript of the D in the numerator is the age at which a pure endowment would be paid, and the subscript of the D in the denominator is the age at which this accumulated cost of insurance is evaluated. (In this particular case, the D in the numerator does not appear, because no pure endowment is involved in the first 4 years.)

Equation given above

$$\begin{pmatrix}\text{Accumulated Cost}\\ \text{of Insurance}\end{pmatrix} = \$1{,}000\left(\frac{M_{21} - M_{25}}{D_{25}}\right)$$

Substituting values from Table IV

$$= \$1{,}000\left(\frac{1{,}312{,}569 - 1{,}276{,}590}{4{,}573{,}377}\right)$$

$$= \$1{,}000\left(\frac{35{,}979}{4{,}573{,}377}\right)$$

$$= \$7.87$$

Hence, as calculated by the retrospective method, the 4th terminal reserve is

Basic equation

$$\begin{pmatrix}\text{Terminal}\\ \text{Reserve}\end{pmatrix} = \begin{pmatrix}\text{Accumulated Value}\\ \text{of Net Premiums}\\ \text{Received}\end{pmatrix} - \begin{pmatrix}\text{Accumulated}\\ \text{Cost of}\\ \text{Insurance}\end{pmatrix}$$

Substituting values calculated above

$$= \$68727 - \$7.87$$

$$= \$679.50$$

This answer agrees with that calculated in Sections 11.2 and 11.3. In the equation for the *prospective* method, namely,

$$\begin{pmatrix}\text{Terminal}\\ \text{Reserve}\end{pmatrix}=\begin{pmatrix}\text{Present Value}\\ \text{of Future Benefits}\end{pmatrix}-\begin{pmatrix}\text{Present Value of}\\ \text{Future Net Premiums}\end{pmatrix}$$

The items on the right side make use of principles previously presented. The "present value of future benefits" is the *net single premium* at the attained age for those benefits then remaining. Such a calculation, using commutation functions, was described in Section 9.8. The "present value of future net premiums" represents the present value of a life annuity due. The use of commutation functions to calculate life annuities was described in Section 8.7.

To Illustrate- Using Table IV, calculate the 4th terminal reserve for a $1,000 5-payment 10-year endowment policy issued at age 21. The net annual premium *is* $158752.

Solution

This is the same problem presented in the illustrations in Sections 11.2 and 11.3 and above in this section. The solution will now be given by using the prospective method and commutation functions.

The 'present value of future benefits" (as of the end of the 4th year) may he written

$$\begin{pmatrix}\text{Present Value of}\\ \text{Future Benefits}\end{pmatrix}=\$1{,}000\left(\frac{M_{25}-M_{31}+D_{31}}{D_{25}}\right)$$

or

$$\begin{pmatrix}\text{Net Single Premium}\\ \text{at Attained Age}\end{pmatrix}$$

This follows from the general statement made in Chapter 9 that the factor to use in calculating a net single premium will be of the form $\frac{M - M + D}{D}$ where the subscript of the first M is the age when the insurance coverage begins, the subscript of the second M is the age at which the insurance coverage stops, the subscript of the D in the numerator is the age at which a pure endowment would be pad, and the subscript of the D in the denominator is the age at which this net single premium is evaluated.

Equation given Above

$$\begin{pmatrix}\text{Present Value of}\\ \text{Future Benefits}\end{pmatrix} = \$1{,}000\left(\frac{M_{25} - M_{31} + D_{31}}{D_{25}}\right)$$

or

$$\begin{pmatrix}\text{Net Single Premium}\\ \text{at Attained Age}\end{pmatrix}$$

Substituting values from Table IV

$$= \$1{,}000\left(\frac{1{,}276{,}590 - 1{,}226{,}873 + 3{,}783{,}944}{4{,}573{,}377}\right)$$

$$= \$1{,}000\left(\frac{3{,}833{,}661}{4{,}573{,}377}\right)$$

$$= \$838.256$$

The "present value of future net premiums" (as of the end of the 4th year) may be written:

$$\begin{pmatrix}\text{Present Value of}\\ \text{Future Net Premiums}\end{pmatrix} = \$158.752\left(\frac{N_{25} - N_{26}}{D_{25}}\right)$$

This follows from the general statement in Chapter 8 that the factor to use in evaluating a life annuity will be of the form $\frac{N-N}{D}$, where the subscript of the first N is the age when the first payment is due, the subscript of the second N is the first age when there are no more payments due, and the subscript of the D is the age at which the annuity is being evaluated.

Equation given above

$$\begin{pmatrix}\text{Present Value of}\\ \text{Future Net Premiums}\end{pmatrix} = \$158.752\left(\frac{N_{25}-N_{26}}{D_{25}}\right)$$

Substituting values from Table IV

$$= \$158.752\left(\frac{113{,}189{,}600 - 108{,}616{,}223}{4{,}573{,}377}\right)$$

$$= \$158.752\left(\frac{4{,}573{,}377}{4{,}573{,}377}\right)$$

Hence, as calculated by the prospective method, the 4th terminal reserve is

Basic equation

$$\begin{pmatrix}\text{Terminal}\\ \text{Reserve}\end{pmatrix} = \begin{pmatrix}\text{Present Value}\\ \text{of Future Benefits}\end{pmatrix} - \begin{pmatrix}\text{Present Value of}\\ \text{Future Net Premiums}\end{pmatrix}$$

$= \$838.256 - \$158.752$

$= \$679.50$

This answer agrees with that calculated in the previous illustrations.

EXERCISES

(Use Table IV for all of the following)

1- Write an expression (using commutation functions) for each of the following terminal reserves, using both the retrospective and prospective methods. (If the student wishes to practice, he can calculate the value of each reserve.)

a) Tenth terminal reserve on a $1,000 15-year term insurance policy issued at age 40 (Net annual premium = $6308).

b) Terminal reserve at age 65 on a $1,000 ordinary life policy issued at age 25 (Net annual premium = $11278).

c) Fifth terminal reserve on a $1,000 20-payment life policy issued at age 30 (Net annual premium = $21145).

d) Eighth terminal reserve on a $1,000 20-year endowment insurance policy issued at age 0 (Net annual premium = $37266).

2- Calculate the 10th terminal reserve for a $1,000 term-to-age-65 insurance policy issued at age 35.

3- The net annual premium per $1,000 for a 30-payment life insurance policy issued at age 25 is $14.32. Calculate the 15th terminal reserve per $1,000 using the prospective method.

4- Calculate the 5th terminal reserve for a $1,000 20-payment 35-year endowment policy issued at age 30. What would this reserve be for a $3,000 policy?

The reserve at any particular time for any particular policy is the amount the company must then have on hand for that policy. Some of the special instances of this, which have been described previously in this chapter, are:

1- *For any policy,* the terminal reserve at the end of *zero years* (at the time the policy is issued) equals zero.

2- *For term insurance policies,* the *final* terminal reserve (at the end of the term) equals zero. This is because all of the net premiums have been used to pay death claims. The company owes nothing further in benefits.

3- *For endowment insurance policies*, the *final* terminal reserve (on the date the pure endowment is payable) is equal to the amount of the pure endowment. This provides the exact amount which the company will have to pay on that date.

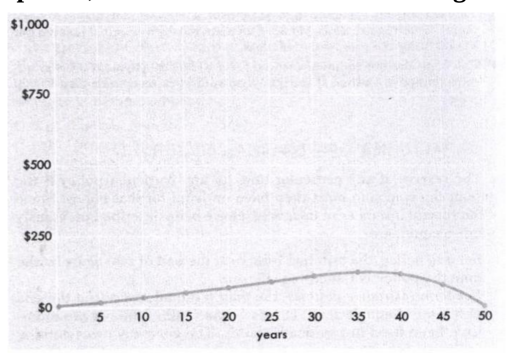
4- *For whole life insurance policies,* the *final* terminal reserve (at the end of the mortality table) equals zero. This is because, according to the modality table, none are then living; hence, the company should owe nothing further in benefits. However, the terminal reserve *one year prior* is nearly equal to the full amount of the death benefit. This helps to provide for payment of the full death benefit to *everybody* that final year (when all are presumed to die).

Typical patterns of terminal reserves, on the common types of policies, can be seen in the graphs that follow. The number of years since the policy was issued is shown along the bottom of the graphs. For each such year, the distance up to the line indicates the size of the reserve at the end of that year.

For a *term insurance policy,* the reserve rises up to a high-point near the middle of the term, and then decreases back to zero. The magnitude of the reserve never gets very large compared to the amount of insurance. (See Figure 13-1.)

FIGURE-1

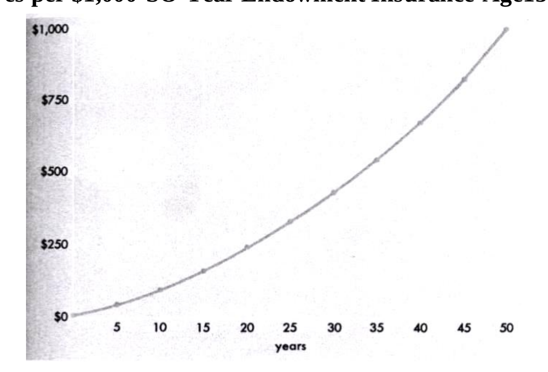
Terminal Reserves per $1,000-50-Year Term insurance-Age 15 (1958 C.S.O. 3%)



For an *endowment insurance policy,* the reserve rises up to equal the amount of the pure endowment on the date the pure endowment is payable. (See Figure-2.)

FIGURE-2

Terminal Reserves per $1,000-SO-Year Endowment Insurance-Age15 (195S C.S.O. 3%)



For a *whole life insurance policy,* the reserve rises up to nearly equal the amount of insurance one year prior to the end of the mortality table. One year later, at the end of the table, the reserve is zero. It was pointed out earlier that it is customary for the company to pay the amount of insurance to persons who actually are still alive at the end of the mortality table. For this reason, some published tables of terminal reserves per $1,000 show the final whole life reserve as being $1,000, instead of zero. (See Figure-3.)

Some life insurance policies have premiums payable for a shorter time than the period of insurance, such as 20-payment life policies. For such policies, the reserve will increase much faster during the time premiums are being paid than in later years. Another example would be

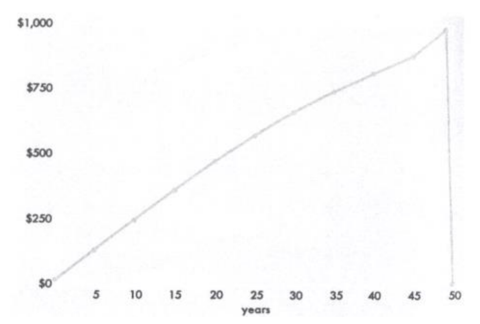
the reserve for the 5-payment 10-year endowment policy used as an illustration in this chapter. The graph in Figure-4 shows this reserve, using the values from column (8) of the schedule in Section 13.1.

INITIAL AND MEAN RESERVES

The premiums for a life insurance policy are paid at the *beginning* of each year. The terminal reserves for a life insurance policy are calculated **as** of the *end* of each year (after that year's death benefits are paid). It is assumed that the end of any year falls upon the same date as the beginning of the following year. For example, for a policy issued on August 17, 1971, the fifth terminal reserve would be the amount of liability the insurance company would have on August 17, 1976. Also, the premium for the sixth year would be due on August 17, 1976.

FIGURE-3

Terminal Reserves per $1,000-Ordinary Life-Age 50 (1958 C.S.O. 3%)



The *initial reserve* for a policy is the amount that the company has at the *beginning* of a given year. It equals the terminal reserve at the end of the previous year, plus the net premium collected at the beginning of the current year. In the example given in the above paragraph, the initial reserve for the sixth year would be the amount on August 17, 1976, after the collection of the net annual premium due on that date.

To Illustrate- Calculate the initial reserves per $1,000 for each year for a 5-payment 10-year endowment policy issued at age 21 (using the 1958 C.S.O. Table and 3% interest).

Solution

This is the same policy for which terminal reserves were calculated in Section 11.1. The net annual premium was given as $158.752, which will be rounded off to $158.75. The terminal reserves

appear in column (8) of the schedule in Section 11.1. Each year's initial reserve equals the terminal reserve at the end of the previous year, plus the net premium for the current year:

1st Initial Reserve = 0 Terminal Reserve + 1st Year Net Premium

= 0 + $158.75

= $158.75

2nd Initial Reserve = 1st Terminal Reserve+2nd Year Net Premium

= $161.98 + $158.75

= $320.73

3rd Initial Reserve = 2nd Terminal Reserve+ 3rd Year Net Premium

= $329.11 I- $158.75

= $487.86

4th Initial Reserve = 3rd Terminal Reserve + 4th Year Net Premium

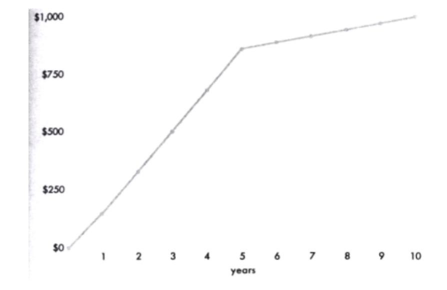
=$501.55 + $158.75

= $660.30

FIGURE-4

Terminal Reserves per $1,000-a-Pay 10-Year Endowment-Age 21

(1958 C.S.O. 3%)



5th Initial Reserve = 4th Terminal Reserve + 5th Year Net Premium

= $679.50 + $158.75

= $838.25

6th Initial Reserve = 5th Terminal Reserve + 6th Year Net Premium

= $863.14 + 0

= $863.14

7th Initial Reserve = 6th Terminal Reserve + 7th Year Net Premium
= $888.81 + 0
= $888.81

8th Initial Reserve = 7th Terminal Reserve + 8th Year Net Premium
= $915.31 + 0
= $915.31

9th Initial Reserve = 8th Terminal Reserve + 9th year Net Premium
= $942.65 + 0
= $942.65

10th Initial Reserve =9th Terminal Reserve+10th Year Net Premium
= $970.87 + 0
= $970.87

The *first* initial reserve always equals the first year net premium, because there is no previous year's terminal reserve to add to it. It should also be observed that in a year in which no premiums are payable (after the 5th year in the above illustration), the initial reserve is equal to the previous year's terminal reserve.

In actual practice, life insurance companies are required by government regulatory authorities to determine the total reserves for all policies each December 31. To make this calculation, it is assumed that all policies are issued in the *middle of the calendar year.* For example, all policies issued in 1970 are assumed to be issued in the middle of 1970 (July 1,1970). This assumption is reasonably accurate, the policies issued before July 1 each year counterbalancing those issued after July 1. Therefore, on December 31, all policies which were issued in the calendar year just ended are assumed to have been in effect for $\frac{1}{2}$ year. All policies issued in the calendar year *prior* to the calendar year just ended are assumed to have been in effect $1\frac{1}{2}$ years; and so forth,

For the December 31 reserve, *mean reserves* are used. The mean reserve for a policy is the amount that the company has on band in the *middle* of a given policy year. It equals one half of the total of the initial and terminal reserves for that policy year:

$$\text{Mean Reserve} = \frac{\text{Initial Reserve} + \text{Terminal Reserve}}{2}$$

The *answer is* generally then rounded off to two decimal places. It should be noted that this calculation will often result in answers having a 5 in the third decimal place, such as $148.725. The rule given in Section 1.4 would require that the 5 be dropped off and that I be added to the digit which is then in last place. That is, $148.725 would be rounded off to $148.73. However, published tables sometimes round off mean reserves to the nearest dollar.

To Illustrate - Calculate the mean reserves per $1,000 for each year for a 5-payment 10-year endowment policy issued at age 21 *(using* the 1958 C.S.O. Table and 3% interest).

Solution

This is the same policy for which terminal reserves were calculated in Section 11.1 and initial reserves were calculated in the illustration above. For convenience, these figures are repeated here:

Policy Year	*Initial Reserve*	*Terminal Reserve*
1	$158.75	S 161.98
2	320.73	329.11
3	487.86	501.55
4	660.30	679.50
5	838.25	863.14
6	863,14	888.81
7	888.81	915.31
3	915.31	942.65
9	942.65	970.87
10	970.87	1,000.00

Each year's *mean reserve equals* one half of the total of the initial and terminal reserves for that policy year:

$$1^{st}\text{ Mean Reserve} = \frac{\text{1st Initial Reserve + 1st Terminal Reserve}}{2}$$

$$= \frac{\$158.75 + \$161.98}{2}$$

$$= \frac{\$320.73}{2}$$

$$= \$160.37$$

$$2^{nd}\text{ Mean Reserve} = \frac{\text{2nd Initial Reserve + 2nd Terminal reserve}}{2}$$

$$= \frac{\$320.73 + \$329.11}{2}$$

$$= \frac{\$649.84}{2}$$

$$= \$324.92$$

The remaining mean reserves, calculated in the same manner, are as follows:

3rd Mean Reserve = $494.71
4th Mean Reserve = 669.90
5th Mean Reserve = 850.70
6th Mean Reserve = 875.98
7th Mean Reserve = 902.06
8th Mean Reserve = 928.98
9th Mean Reserve = 956.76
10th Mean Reserve= 985.44

It should be noted that mean reserves can be calculated if the net premium and appropriate terminal reserves are known, because the “initial reserve” in the equation can be replaced by “net premium + previous terminal reserve”:

$$\text{Mean Reserve} = \frac{\text{Initial Reserve} + \text{Terminal Reserve}}{2}$$

$$= \frac{\text{Net Premium} + \text{Previous Terminal Reserve} + \text{Terminal Reserve}}{2}$$

To Illustrate- Calculate the reserve used on December 31, 1975, for a policy issued in 1972. The following values are given:

Net level annual premium…….	$ 27.65
Terminal reserve	
1st year ………………………	22.42
2nd year………………………	49.99
3rd year……………………….	**78.67**
4th year ……………………….	108.30
5th year ……………………….	138.92

Solution

If this policy is assumed to have been issued in the *middle* of 1972, then on December 31, 1975, it has been in effect for 31 years. That is, it completed 3 policy years in the middle of 1975, and is half-way through its 4th policy year. The 4th mean reserve is therefore required.

$$\text{4th Mean Reserve} = \frac{\text{Net Premium} + \text{3rd Initial Reserve} + \text{3rd Terminal Reserve}}{2}$$

$$= \frac{\$27.65 + \$78.67 + \$108.30}{2}$$

$$= \frac{\$214.62}{2}$$

$$= \$107.31$$

NET AMOUNT AT RISK

As explained before, each policy has its own reserve, which is the amount the company has on hand at any particular time for that policy. Therefore, for each death claim paid at die end of the year, a certain portion of the claim payment is available from that policy's terminal reserve. The remainder of the claim payment, known as the *net amount at risk*, is that portion which must be paid from the funds of the other policies. A policy's net amount at risk for any year is, therefore, the amount of insurance less the terminal reserve for that year:

$$\begin{pmatrix}\text{Net Amount}\\ \text{at risk}\end{pmatrix} = \begin{pmatrix}\text{Amount of}\\ \text{Insurance}\end{pmatrix} - \begin{pmatrix}\text{Terminal}\\ \text{Reserve}\end{pmatrix}$$

To Illustrate- Calculate the net amount at risk each year for a $1,000 5-payment 10-year endowment policy issued at age 21 (using the 1958 C.S,O. Table and 3% interest).

Solution

This is the same policy for which terminal reserves were previously calculated and tabulated (Section 11.6). Each year's net amount at risk equals $1,000 less the terminal reserve for that year:

1st Year Net Amount at Risk = $1,000- 1st Terminal Reserve
= $1,000 - $16198
= $838.02

2nd Year Net Amount at Risk = $1,000- 2nd Terminal Reserve
= $1,000 -$329.11
= $670.89

Continuing such calculations for the remaining years gives the following figures:

3rd Year Net Amount at Risk = $498.45
4th Year Net Amount at Risk = 320.50
5th Year Net Amount at Risk = 136.86
6th Year Net Amount at Risk = 111.19
7th Year Net Amount at Risk = 84.69
8th Year Net Amount at Risk = 57.35
9th Year Net Amount at Risk = 29.13
10th Year Net Amount at Risk = 0

For the final year of any endowment policy, such as this one, the net amount at risk is always zero. This is because the final year terminal reserve is the same as the amount of insurance. Accordingly, subtracting one from the other yields an answer of zero. This leads to the statement that, in the final year ohm endowment insurance policy, it makes no difference financially to the company whether the person insured lives or dies, The full amount is paid at the end of the year in either instance.

TABULAR COST OF INSURANCE

The progress of the reserve for a policy, for any one-year period, can be described as follows: The initial reserve (at the beginning of the year) accumulates at interest for one year, At the end of the year, an amount necessary to help pay death claims is deducted. The remaining amount is the terminal reserve.

The amount so deducted is called the *tabular cost of insurance*. The word "tabular" means that this cost is calculated using the same mortality table and interest rate as used in the calculation of net premiums and reserves.

For all those policies where the persons insured die during this one-year period, the amount necessary to pay these total death claims is made up of the terminal reserves for those particular policies plus the tabular cost of insurance from *all* policies.

The progress of the reserve for one year, as described above, can be written in equation form:

$$\begin{pmatrix}\text{Initial}\\\text{Reserve}\end{pmatrix}(1+i)-\begin{pmatrix}\text{Tabular Cost}\\\text{of Insurance}\end{pmatrix}=\begin{pmatrix}\text{Terminal}\\\text{Reserve}\end{pmatrix}$$

When the above equation is solved for the tabular cost of insurance, the following equation results:

$$\begin{pmatrix}\text{Tabular Cost}\\\text{of Insurance}\end{pmatrix}=\begin{pmatrix}\text{Initial}\\\text{Reserve}\end{pmatrix}(1+i)-\begin{pmatrix}\text{Terminal}\\\text{Reserve}\end{pmatrix}$$

This equation can be used to calculate tabular costs of insurance, provided initial and terminal reserves are known.

To Illustrate- Calculate the tabular cost of insurance each year for a $1,000 5-payment 10-year endowment policy issued at age 21 (using the 1958 C.S.O. Table and 3% interest).

Solution

This is the same policy for which terminal and initial reserves were previously calculated and tabulated (Section 13.6).

$$\begin{pmatrix}\text{1st Year}\\\text{Tabular Cost}\\\text{of Insurance}\end{pmatrix}=\begin{pmatrix}\text{1st Year}\\\text{Initial}\\\text{Reserve}\end{pmatrix}(1+i)-\begin{pmatrix}\text{1st Year}\\\text{Terminal}\\\text{Reserve}\end{pmatrix}$$

= $158.75(l.03) - $161.98

= $163.51 - $161.98

= $1.53

$$\begin{pmatrix}\text{2nd Year}\\ \text{Tabular Cost}\\ \text{of Insurance}\end{pmatrix} = \begin{pmatrix}\text{2nd Year}\\ \text{Initial}\\ \text{Reserve}\end{pmatrix}(1+i) - \begin{pmatrix}\text{2nd Year}\\ \text{Terminal}\\ \text{Reserve}\end{pmatrix}$$

= \$320.73(1.03) - \$329.11

= \$330.35 -\$329.11

= \$1.24

If the calculations are continued in the same manner, the following results are obtained:

3rd Year Tabular Cost of Insurance	=	\$0.95
4th Year Tabular Cost of Insurance	=	.61
5th Year Tabular Cost of Insurance	=	.26
6th Year Tabular Cost of Insurance	=	.22
7th Year Tabular Cost of Insurance	=	.16
8th Year Tabular Cost of Insurance	=	.12
9th Year Tabular Cost of insurance	=	.06
10th Year Taint hr Cost of Insurance	=	0

For the final year of any endowment policy, such as this one, the tabular cost of insurance is always zero. This would be expected since as mentioned in the previous section, it then makes no difference financially to the company whether the insured lives or dies. That is, for endowment policies the final year initial reserve accumulates at interest to the final year terminal reserve, without subtracting anything for tabular cost of insurance.

It should be noted that tile above equation can be used when only the net premium and appropriate terminal reserves are given, because the “initial reserve” in the equation can be replaced by “net premium + previous terminal reserve.

$$\begin{pmatrix}\text{Tabular Cost}\\ \text{of Insurance}\end{pmatrix}=\begin{pmatrix}\text{Initial}\\ \text{Reserve}\end{pmatrix}(1+i)-\begin{pmatrix}\text{Terminal}\\ \text{Reserve}\end{pmatrix}$$

$$=\left[\begin{pmatrix}\text{Net}\\ \text{Premium}\end{pmatrix}+\begin{pmatrix}\text{Previous}\\ \text{Terminal Reservr}\end{pmatrix}\right](1+i)-\begin{pmatrix}\text{Terminal}\\ \text{Reserve}\end{pmatrix}$$

Tabular cost of insurance is sometimes referred to as the *tabular cost of insurance based on the net amount at risk.* This description refers to a second method of calculation for this cost, which is as follows: The tabular cost of insurance for a policy, for any one-year period, equals the net amount at risk multiplied by the rate of mortality for that year. In equation form, this ma)' be written

$$\begin{pmatrix}\text{Tabular Cost}\\ \text{of Insurance}\end{pmatrix}=\begin{pmatrix}\text{Net Amount}\\ \text{at Risk}\end{pmatrix}q_x$$

where x is the attained age at the *beginning of the year*.

To Illustrate- Using the second method, verify the tabular cost of insurance calculated in the above illustration for the 1st, 2nd, and 10th years.

Solution

For the 1st year, the attained age is 21, and q_{21} = .00183 from Table III. The net amount at risk was calculated in Section 11.7 to be $838.02:

$$\begin{pmatrix}\text{1st Year}\\ \text{Tabular Cost}\\ \text{of Insurance}\end{pmatrix}=\begin{pmatrix}\text{1st Year}\\ \text{Net Amount}\\ \text{at Risk}\end{pmatrix}q_x$$

= ($838.02)(.00183)

= $1.53

For the 2nd year, the attained age is 22, and q_{22} = .00186 from Table III. The net amount at risk was calculated in Section 11.7 to be $670.89:

$$\begin{pmatrix} \text{2nd Year} \\ \text{Tabular Cost} \\ \text{of Insurance} \end{pmatrix} = \begin{pmatrix} \text{2nd Year} \\ \text{Net Amount} \\ \text{at Risk} \end{pmatrix} q_x$$

= ($670.89)(.00186)

= $1.25

For the 10th year, the attained age is 30, and q_{30} = .00213 from Table III. The net amount at risk was calculated in Section 11.7 to be zero

$$\begin{pmatrix} \text{10th Year} \\ \text{Tabular Cost} \\ \text{of Insurance} \end{pmatrix} = \begin{pmatrix} \text{10th Year} \\ \text{Net Amount} \\ \text{at Risk} \end{pmatrix} q_x$$

= (zero)(.00213)

= 0

These agree with the answers obtained in the first illustration in this section, except for a difference of 1 cent for the 2nd year. This is due to rounding off the reserves to two decimal places.

As mentioned above, the tabular cost is always zero for the final year of any endowment policy, such as this one. Here it is the result of the fact that the final year net amount at risk is always zero. Multiplying zero by any rate of modality yields an answer of zero.

It should be noted that this second method can be used when only the appropriate terminal reserves are known. This is because the "net amount at risk" in the equation can be replaced by "amount of insurance-terminal reserve":

$$\begin{pmatrix}\text{Tabular Cost}\\ \text{of Insurance}\end{pmatrix}=\begin{pmatrix}\text{Net Amount}\\ \text{at Risk}\end{pmatrix}q_x$$

$$=\left[\begin{pmatrix}\text{Amount of}\\ \text{Insurance}\end{pmatrix}-\begin{pmatrix}\text{Terminal}\\ \text{Reserve}\end{pmatrix}\right]q_x$$

To Illustrate- Calculate the 6th year tabular cost of insurance on a $5,000 policy issued at age 10, if the 6th terminal reserve is $107.82 per $1,000. Use the 1958 C.S.O. Table.

Solution

Since the amount of insurance is $5,000, the total 6th terminal reserve is

(5)($107.82) = $539.10

Since the policy was issued at age 10, the attained age at the *beginning* of the 6th year is 15.

$$\begin{pmatrix}\text{Tabular Cost}\\ \text{of Insurance}\end{pmatrix}=\left[\begin{pmatrix}\text{Amount of}\\ \text{Insurance}\end{pmatrix}-\begin{pmatrix}\text{Terminal}\\ \text{Reserve}\end{pmatrix}\right]q_x$$

Substituting values calculated

$= (\$5,000 - \$539.10)q_{15}$

Substituting value from Table III

= ($5,000- $539.10)(.00146)

= ($4,460.90)(.00146)

= $6.51

PREMIUM DEFICIENCY RESERVES

The mortality table and interest rate which are used to calculate the reserves for any particular policy are generally identified in the printed policy itself In some circumstances it may happen that the *net* annual premium, calculated using this mortality table and interest rate, is larger than the *gross* annual premium for the policy, that is, there is a *negative loading*. This would generally happen only on nonparticipating policies where the modality table used to calculate reserves exhibits mortality rates considerably larger than the company expects it will actually experience. This situation is most likely to occur when the modality table is one which has been used for a considerable number of years. The existence of a number of policies being offered for sale with gross premiums less than the net premiums is usually regarded as evidence of the need to establish a new mortality table for calculating reserves.

The laws of all states require that, for such policies, a special reserve be added to the policy's regular reserve. This special reserve is known as a *premium deficiency reserve*. It must be calculated as the present value (with benefit of survivorship) of the future differences between the net premiums and gross premiums.

To Illustrate- Using the 1958 C.S.O. Table and 3% interest, calculate the premium deficiency reserve at the end of 17 years for a $10,000 20-year term insurance policy issued at age 30, given the following:

Gross annual premium = $34.20
Net annual premium = 35.30

Solution

The annual "premium deficiency" would be the difference between the two premiums:

$35.30 -$34.20 = $1.10

At the end of 17 years, the attained age is 30 + 17 = 47. At that time, there are 3 premiums remaining. The premium deficiency reserve equals the present value of those 3 years' deficiencies. Inline diagram form, they appear as follows:

```
 $1.10        $1.10       $1.10
   *------------+-----------+--------|
age 47         48          49       50
                1           2        3years
```

Basic equation

$$\begin{pmatrix}\text{Premium}\\\text{Deficiency}\\\text{Reserve}\end{pmatrix} = \$1.10\left[\frac{l_{47} + l_{48}\upsilon + l_{49}\upsilon^2}{l_{47}}\right]$$

Substituting values from the tables

$$= \$1.10\left(\frac{\begin{matrix}8{,}948{,}114\\+8{,}891{,}204(.970874)\\+8{,}829{,}410(.942596)\end{matrix}}{8{,}948{,}114}\right)$$

$$= \$1.10\left[\frac{8{,}948{,}114 + 8{,}632{,}239 + 8{,}322{,}567}{8{,}948{,}114}\right]$$

$$= \$3.18$$

For December 31 annual statement purposes, deficiency reserves are generally calculated for a point-in-time half-way between the policy's anniversaries, similar to the procedure for the regular mean reserve.

RESERVES FOR DECREASING TERM POLICIES

A popular type of life insurance policy is one wherein the death benefit decreases each year, although the annual premium stays the Same. Normally, these are term-type policies, that is, the death benefit decreases to zero in a relatively few years. This type of policy has certain recognized advantages:

1- The death benefit is highest when the mortality rates are lowest (in the early years), and lowest when the mortality rates are highest (in the late years) resulting in a very low net level annual premium;

2- There are many situations where the insured person s need for insurance is a decreasing one, such as insurance to repay the balance of a mortgage loan; and

3- The reserves which the company needs to have are very small, and the "non forfeiture values" (to be presented in Chapter 13) are usually nonexistent.

In the calculation of the net annual premiums and reserves for decreasing term policies, the present value of future benefits will usually take the form

$$A\left(\frac{d_x \upsilon}{l_x}\right)+B\left(\frac{d_{x+1}\upsilon^2}{l_x}\right)+C\left(\frac{d_{x+2}\upsilon^3}{l_x}\right)+D\left(\frac{d_{x+3}\upsilon^4}{l_x}\right)+...etc$$

where *A, B, C, D,* etc., are the respective death benefits in the first year, second year, third year, fourth year, etc. The present value of future net premiums will take the *usual form*

$$\begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}\left(\frac{l_x + l_{x+1}\upsilon + l_{x+2}\upsilon^2 + ...etc}{l_x}\right)$$

because the premiums do not change from year to year.

The unusual feature of the terminal reserves is that, in many instances, they are *negative*. That is, in the prospective calculation, the present value of future net premiums is sometimes greater than the present value of future benefits. In the retrospective calculation, the accumulated cost of insurance is sometimes greater than the accumulated value of net premiums received. The terminal reserve at the end of the term of the policy, however, is always zero.

Special attention must be directed to the calculation of *mean reserves* for such policies. In Section 11.6 the following equation was given:

$$\text{Mean Reserve} = \frac{\text{Net Premium} + \text{Previous Terminal Reserve} + \text{Terminal Reserve}}{2}$$

In the case of decreasing term insurance, either of the two terminal reserves (or both) may be negative. The usual rule followed is that if the total of Previous Terminal Reserve + Terminal Reserve is negative, this total is assumed to be zero. In the event that zero is so used, the equation can be written as

$$\text{Mean Reserve} = \frac{\text{Net Premium}}{2}$$

To Illustrate- Calculate the current mean reserve for a decreasing term policy, given the following information:

Net level annual premium = \$89.10

Previous terminal reserve = \$2.91

Terminal reserve = -\$5.12

Solution

The total of the terminal reserves is

Previous Terminal Reserve+Terminal Reserve =\$2.91+(-\$5.12)

= (-\$2.21)

Since this total is *negative*, the rule would require it to be used as zero. Hence, the equation becomes

$$\text{Mean Reserve} = \frac{\text{Net Premium}}{2}$$

$$= \frac{\$89.10}{2}$$

$$= \$44.55$$

It is a widely used practice to calculate the mean reserve on decreasing term policies as being equal to one-half the *gross* annual premium, instead of one-half the *net* annual premium. *This is* done for several reasons:

1- It is usually a simpler calculation, because gross premiums are always known exactly, whereas net premiums on these policies are often estimated or only calculated for a few sample ages;
2- This injects an element of conservatism into the company's financial statement by slightly overstating this liability; and
3- This helps offset the slight understatement in these reserves which the use of one-half the net premium would produce (because in a few instances the true mean reserve would exceed this).

RESERVES USING CONTINOUS FUNCTIONS

In Section 10.6, "life annuities payable continuously" was presented. In Section 11.7, "insurance payable at the moment of death" was presented. The term continuous *functions* is used to describe the combination of the two concepts, that is, the assumptions of the death benefit payable immediately and the premiums payable continuously.

These assumptions came into wider use by insurance companies in calculating premiums and reserves at the time the 1958 C.S.O. Table was adopted. Use of the 1958 C.S.O. Table frequently produced lower reserves than the previously used table, a result which was unwelcome for

some companies for a number of reasons, The use of continuous functions helped to increase the reserves to a more desired level.

It should be remembered that the equations given in Sections 10.6 and 10.7 are all *approximate* equalities. In actual practice, companies use more complicated equations which give the relationships exactly.

However, the approximations yield answers very close to the true figures.

To calculate the approximate present value of future benefits using continuous functions, the death benefit is multiplied by $(1+\frac{1}{2}i)$, as explained in Section 9.7. In practice, this means that regular whole life net single premiums which are used to calculate net annual premiums and reserves (by the prospective method) may be multiplied by $(1+\frac{1}{2}i)$.

To calculate the approximate present value of a whole life annuity due using continuous functions, one half of a year's payments must be subtracted from the regular present value, as explained in Section 9.6. In practice, this means that whole life annuity due factors which are used to calculate ordinary life net annual premiums and reserves (by the prospective method) may be diminished by d. for example, if the present value of a whole life annuity due of 1 per year were 22.7843 this factor would be diminished, as follows:

$$22.7843 - \frac{1}{2} = 22.7843 - .5000$$

$$= 22.2843$$

In the calculation of mean reserves by the usual equation:

$$\text{Mean Reserve} = \frac{\text{Net Premium + Previous Terminal Reserve + Terminal Reserve}}{2}$$

The year's net premium (if it is payable continuously) must be discounted to the beginning of the year. The methods used for doing this are complex and beyond the scope of this book.

EXERCISES

For Exercises 1 through 4, use the following information Concerning a $10,000 policy.

Net annual premium ……. $1,107.65

Terminal reserve

1st year ……….	439.40	2nd year ……….	87060
3rd year ……….	1,291.30	4th year ……….	1,698.30
5th year ……….	2,089.70		

1- Calculate the 1st year initial reserve.

2- Calculate the 6th year initial reserve.

3- Calculate the 1st year mean reserve.

4- Calculate the net amount at risk for the 3rd year.

5- Calculate this year's tabular cost of insurance for a certain policy, given the following information:

Reserve basis =1969 United Mortality Table at $3\frac{1}{2}$%

This year's terminal reserve = $8,401.40
This year's initial reserve = $8,502.00

6- Using Table III, calculate the 1st year tabular cost of insurance for a policy issued at age 21. The 1st year net amount at risk is $5,100.

7- Using Table III, calculate the 5th year tabular cost of insurance for a $10,000 policy issued at age 32. The 5th year terminal reserve is $112.40 per $1,000.

8- Write an expression (using symbols) for the deficiency reserve at age 63 for a term-to-age-65 policy, given the following information:

Gross annual premium = $207.80
Net annual premium = 20995
Using Table III and 3% interest calculate the value,

9- Calculate the mean reserves which a company should have for each policy year on a 5-year decreasing term policy, given the following information:

Net annual premium = $10.36
Terminal reserves = $28,-$1.79,-$1.02,$1.12,0, respectively

10- Using the following table based oh the 1958 C.S.O. Table and 3% interest, calculate the net annual premium (per $1,000) and the 5th terminal reserve (per $1,000) for an ordinary life policy issued at age 23. *Use continuous functions.*

Age	Net Single Premium for $1,000 Whole Life Insurance	Present Value of a Whole Life Annuity Due of 1 per Year
23	$265.75	25.209
24	272.34	24.983
25	279.14	24.750
26	286.13	24,509
27	293.33	24.262
28	300.74	24.008

Chart 13-3 displays certain internationally used symbols, each of which represents the terminal reserve at the end of t years for $1 of life insurance issued at age x.

In general, the symbol is the capital letter *"V."* The subscripts at the lower right may be said to describe the benefits, and are identical to those shown in Section 9.9 which appear with "*A*" for the various types of net single premiums. The letters denoting the premium-paying period (if shorter than the benefit period) and the Particular policy year to which the reserve applies both appears at the left side of the "*V*." For example, the symbol ${}^{m}_{t}V_{x:n}$ represents the t^{th}-year terminal reserve for a $1 *m*-payment *n*-year endowment policy issued at age x.

CHART-3

t^{th}-Year Terminal Reserves for $1 of Life Insurance Issued at Age x

Type of Life Insurance	*Symbol for Terminal Reserve*
Ordinary Life	${}_{t}V_{x}$
m-Payment Life	${}^{m}_{t}V_{x}$
n-Year Term Insurance	${}_{t}V^{1}_{x:\overline{n}\rceil}$
m-Payment n-Year Term Insurance	${}^{m}_{t}V^{1}_{x:\overline{n}\rceil}$
n-Year Endowment Insurance	${}_{t}V_{x:\overline{n}\rceil}$
m-Payment n-Year Endowment Insurance	${}^{m}_{t}V_{x:\overline{n}\rceil}$

Table (1) Expectation of life

Age x	Male $\mathring{e}_x$	Female $\overset{\bullet}{e}_x$	Age x	Male $\mathring{e}_x$	Female $\mathring{e}_x$	Age x	Male $\mathring{e}_x$	Female $\mathring{e}_x$
0	70.83	75.83	18	54.18	58.93	36	37.69	42.05
1	70.13	75.04	19	53.27	57.98	37	36.78	41.12
2	69.20	74.11	20	52.37	57.04	38	35.87	40.20
3	68.27	73.17	21	51.47	65.10	39	34.96	39.28
4	67.34	72.23	22	50.57	55.16	40	34.05	38.36
5	66.44	71.28	23	49.66	54.22	41	33.16	37.46
6	65.46	70.34	24	48.75	53.28	42	32.26	36.46
7	64.52	69.39	25	47.84	52.34	43	31.38	35.66
8	63.57	68.44	26	46.93	51.40	44	30.50	34.77
9	62.62	67.48	27	64.01	50.46	45	29.62	33.88
10	61.66	66.53	28	45.09	49.52	46	28.70	33.00
11	60.71	65.58	29	44.16	48.59	47	27.90	32.12
12	59.75	64.62	30	43.24	47.65	48	27.04	31.12
13	58.80	63.67	31	42.31	46.71	49	26.20	30.39
14	57.86	62.71	32	41.38	45.78	50	25.36	29.53
15	56.93	61.76	33	40.46	44.84	51	24.52	28.67
16	56.00	60.82	34	39.54	43.91	52	23.70	27.82
17	55.09	59.87	35	38.61	42.98	53	22.89	26.98

Age x	Male $\mathring{e}_x$	Female $\mathring{e}_x$	Age x	Male $\mathring{e}_x$	Female $\mathring{e}_x$	Age x	Male $\mathring{e}_x$	Female $\mathring{e}_x$
54	22.08	26.14	72	9.84	12.28	90	3.18	3.45
55	21.29	25.31	73	9.30	11.60	91	2.94	3.15
56	20.51	24.49	74	8.79	10.95	92	2.70	2.85
57	19.74	23.67	75	8.31	10.32	93	2.44	2.55
58	18.99	22.86	76	7.84	9.71	94	2.17	2.24
59	18.24	22.05	77	7.40	9.12	95	1.87	1.91
60	17.51	21.25	78	6.97	8.55	96	1.54	1.56
61	16.79	20.44	79	6.57	8.01	97	1.20	1.21
62	16.08	19.65	80	6.18	7.48	98	.84	.84
63	15.38	18.80	81	5.80	6.98	99	.50	.50
64	14.70	18.08	82	5.44	6.46			
65	14.04	17.32	83	5.09	6.03			
66	13.39	16.57	84	4.77	5.59			
67	12.67	15.83	85	4.46	5.18			
68	12.14	15.10	86	4.18	4.80			
69	11.54	14.38	87	3.91	4.43			
70	10.96	13.67	88	3.66	4.09			
71	10.39	12.97	89	3.41	3.77			

Source: Reproduced from Society of Actuaries tables.

(Males)

Age x	Number Living l_x	Number Dying d_x	Prob. Of Death q_x	Age x	Number Living l_x	Number Dying d_x	Prob. of Death q_x
0	185890	777	0.00418	24	179955	328	0.00182
1	185113	200	0.00108	25	179627	318	0.00177
2	184913	181	0.00098	26	179309	310	0.00173
3	184732	181	0.00098	27	178999	306	0.00171
4	184551	175	0.00095	28	178693	304	0.00170
5	184376	166	0.00095	29	178389	305	0.00171
6	184210	158	0.00086	30	178084	308	0.00178
7	184052	147	0.00080	31	177776	316	0.00183
8	183905	140	0.00076	32	177460	325	0.00183
9	183765	136	0.00074	33	177135	338	0.00191
10	183629	134	0.00073	34	176797	354	0.00200
11	183459	141	0.00077	35	176443	372	0.00211
12	183354	156	0.00085	36	176071	394	0.00224
13	183198	181	0.00099	37	175677	422	0.00240
14	183017	210	0.00115	38	175255	452	0.00285
15	182807	243	0.00133	39	174803	488	0.00279
16	182564	276	0.00151	40	174315	526	0.00302
17	182288	304	0.00167	41	173789	572	0.00329
18	181984	324	0.00178	42	173217	617	0.00356
19	181660	338	0.00186	43	172600	668	0.00387
20	181322	345	0.00190	44	171932	720	0.00419
21	181322	346	0.00191	45	171212	779	0.00455
22	180631	341	0.00189	46	170433	839	0.00492
23	180290	335	0.00186	47	169594	902	0.00532

Table (Males)

Age x	Number Living l_x	Number Dying d_x	Prob. Of Death q_x	Age x	Number Living l_x	Number Dying d_x	Prob. of Death q_x
48	168692	968	0.00574	74	96691	5626	0.05819
49	167724	1042	0.00621	75	91065	5845	0.06418
50	166682	1118	0.00671	76	85220	6011	0.07054
51	165564	1209	0.00730	77	79209	6109	0.07713
52	164355	1308	0.00796	78	73100	6133	0.08390
53	163047	1420	0.00871	79	66967	6097	0.09104
54	161627	1545	0.00956	80	60870	6016	0.09883
55	160082	1676	0.01047	81	54854	5898	0.10749
56	158406	1515	0.01146	82	48958	5740	0.11724
57	156591	1956	0.01249	83	43218	5543	0.12826
58	154635	2101	0.01359	84	37675	5284	0.14025
59	152534	2253	0.01477	85	32391	4954	0.15294
60	150281	2417	0.01608	86	27437	4557	0.16609
61	147864	2594	0.01754	87	22880	4108	0.17955
62	145270	2788	0.01919	88	18772	3628	0.19327
63	142482	3001	0.02106	89	15144	3139	0.20728
64	139481	3228	0.02314	90	12005	2662	0.22174
65	136253	3464	0.02542	91	9343	2214	0.23697
66	132789	3698	0.02785	92	7129	1807	0.25347
67	129091	3930	0.03044	93	5322	1448	0.27208
68	125161	4154	0.03319	94	3874	1146	0.29582
69	121007	4377	0.03617	95	2728	900	0.32991
70	116630	4608	0.03951	96	1828	703	0.38457
71	112022	4851	0.04330	97	1125	540	0.48000
72	107171	5107	0.04765	98	585	385	0.65812
73	102064	5373	0.05264	99	200	200	1.00000

(Females)

Age x	Number Living l_x	Number Dying d_x	Prob. Of Death q_x	Age x	Number Living l_x	Number Dying d_x	Prob. of Death q_x
0	65135	188	0.00289	24	63692	73	0.00115
1	64947	57	0.00088	25	63619	74	0.00116
2	64890	53	0.00082	26	63545	76	0.00120
3	64837	51	0.00079	27	63469	77	0.00121
4	64786	50	0.00077	28	63392	80	0.00126
5	64736	49	0.00076	29	63312	82	0.00130
6	64687	47	0.00073	30	63230	85	0.00134
7	64640	47	0.00073	31	63145	88	0.00139
8	64593	45	0.00070	32	63057	91	0.00144
9	64548	45	0.00070	33	62966	94	0.00149
10	64503	44	0.00068	34	62872	99	0.00157
11	64459	44	0.00068	35	62773	104	0.00166
12	64415	64	0.00071	36	62669	110	0.00176
13	64369	48	0.00075	37	62559	118	0.00203
14	64321	51	0.00079	38	62441	127	0.00203
15	64270	55	0.00086	39	62314	138	0.00221
16	84215	58	0.00090	40	62176	150	0.00241
17	64157	61	0.00095	41	62026	164	0.00264
18	64095	63	0.00098	42	61862	178	0.00288
19	64033	65	0.00102	43	61684	191	0.00310
20	63968	67	0.00105	44	61493	204	0.00332
21	63901	68	0.00106	45	61289	218	0.00356
22	63833	70	0.00110	46	61071	232	0.00380
23	63763	71	0.00111	47	60839	246	0.00404

(Females)

Age x	Number Living l_x	Number Dying d_x	Prob. Of Death q_x	Age x	Number Living l_x	Number Dying d_x	Prob. of Death q_x
48	60593	262	0.00432	74	43698	1483	0.03394
49	60331	279	0.00642	75	42215	1614	0.03823
50	60052	298	0.00496	76	40601	1745	0.04298
51	59754	317	0.00531	77	38856	1867	0.04805
52	59437	339	0.00570	78	36989	1977	0.05345
53	59098	363	0.00614	79	35012	2078	0.05935
54	58735	388	0.00661	80	32934	2173	0.06598
55	58347	414	0.00710	81	30761	2264	0.07360
56	57933	439	0.00758	82	28497	2348	0.08239
57	57494	462	0.00804	83	26149	2420	0.09255
58	57032	483	0.00847	84	23729	2463	0.10380
59	56549	506	0.00895	85	21266	2469	0.11610
60	56043	531	0.00947	86	18797	2430	0.12928
61	55512	562	0.01012	87	16367	2346	0.14334
62	54950	602	0.01096	88	14021	2218	0.15819
63	54348	653	0.01202	89	11803	2053	0.17390
64	53695	711	0.01324	90	9750	1860	0.19077
65	52948	773	0.01459	91	7890	1648	0.20887
66	52211	835	0.01599	92	6242	1428	0.22877
67	51376	895	0.01742	93	4814	1211	0.25156
68	50481	951	0.01884	94	3603	1006	0.27921
69	49530	1008	0.02035	95	2597	824	0.31729
70	48522	1073	0.02211	96	1773	666	0.37563
71	47449	1150	0.002424	97	1107	526	0.47516
72	46299	1244	0.02687	98	581	381	0.65577
73	45055	1357	0.03012	99	200	200	1.0000

Age x	D_x	N_x	C_x	M_x	ä	A_x
0	185890.0	4026216.0	743.541	12512.420	21.659132	0.06731088
1	177141.6	3840326.0	183.146	11768.878	21.679409	0.06643767
2	169330.4	366184.3	158.610	11585.732	21.633356	0.06842087
3	161880.3	3493853.5	151.780	11427.122	21.582978	0.07059006
4	154757.3	3331973.8	140.429	11275.342	21.530311	0.07285820
5	147952.7	3177216.3	127.471	11134.912	21.474536	0.07525993
6	141454.1	3029203.5	116.103	11007.441	21.415170	0.07781635
7	135246.7	2887809.3	103.368	10891.339	21.352165	0.08052944
8	129319.3	2752562.5	94.207	10787.971	21.285011	0.08342121
9	123656.3	2623243.3	87.574	10693.765	21.213986	0.08647973
10	118243.8	2499587.0	82.571	10606.190	21.139260	0.08969762
11	113069.4	2381343.0	83.143	10523.620	21.060892	0.09307219
12	108117.3	2268273.8	88.026	10440.476	20.979757	0.9656625
13	103373.5	2160156.0	97.735	10352.451	20.896618	0.10014610
14	98824.3	2056782.4	108.511	10254.716	20.812527	0.10376720
15	94460.1	1957958.3	120.156	10146.205	20.727876	0.10741255
16	90272.3	186398.0	130.597	10026.049	20.643068	0.11106447
17	86254.4	1773225.8	137.651	9895.452	20.558087	0.11472401
18	82402.5	1686791.4	140.390	9757.801	20.472342	0.11841638
19	78713.6	1604568.9	140.149	9617.411	20.384890	0.12218227
20	75183.9	1525855.4	136.892	9477.262	20.294973	0.12605440
21	71809.4	1450671.6	131.377	9340.370	20.201685	0.13007162
22	68585.8	1378862.1	123.902	9208.994	20.104196	0.13426971
23	65508.4	1310276.3	116.481	9085.092	20.001641	0.13868583
24	62571.0	1244767.8	109.136	8958.610	19.893679	0.14333489
25	59767.4	1182196.8	101.252	8859.475	19.779946	0.14823246
26	57092.5	1122429.3	95.454	8758.222	19.659849	0.15340417
27	54539.5	1065337.0	89.221	855.768	19.533315	0.15885312
28	52101.7	1010797.3	84.821	8574.547	19.400475	0.16457333
29	49773.3	958695.6	81.435	8489.726	19.261262	0.17056805
30	47548.5	908922.4	78.695	8408.291	19.115702	0.17683621
31	45422.2	861373.9	77.282	8329.596	18.963707	0.18338147
32	43389.0	815951.6	76.041	8252.334	18.805498	0.19019418

(Cont.) Commutation columns-Interest at 4.5% -Males						
Age x	**D_x**	**N_x**	**C_x**	**M_x**	ä	**A_x**
33	41444.5	772562.7	75.677	8167.293	18.640885	0.19728281
34	39584.2	731118.1	75.846	8100.616	18.469969	0.20464289
35	37803.7	691534.0	76.271	8024.770	18.292745	0.21227455
36	36099.5	653730.2	77.303	7984.500	18.109097	0.22018281
37	34467.7	617630.7	79.231	7871.197	17.919103	0.22836429
38	32904.2	583163.0	81.209	7791.966	17.723038	0.23680741
39	31406.1	550258.8	83.901	7710.757	17.520759	0.24551782
40	29969.8	518852.7	86.540	7626.856	17.312527	0.25448486
41	28592.7	588882.9	90.056	7540.315	17.098185	0.26371491
42	27271.4	460290.2	92.958	7450.260	16.878156	0.27318992
43	26004.0	433018.9	96.308	7357.302	16.651984	0.28292918
44	24787.9	407015.8	99.334	7260.995	16.419874	0.29292449
45	23621.2	382226.9	102.846	7161.660	16.181529	0.30318802
46	22501.2	358605.7	105.998	7058.813	15.937210	0.31370890
47	21426.2	336104.5	109.050	6952.816	15.686606	0.32450049
48	20394.5	314678.3	111.990	6843.767	15.429567	0.33556922
49	19404.3	294283.8	115.360	6731.777	15.165925	0.34692232
50	18453.3	274879.5	118.444	5516.417	14.895933	0.35854874
51	17540.2	256426.2	122.569	6497.974	14.619306	0.37046084
52	16662.4	238886.0	126.895	6375.405	14.336866	0.38262667
53	15817.9	222223.6	131.828	6248.509	14.048831	0.39502667
54	15005.0	206405.7	137.257	6116.681	13.755829	0.40764394
55	14221.6	191400.7	142.483	5979.425	13.458490	0.42044797
56	13466.7	177179.1	147.655	5836.942	13.156869	0.43343640
57	12739.1	163712.5	152.274	5689.287	12.851176	0.44660023
58	12038.3	150973.4	156.518	5537.013	12.541132	0.45995143
59	11363.3	138935.1	160.514	5380.495	12.226603	0.47349567
60	10713.4	127571.8	164.886	5219.880	11.907684	0.48722915
61	10087.2	116858.4	169.340	5054.995	11.584851	0.50113100
62	9483.5	106771.2	174.168	4885.654	11.258681	0.51517658
63	8900.9	97287.8	179.401	4711.486	10.930092	0.52932643
64	8338.2	88386.9	184.661	4532.086	10.600212	0.54353187
65	7794.5	80048.6	189.628	4347.424	10.269896	0.55775589

(Cont.) Commutation columns-Interest at 4.5% -Males						
Age x	**D_x**	**N_x**	**C_x**	**M_x**	ä	**A_x**
67	6762.5	64984.9	197.009	3964.075	9.609646	0.58618763
68	6274.3	58222.5	199.271	3767.067	9.279584	0.60040089
69	5804.8	51948.2	200.926	3567.796	8.949182	0.61462870
70	5353.9	46143.4	202.421	3366.870	8.618643	0.62886235
71	4920.9	40789.5	203.920	3164.448	8.288977	0.64305858
72	4505.1	35868.6	205.436	2950.529	7.961756	0.65714937
73	4105.7	31363.5	206.829	2755.092	7.639057	0.67104542
74	3722.0	27257.8	207.242	2548.263	7.323340	0.68464099
75	3354.5	23535.7	206.038	2341.021	7.016127	0.69787028
76	3004.0	20181.2	202.765	2134.983	6.718050	0.71070628
77	2671.9	17177.2	197.197	1932.218	6.428819	0.72316106
78	2359.6	14505.3	189.447	1735.021	6.147220	0.73528731
79	2068.6	12145.6	180.224	1545.574	5.871453	0.74716248
80	1799.3	10077.0	170.172	1365.349	5.600572	0.75882731
81	1551.6	8277.8	159.596	1195.177	5.334860	0.77026916
82	1325.2	6726.1	148.683	1035.581	5.075467	0.78142923
83	1119.5	5400.9	137.397	886.898	4.824505	0.97224638
84	933.9	4281.4	125.337	749.501	4.584615	0.80257663
85	768.3	3347.6	112.449	624.164	4.357000	0.81237811
86	622.8	2579.2	98.983	511.715	4.141478	0.82165896
87	497.0	1956.5	85.388	512.732	3.936687	0.83047760
88	390.2	1459.5	72.164	327.343	3.740412	0.83892963
89	301.2	1069.3	59.784	255.180	3.549785	0.84713838
90	228.5	768.1	48.487	195.431	3.361213	0.85525802
91	170.2	539.6	38.590	146.944	3.170520	0.86347039
92	124.3	369.4	30.140	108.354	2.972609	0.87199295
93	88.8	245.1	23.112	78.214	2.761284	0.88109319
94	61.8	156.3	17.504	55.102	2.528488	0.89111786
95	41.7	94.5	13.155	37.598	2.268264	0.90232348
96	26.7	52.8	9.833	24.443	1.977854	0.91482941
97	15.7	26.1	7.228	14.610	1.660404	0.92849942
98	7.8	10.4	4.931	7.382	1.327158	0.94284966
99	2.6	2.6	2.451	2.451	1.000000	0.95693780

(Cont.) Commutation columns-Interest at 4.5% -Females						
Age x	**D_x**	**N_x**	**C_x**	**M_x**	ä	**A_x**
0	65135.0	1430336.9	179.904	3541.571	21.959574	0.05437277
1	62150.2	1365210.9	52.197	3361.666	21.966155	0.05408934
2	59421.7	1303051.6	46.444	3309.469	2.928878	0.05569461
3	56816.4	1243630.0	42.767	3263.026	21.888555	0.05743101
4	54327.0	1186813.4	40.123	3220.259	21.845722	0.05927544
5	51947.5	1132486.5	37.627	3180.137	21.800605	0.06121831
6	49672.9	1080539.1	34.537	3142.510	21.753100	0.06326410
7	47499.3	1030866.4	33.050	3107.973	21.702761	0.06543195
8	45420.8	983367.1	30.281	3074.923	21.650128	0.06769850
9	43434.6	937946.3	28.977	3044.642	21.594425	0.07009709
10	41535.3	894511.6	27.113	3015.66	21.536186	0.07260491
11	39719.6	852976.3	25.945	2988.553	21.474962	0.07524132
12	37983.2	813256.8	25.957	2962.607	21.410953	0.07799782
13	36321.6	775273.5	25.919	2936.651	21.344686	0.08085133
14	34731.6	738951.8	26.353	2910.732	21.276060	0.08380643
15	33209.6	704220.3	27.196	2884.380	21.205298	0.08685370
16	31752.4	6719019.6	27.444	2857.184	21.132621	0.08998335
17	30357.6	639358.3	27.621	2829.740	21.057609	0.09321357
18	29022.7	608900.6	27.298	2802.119	20.990147	0.09654919
19	27745.6	579877.9	26.952	2774.821	20.899792	0.10000928
20	26523.9	552132.3	26.585	2747.869	20.816416	0.10359979
21	25355.1	525608.5	25.820	2721.284	20.729871	0.10732678
22	24237.5	500253.3	25.435	2695.464	20.639675	0.11121067
23	23168.3	47015.8	24.687	2670.030	20.545990	0.11524493
24	22145.9	452847.6	24.289	2645.343	20.448330	0.11945045
25	21168.0	430701.6	23.562	2621.054	20.346823	0.12382150
26	20232.9	409533.6	23.157	2597.492	20.240975	0.12837963
27	19338.5	389300.7	22.451	2574.335	20.130893	0.13311991
28	18483.3	269962.2	22.321	2551.885	20.016068	0.13806462
29	17665.0	351479.0	21.894	2529.563	19.896898	0.14319622
30	16882.4	333813.9	21.718	2507.669	19.772868	0.14319622
31	16133.7	316831.5	21.516	2485.952	19.644055	0.15408430
32	15417.4	300797.8	21.291	2464.436	19.510225	0.15984722

(Cont.) Commutation columns-Interest at 4.5% -Males						
Age x	D_x	N_x	C_x	M_x	ä	A_x
33	14732.2	285380.3	21.046	2443.144	19.371137	0.16583653
34	14076.8	270648.1	21.211	2422.098	19.226542	0.17206319
35	13449.4	256571.4	21.323	2400.887	19.076781	0.17851247
36	12848.9	24122.0	21.582	2379.564	18.921586	0.18519562
37	12274.0	230273.0	22.155	2357.982	18.760985	0.19211135
38	11723.3	217999.0	22.818	2335.827	18.595306	0.19924598
39	11195.7	206275.7	23.726	2313.010	18.424565	0.20659828
40	10689.9	195080.0	24.679	2289.283	18.249084	0.21415486
41	10204.8	184390.1	25.820	2265.605	18.068886	0.22191471
42	9739.6	174185.3	26.818	2238.784	17.884275	0.22986460
43	9293.4	164445.7	27.537	2211.967	17.694978	0.23801592
44	8865.6	155152.4	28.145	2184.430	17.500444	0.24639320
45	8455.7	146286.7	28.781	2156.285	17.300353	0.25500940
46	8062.8	137831.0	29.310	2127.504	17.094669	0.26386645
47	7686.3	129768.1	29.741	2098.193	16.883058	0.27297855
48	7325.6	122081.8	30.311	2068.453	16.665182	0.28236091
49	6979.8	114756.3	30.888	2038.141	16.441206	0.29200583
50	6648.3	107776.5	31.571	2007.253	16.211028	0.30191779
51	6330.5	101128.1	32.138	1975.683	15.974796	0.31209045
52	6025.7	94797.7	32.888	1943.545	15.732121	0.32254053
53	5733.4	88771.9	33.700	1910.657	15.400375	0.33325201
54	5452.8	83038.6	34.470	1876.957	15.228667	0.34422032
55	5183.5	77585.8	35.196	1842.488	14.967832	0.35545233
56	4925.1	72402.3	35.714	1807.292	14.700695	0.36695597
57	4677.3	67477.2	35.966	1771.578	14.426547	0.37876141
58	4439.9	62799.9	35.982	1735.612	14.144403	0.39091131
59	4212.7	58360.0	36.072	1699.630	13.853223	0.40345013
60	3995.3	54147.3	36.224	1663.557	13.552888	0.41638319
61	3787.0	50152.0	36.688	1627.333	13.243246	0.42971707
62	3587.2	46365.0	37.607	1590.645	12.925044	0.44341948
63	3395.1	42777.8	39.037	1553.037	12.599705	0.45742931
64	3209.9	39382.6	40.673	1514.001	12.269107	0.47166564
65	3031.0	36172.7	42.316	1473.327	11.934243	0.48608553

(Cont.) Commutation columns-Interest at 4.5% -Males						
Age x	**D_x**	**N_x**	**C_x**	**M_x**	ä	**A_x**
67	2691.3	30283.6	44.866	1387.270	11.252202	0.51545566
68	2530.6	27592.2	45.620	1342.404	10.903498	0.53047177
69	2376.0	25061.6	46.272	1396.784	10.547861	054578609
70	2227.4	22685.6	47.135	1250.511	10.184790	0.56142086
71	2084.4	20458.2	48.342	1203.376	9.815154	0.57733822
72	1946.3	18373.9	50.042	1155.034	9.440645	0.59346532
73	1812.4	16427.6	52.237	1104.993	9.064013	0.60968388
74	1682.1	14615.2	54.629	1052.756	8.688582	0.62585075
75	1555.1	12933.1	56.894	998.127	8.316820	0.64185961
76	1431.2	11378.1	58.863	491.233	7.950028	0.65765445
77	1310.7	9946.9	60.266	882.370	7.588946	0.67320353
78	1194.0	8636.2	61.069	822.104	7.232987	0.68853183
79	1081.5	7442.2	61.425	761.035	6.881263	0.70367782
80	973.5	6360.6	61.467	699.610	6.533703	0.71864470
81	870.1	5387.1	61.283	638.144	6.191218	0.73339272
82	771.4	4517.0	60.820	576.860	5.855809	0.74783619
83	677.3	3745.6	59.986	516.040	5.529959	0.66186795
84	588.2	3068.3	58.423	456.055	5.216584	0.77536257
85	504.4	2480.1	56.043	397.632	4.916666	0.78827768
86	426.7	1975.7	52.783	441.589	4.630524	0.80059962
87	355.5	1549.0	48.764	288.806	4.357175	0.81237051
88	291.4	1193.5	44.118	240.043	4.095249	0.82364959
89	234.8	902.1	39.077	195.925	3.842362	0.83453933
90	185.6	667.3	33.879	156.484	3.595701	0.84516115
91	143.7	481.7	28.725	122.969	3.351958	0.85565730
92	108.8	338.0	23.818	94.244	3.106699	0.86621877
93	80.3	229.2	19.329	70.425	2.854542	0.87707735
94	57.5	148.9	15.366	51.096	2.589374	0.88849590
95	39.7	91.4	12.044	35.731	2.304267	0.90077273
96	25.9	51.7	9.315	23.687	1.995407	0.91403051
97	15.5	25.8	7.040	14.372	1.687685	0.92818579
98	7.8	10.3	4.880	7.331	1.329411	0.94275266
99	2.6	2.6	2.451	2.451	1.000000	0.95693780